Inspecting the Mechanism Leverage and the Great Recession in the Eurozone

Philippe Martin
Thomas Philippon
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Philippe Martin and Thomas Philippon†

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Abstract
We provide a first comprehensive account of the dynamics of Eurozone countries from the creation of the Euro to the Great recession. We model each country as an open economy within a monetary union and analyze the dynamics of private leverage, fiscal policy and spreads. Our parsimonious model can replicate the time-series for nominal GDP, employment, and net exports of Eurozone countries between 2000 and 2012. We then ask how periphery countries would have fared with: (i) more conservative fiscal policies; (ii) macro-prudential tools to control private leverage; (iii) a central bank acting earlier to limit sovereign spreads; and (iv) the possibility to recoup the competitiveness they lost in the boom. To perform these counterfactual experiments, we use U.S. states as a control group that did not suffer from a sudden stop. We find that periphery countries could have stabilized their employment if they had followed more conservative fiscal policies during the boom. This is especially true in Greece. For Ireland, however, given the size of the private leverage boom, such a policy would have required buying back almost all of the public debt. Macro-prudential policy would have been helpful, especially in Ireland and Spain. However, in presence of a spending bias in fiscal rules, macro-prudential policies would have led to less prudent fiscal policies in the boom. Central bank actions would have stabilized employment during the bust but not public debt. Finally, if these countries had been able to regain in the bust the competitiveness they lost in the boom, they would have experienced a shorter and milder recession.

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There is wide disagreement about the nature of the eurozone crisis. Some see the crisis as driven by fiscal indiscipline and some by fiscal austerity, some emphasize excessive private leverage, while others focus on external imbalances, sudden stops or competitiveness divergence due to fixed exchange rates. Most observers understand that all these “usual suspects” have played a role, but do not offer a way to quantify their respective importance. In this context it is difficult to frame policy prescriptions on macroeconomic policies and on reforms of the eurozone. Moreover, given the scale of the crisis, understanding the dynamics of the Eurozone is one of the major challenge for macroeconomics today. We argue that we need a quantitative framework to identify the various mechanisms and, ultimately, to run counterfactual experiments. The objective of this paper is to make progress on these fronts. To do this we propose a simple model that focuses on three types of shocks: household leverage, fiscal policy, interest rate spreads and exports. A key challenge is then to empirically identify private leverage shocks that are orthogonal to shocks on fiscal policy and shocks on spreads. To help us identify the eurozone shocks, we use the US as a control. The US experience is of great use for us because of both its similarities and its differences with the eurozone experience.

Figure 1: Employment Rates in Ireland, Arizona, Spain and Florida.

To illustrate this, we take the example of Arizona and Ireland because their increase in household debt to income ratio during the boom years were very large and similar. Figure 1 shows the evolutions of the employment rates, normalized to zero in 2005. The employment boom and bust are almost identical up to 2010 but diverge afterwards. This suggests that the fundamental mechanisms at work in both regions were similar up to 2010, but different in later years. We argue that the key difference between Ireland and Arizona is that Arizona did not experience a sudden stop after 2010. There was no concern on its public
debt let alone on its remaining in the dollar zone. A similar, although less striking, pattern emerges when we compare Spain and Florida. Again, divergence is clear after 2010.

A salient feature of the great recession in both the US and the eurozone is that regions that have experienced the largest swings in household borrowing have also experienced the largest declines in employment and output. Figure 2 illustrates this feature of the data, by plotting the change in employment during the credit crunch (2007-2010) against the change in household debt-to-income ratios during the preceding boom (2003-2007) for the largest US states and Eurozone countries.

Figure 2: First Stage of the Great Recession: Household Borrowing predicts Employment Bust in the US and the EZ

The American and European cross-sectional experiences look strikingly similar in this respect on the period 2007-2010. This suggests that the shock faced by these two economies were similar in nature on that period. Moreover this suggests that the structural parameters that govern the way the economy reacts to a deleveraging shock may also be similar in the two monetary zones.

The key difference between the US and the eurozone experience is the sudden stop in capital flows starting in 2010 in the later. The eurozone stands apart from the US but also historically as we do not know of any other historical example of a sudden stop among countries or states inside a monetary union although sudden stops have been frequent in the 19th and 20th centuries (see Accominotti and Eichengreen (2013)).

1State level household debt for the US comes from the Federal Reserve Bank of New York, see Midrigan and Philippon (2010).
Contrary to the eurozone, the US states did not experience any shock on spreads in borrowing costs and no fear of a potential exit of the dollar zone. This allows us, for the eurozone, to identify the part of the private deleverage dynamics that is not due to the spreads shocks by the private deleveraging predicted in the US on the period 2008-2012. We call this the “structural” private leverage shock.

Figure 3 illustrates the differences between the American and European experiences during the later stage of the recession. Starting in the Spring of 2010, sovereign spreads widen and several European countries find it difficult to borrow on financial markets. The US and EZ experiences then start to diverge. While US states grow (slowly) together, eurozone countries experience drastically different growth rates and employment. A state variable that correlates well with labor markets performance in 2010-2011 in the Eurozone is the change in social transfers during the boom. Eurozone countries where spending on transfers (and also government expenditures) increased the most from 2003 to 2008 are those that are now experiencing severe recessions in the later stage. This suggests that in the second stage past fiscal policy, because of its effect on accumulated debt, had an impact on the economy through spreads and the constraint on fiscal policy it generated after 2010. This is an hypothesis we will analyze.

As noted in Midrigan and Philippon (2010), the pattern of figure 2 is at odds with the predictions of standard models of financing frictions. Such models predict that a tightening of borrowing constraints at the household level leads to a decline in consumption but, due to wealth effects, to an increase in the supply of labor. In this paper, we analyze a model where borrowing limits on “impatient” agents drive consumption, income, the saving decisions of “patient” agents and employment in small open economies belonging to a monetary union. We introduce nominal wage rigidities which translate the change of nominal expenditures
into employment. We first consider the predictions of the model taking as given the observed series for private debt, fiscal policy and interest rate spreads between 2000 and 2012. This reduced form simulation reproduces very well what was observed across Eurozone countries both during the boom and the recession starting in 2008 for employment, nominal GDP, consumption and wages. We then identify structural shocks for household debt, interest rate spreads and fiscal policy using the US experience to predict the household debt shock. Finally we feed our model with these structural shocks and run counterfactual experiments on fiscal policy, macro prudential policies, actions of the central bank to limit spreads and competitiveness.

We first ask how periphery countries would have fared if they had followed more conservative fiscal policies during the boom than they actually pursued. Such policies would have reduced the spreads and fiscal austerity during the bust. We find that periphery countries would then have stabilized their employment. This is especially true for Greece and Ireland, less so for Spain and Portugal. For Ireland however such policy would have entailed entering the bust with no public debt. This suggests that fiscal policy alone cannot act as a stabilization tool in presence of a massive private credit boom. We then ask how these countries would have fared if they had successfully conducted macro-prudential policies to limit private leverage during the boom. This would have successfully stabilized employment especially in Ireland, in particular because it would have entailed lower recapitalization during the bust and would have reduced spreads and allowed for a more countercyclical fiscal policy in the bust. In Spain, we find that given a spending bias in the fiscal rule they would have substituted public debt to private debt. This suggests that macro-prudential policy alone in this country would not have successfully stabilized employment in presence of a spending bias in the fiscal rule. Finally, the sudden stop episode worsened the crisis by further constraining the fiscal reaction of governments during the bust. In a third counterfactual, we find that if the ECB words and actions (Mario Draghi’s declaration “Whatever it takes” and the OMT program) had come in 2008 rather than 2012 and had been successful in reducing the spreads, the four countries would have been able to avoid the latest part of the slump but not the large buildup of public debt. Irish employment in particular would look very much like Arizona in Figure (1) with a rebound in 2011-2012: in this counterfactual the eurozone and the US are both monetary unions where central banks are successful in eliminating the risk of exit. Finally, we analyze how different the bust would have looked like if these countries had been able to regain in the bust the competitiveness they had lost in the boom. One can think of this counterfactual as close to a situation of flexible exchange rates where the exchange rate can depreciate quickly during a recession. We find that they would have experienced a shorter and milder bust and a much smaller buildup in public debt.

Our paper is related to three lines of research: (i) macroeconomic models with credit frictions, (ii)
monetary economics, (iii) sudden stops and sovereign defaults. We discuss the connections of our paper to each topic. Following Bernanke and Gertler (1989), many macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke et al. (1999), or Cooley et al. (2004)). In all these models, the availability of credit limits corporate investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. Curdia and Woodford (2009) analyze the implication for monetary policy of imperfect intermediation between borrowers and lenders. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, this makes a difference for the model’s cross-sectional implications. Models that emphasize firm-level frictions cannot reproduce the strong correlation between household-leverage and employment at the micro-level, unless the banking sector is island-specific, as in the small open economy “Sudden Stop” literature (Chari et al. (2005), Mendoza (2010)). This “local lending channel” does not appear to be operative across U.S. states, however, presumably because business lending is not very localized. Our framework is also related to heterogeneous-agent macroeconomic models such as Krusell and Smith (1998), and models in the tradition of Campbell and Mankiw (1989), that feature impatient and patient consumers. This type of models has been used by Gali et al. (2007) to analyze the impact of fiscal policy on consumption and by Eggertsson and Krugman (2012) to analyze macroeconomic dynamics during the Great Recession.

Papers in the sudden stop literature have aimed at reproducing the stylized facts of these crises in emerging markets. According to Korinek and Mendoza (2013) the key characteristics of a sudden stop are 1) a sharp, sudden reversal in international capital flows, which is typically measured as a sudden increase in the current account 2) a deep recessions and 3) sharp changes in relative prices, including exchange rate depreciations. The eurozone crisis shares the two first characteristics even if the pace of current account adjustment in the euro area is slower than for non euro area countries (such as Bulgaria, Latvia and Lithuania) and past experiences of emerging markets crises (see Merler and Pisani-Ferry (2012) for a discussion). Substitution of private-capital inflows by public inflows, especially Eurosystem financing, partly explains this difference. The third characteristic of an emerging market sudden stop has been absent in the eurozone crisis: there has been (so far) no currency depreciation and no sudden and large change in goods relative prices between countries hit at different degrees by a sudden stop (Greece, Spain, Ireland, Portugal and Italy) and the rest of the eurozone. That these countries belong to a monetary union means the eurozone sudden stop stands 2

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2For instance, Mian and Sufi (2010) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balanced sheets and were able to draw on existing credit lines Ivashina and Scharfstein (2008).
apart. These differences are important for the choice of modeling approach. The sudden stop literature on emerging markets (see Mendoza (2010) and Korinek and Mendoza (2013) for example) has focused on a Fisherian amplification mechanism where debts are denominated in different units than incomes and collateral. This is not the case in our model as we study countries that belong to a monetary union. Another difference is that the sudden stop literature in emerging markets has focused on the sudden imposition of an external credit constraint (see Mendoza and Smith (2006) and Christiano and Roldos (2004) for example) or on transaction costs on international financial markets with multiple equilibria, as in Martin and Rey (2006).

Our model integrates, for the first time to our knowledge, both a domestic credit crunch and a sudden stop produced by a spike in interest rate so that we can compare the impact of both on macroeconomic aggregates. The role of interest rates in our model relates our work to the paper of Neumeyer and Perri (2005). In their paper, as in ours, the economy is subject to interest rate shocks that generate a sudden stop in the form of a current account reversal. However, the mechanism is very different. In Neumeyer and Perri (2005), real interest rates movements either exogenous or induced by productivity shocks amplify the effect of the latter on production because they induce a working capital shortage. In our model, the increase in interest rate generates a demand shock through a fall in consumption.

Even if the bulk of the literature on sudden stops has put credit constraints at the center of the story, Gopinath (2004) and Aguiar and Gopinath (2007) have focused on an alternative explanation with TFP shocks taking center stage. Gopinath (2004) proposes a model with a search friction to generate asymmetric responses to symmetric shocks. A search friction in foreign investors entry decision into emerging markets creates an asymmetry in the adjustment process of the economy: An increase in traded sector productivity raises GDP on impact and it continues to grow to a higher long-run level. On the other hand, a decline in traded sector productivity causes GDP to contract in the short run by more than it does in the long-run. A related approach is the possibility of growth shocks as explored in Aguiar and Gopinath (2007). Because of the income effect, a negative shock leads to a fall in consumption and an increase in the trade balance. Aguiar and Gopinath (2007) do not study the response of the labor market but it is well known that income effects tend move consumption and hours in opposite directions.

Shocks to trend TFP growth might be important in emerging markets, but they do not seem to explain the dynamics of euro area countries over the past 5 years. With the exception of Greece, countries that were hit by a sudden stop (Greece, Ireland, Italy, Spain, Portugal) are not those for which the reversal in TFP growth is the largest\footnote{If the differential of TFP is computed on the period 2010-2012, this conclusion remains unchanged.} Moreover, as illustrated by figure 4, there is no correlation between the differential in
TFP growth (between the periods 2008-2012 and 2000-2007) and employment growth during the bust (2008-2012)\(^4\). Spain is a prime example. It is the only eurozone country that experienced an acceleration of its TFP growth during the crisis. Sanguinetti and Fuentes (2012) show that “The Spanish economy experienced significantly weaker labour productivity growth than other OECD economies and failed to catch up with the most advanced economies in the period 1996-2007... The relatively weak performance largely reflects the low growth of total factor productivity within a wide range of sectors, with very limited impact of composition effects, while the capital stock and educational attainment of the workforce have grown relatively strongly.”

We conclude that we need to look somewhere else for an explanation to the business cycles and to sudden stops in the eurozone. We focus on leverage, interest rates and fiscal policy dynamics.

Most closely connected to our paper is the work of Midrigan and Philippon (2010), Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2012) who also study the responses of an economy to a household-level credit crunch. Consistent with our results, Mian and Sufi (2012) show that differences in the debt overhang of households across U.S. counties partly explain why unemployment is higher in some regions than others. Schmitt-Grohe and Uribe (2012) emphasize the role of downward wage rigidity in the Eurozone recession.

Our paper is also related to the literature on sovereign default (see Eaton and Gersovitz (1982), Arellano (2008) and Mendoza and Yue (2012)) that models default as a strategic decision with a tradeoff between gains from forgone repayment and the costs of exclusion from international credit markets. The objective of our paper however is to analyze how the sovereign default risk can affect the real economy through the

\[^4\]A simple panel regression among Eurozone members on the period 2002-2012 of GDP growth on TFP growth, with or without year fixed effects and country fixed effects reveals no significant correlation between GDP and TFP growth.
impact it can have on liquidity available to households. The paper by Corsetti et al. (2013) considers a "sovereign risk channel," through which sovereign default risk spills over to the rest of the economy, raising funding costs in the private sector. Finally the paper is related to the recent research on fiscal multipliers at the regional level (Nakamura and Steinsson (2014), Farhi and Werning (2013)).

In Section 1 we present the model and in section 2 we analyze its dynamic properties. 3 presents the calibration exercise and compares the reduced form model predictions to the data. In Section 4, we present the structural relations between private leverage, fiscal policy and sudden stops. These serve to conduct the final exercise on counterfactual policies presented in section 5. Section 6 concludes.

1 Model

Our model can be interpreted as a large country with a collection of regions (e.g., USA), or a monetary union with a collection of states (e.g., EZ). The key assumption are that these regions share a common currency, and that agents live and work in only one region.

We study a small open economy that trades with other regions of the currency area as in Gali and Monacelli (2008). Each region $j$ produces a tradable domestic good and is populated by households who consume the domestic good and an aggregate of foreign goods. Following Mankiw (2000) and more recently Eggertsson and Krugman (2012), we assume that households are heterogenous in their degree of time preferences. More precisely, in region $j$, there is a fraction $\chi_j$ of impatient households, and $1 - \chi_j$ of patient ones. Patient households (indexed by $i = s$ for savers) have a higher discount factor than borrowers (indexed by $i = b$ for borrowers): $\beta \equiv \beta_s > \beta_b$. Saving and borrowing are in units of the currency of the monetary union.

1.1 Within period trade and production.

Consider household $i$ in region $j$ at time $t$. Within period, all households have the same log preferences over the consumption of home ($h$), foreign goods ($f$), and labor supply:

$$u_{i,j,t} = \alpha_j \log \left( \frac{C_{i,j,t}^h}{\alpha_j} \right) + (1 - \alpha_j) \log \left( \frac{C_{i,j,t}^f}{1 - \alpha_j} \right) - \nu (N_{i,j,t})$$

With these preferences, households of region $j$ spend a fraction $\alpha_j$ of their income on home goods, and $1 - \alpha_j$ on foreign goods. The parameter $\alpha_j$ measures how closed the economy is, because of home bias in
preferences or trade costs. The demand functions are then:

\[ p^h_{j,t} C^h_{i,j,t} = \alpha_j X_{i,j,t}, \]
\[ p^f_{t} C^f_{i,j,t} = (1 - \alpha_j) X_{i,j,t}. \]

where

\[ X_{i,j,t} \equiv p^h_{j,t} C^h_{i,j,t} + p^f_{t} C^f_{i,j,t} \]

measures the spending of household \( i \) in region \( j \) in period \( t \), \( p^h_{j,t} \) is the price of home goods in country \( j \) and \( p^f_{t} \) is the price index of foreign goods. This gives the indirect utility

\[ U(X_{i,j,t}, P_{j,t}) = \log(X_{i,j,t}) - \log(P_{j,t}) - \nu(N_{i,j,t}), \]

where the CPI of country \( j \) is \( \log P_{j,t} = \alpha_j \log p^h_{j,t} + (1 - \alpha_j) \log p^f_{j,t} \), the PPI is \( p^h_{j,t} \), and the terms of trade are \( \frac{p^f_{t}}{p^h_{j,t}} \). From the perspective of country \( j \), foreign demand for home good \( F_{j,t} \) is exogenous, and we assume a unit elasticity with respect to export price \( p^h_{j,t} \). Production is linear in labor \( N_{j,t} \), and competitive, so \( p^h_{j,t} = w_{j,t} \). Market clearing in the goods market requires

\[ N_{j,t} = \chi_j C^h_{b,j,t} + (1 - \chi_j) C^h_{s,j,t} + \frac{F_{j,t}}{p^h_{j,t}} + \frac{G_{j,t}}{p^h_{j,t}}, \]

where \( G_{j,t} \) are nominal government expenditures. Note that we assume that the government spends only on domestic goods. Define nominal domestic product as

\[ Y^h_{j,t} \equiv p^h_{j,t} N_{j,t} \]

and total private expenditures as

\[ \dot{X}_{j,t} \equiv \chi_j X_{b,j,t} + (1 - \chi_j) X_{s,j,t}. \]

It is useful to write the market clearing condition in nominal terms (in euros)

\[ Y_{j,t} = \alpha_j \dot{X}_{j,t} + F_{j,t} + G_{j,t}. \] (1)
Each household supplies labor at the prevailing wage and receives wage income net of taxes \((1 - \tau_{j,t}) w_{j,t} N_{j,t}\). They also receive transfers from the government \(Z_{j,t}\). We assume that wages are sticky and we ration the labor market uniformly across households. This assumption simplifies the analysis because we do not need to keep track separately of the labor income of patient and impatient households within a country. Not much changes if we relax this assumption, except that we lose some tractability.\(^5\)

### 1.2 Inter-temporal budget constraints

Let \(B_{j,t}\) be the face value of the debt issued in period \(t - 1\) by impatient households and due in period \(t\). It will be convenient to define disposable income (after tax and transfers but before interest payments) as

\[
\tilde{Y}_{j,t} = (1 - \tau_{j,t}) Y_{j,t} + Z_{j,t}.
\]

The budget constraint of impatient households in country \(j\) is then

\[
\frac{B_{j,t+1}}{1 + r_{j,t}} + \tilde{Y}_{j,t} = X_{b,j,t} + B_{j,t},
\]

where \(r_{j,t}\) is the nominal borrowing rate between \(t\) and \(t + 1\). We assume that interest rates are time varying and may be country-specific. Borrowing is subject to the exogenous limit \(B_{j,t}^h\):

\[
B_{j,t} \leq B_{j,t}^h.
\]

The savers budget constraint is:

\[
S_{j,t} + \tilde{Y}_{j,t} = X_{s,j,t} + \frac{S_{j,t+1}}{1 + r_{j,t}},
\]

so their Euler equation is

\[
\frac{1}{X_{s,j,t}} = \mathbb{E}_t \left[ \beta \frac{(1 + r_{j,t})}{X_{s,j,t+1}} \right].
\]

\(^5\)In response to a negative shock, impatient households would try to work more. The prediction that hours increase more for credit constrained households appears to be counter-factual however. One can fix this by assuming a low elasticity of labor supply, which essentially boils down to assuming that hours worked are rationed uniformly in response to slack in the labor market. Assuming that the elasticity of labor supply is small (near zero) also means that the natural rate does not depend on fiscal policy. In an extension we study the case where the natural rate is defined by the labor supply condition in the pseudo-steady state \(\nu'(n^*_i) = (1 - \tau) \frac{w_i}{s_i,j}\). We can then ration the labor market relative to their natural rate: \(n_{i,j,t} = \frac{n^*_i(\tau)}{\sum n_{i,j}^*(\tau)} n_{j,t}\) where \(n^*_i(\tau)\) is the natural rate for household \(i\) in country. This ensures consistency and convergence to the correct long run equilibrium. Steady state changes in the natural rate are quantitatively small, however, so the dynamics that we study are virtually unchanged. See Midrigan and Philippon (2010) for a discussion.
Note that financial markets clear in two ways in our model. For the impatient agents, given that they are quantity constrained, interest rates do not affect their borrowing. For the patient agents, their saving is determined by the interest rates through the Euler equation.

The government budget constraint is:

\[ \frac{B^g_{j,t+1}}{1+r_{j,t}} + \tau_{j,t}Y_{j,t} = G_{j,t} + Z_{j,t} + B^g_{j,t}, \]

(6)

where \( B^g_{j,t} \) is public debt issued by government \( j \) at time \( t-1 \).

### 1.3 Exports and foreign assets

Nominal exports are \( F_{j,t} \) and nominal imports are \((1 - \alpha_j)X_{j,t}\) since the government does not buy imported goods while private agents spend a fraction \( 1 - \alpha_j \) on foreign goods. So net exports are:

\[ E_{j,t} = F_{j,t} - (1 - \alpha_j)X_{j,t}. \]

(7)

The net foreign asset position of the country at the end of period \( t \), measured in market value, is:

\[ A_{j,t} \equiv (1 - \chi_j) \frac{S_{j,t+1}}{1+r_{j,t}} - \chi_j \frac{B^h_{j,t+1}}{1+r_{j,t}} - \frac{B^g_{j,t+1}}{1+r_{j,t}}. \]

(8)

Adding up the budget constraints, we have the spending equation

\[ X_{j,t} + p^h_{j,t}G_{j,t} = Y_{j,t} + \chi_j \left( \frac{B^h_{j,t+1}}{1+r_{j,t}} - B^h_{j,t} \right) - (1 - \chi_j) \left( \frac{S_{j,t+1}}{1+r_{j,t}} - S_{j,t} \right) + \frac{B^g_{j,t+1}}{1+r_{j,t}} - B^g_{j,t}, \]

(9)

Total spending (public and private) equals total income (nominal GDP) plus total net borrowing. If we combine with the market clearing condition (1), we get the current account condition

\[ CA_{j,t} \equiv A_{j,t} - A_{j,t-1} = E_{j,t} + \tau_{j,t-1}A_{j,t-1}, \]

It will often be convenient to rewrite (9) with disposable income as

\[ (1 - \alpha_j) \tilde{Y}_{j,t} = \alpha_j \chi_j \left( \frac{B^h_{j,t+1}}{1+r_{j,t}} - B^h_{j,t} \right) - \alpha_j (1 - \chi_j) \left( \frac{S_{j,t+1}}{1+r_{j,t}} - S_{j,t} \right) + F_{j,t} + \frac{B^g_{j,t+1}}{1+r_{j,t}} - B^g_{j,t}. \]

(10)
1.4 Employment and Inflation

The system above completely pins down the dynamics of nominal variables: \( Y_{j,t}, X_{i,j,t}, \) etc. Employment (real output) is given by

\[
N_{j,t} = \frac{Y_{j,t}}{p_{j,t}}.
\]

We need to specify the dynamics of inflation. Letting \( N^* \) denote the natural rate of unemployment, we assume the following Phillips curve:

\[
\frac{p_{h,j,t} - p_{h,j,t-1}}{p_{h,j,t-1}} = \kappa (N_{j,t} - N^*)
\]

(11)

2 Dynamic Properties of the Model

We now study the nominal dynamics of the small open economy. The main challenge is to pin down the behavior of savers. Our first task is to understand why and how savers do, or do not, react to certain shocks.

2.1 Savers’ Inter-temporal Budget

The complete objective function of the savers is \( \sum_{t \geq 0} \beta^t \left( \log (X_{s,j,t}) - \log (P_{j,t}) - \nu (N_{j,t}) \right) \). Prices are additively separable thanks to log preferences. In addition, our rationing rule for labor implies that \( N_{j,t} \) depends only on aggregate variables. The problem of the savers can therefore be written as:

\[
\max \sum_{t \geq 0} \beta^t \log (X_{s,j,t})
\]

\[
X_{s,j,t} + \frac{S_{j,t+1}}{1 + r_{j,t}} = S_{j,t} + \tilde{Y}_{j,t}
\]

where \( \tilde{Y}_{j,t} \) and \( r_{j,t} \) are both random variables.\(^6\) Let us focus first on the budget constraint, and its Ricardian properties. Let us first define the \( k \)-period discount rate from the savers’ perspective as \( R_{j,t,k} \equiv (1 + r_{j,t}) \cdots (1 + r_{j,t+k-1}) \), with the convention \( R_{j,t,0} = 1 \). We can then write the inter-temporal budget constraint of savers as

\[
\mathbb{E}_t \sum_{k=0}^{\infty} \frac{X_{s,j,t+k}}{R_{j,t,k}} = S_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}}.
\]

(12)

\(^6\)This is a well-studied problem but we will not do justice to all of its interesting aspects. In particular, we will neglect (for now) the role of precautionary savings and use a certainty equivalent approach by linearizing the Euler equation. Because of precautionary savings, we know that the interest rate consistent with finite savings must be such that \( \beta (1 + r) < 1 \). We consider the limit where aggregate shocks are small and \( \beta (1 + r) \) is close to one.
Lemma 1. The inter-temporal current account condition for country \( j \) is

\[
(1 - \alpha_j) \left( (1 - \chi_j) S_{j,t} - \chi_j B^h_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}} \right) = (1 - \chi_j) S_{j,t} - \chi_j B^h_{j,t} - B^g_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{j,t+k}}{R_{j,t,k}}.
\]

Proof. See Appendix.

On the left we have private wealth (discounted at the savers’s rate) and \( 1 - \alpha_j \) is the share of wealth spent on imports. On the right we have net foreign assets plus the value of exports. The key point here is that the inter-temporal current-account condition pins down the NPV of disposable income, as a function of current assets, home bias and foreign demand.

If we combine the Lemma with the inter-temporal budget constraint (12), we obtain the following proposition.

Proposition 1. Nominal spending by savers \( (X_{s,j,t}) \) does not react to private credit shocks \( (B^h_{j,t+1}) \) or to fiscal policy \( (\text{neither } G_{j,t} \text{ nor } Z_{j,t}) \). It only reacts to interest rate and foreign demand shocks.

Proof. Lemma 1 shows that the net present value of disposable income is a function of four variables:

\[
\mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}} = V_{j,t} = F \left( S_{j,t}, B^h_{j,t}, B^g_{j,t}, \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{j,t+k}}{R_{j,t,k}} \right)
\]

where the first three variables (saving, household debt, public debt) are pre-determined at time \( t \) and the last one (exports) is exogenous. Therefore, equation (12) is in fact

\[
\mathbb{E}_t \sum_{k=0}^{\infty} \frac{X_{s,j,t+k}}{R_{j,t,k}} = S_{j,t} + V_{j,t}
\]

So current spending of savers can only depend on \( V_{j,t} \) and the path of interest rates. In particular, for given \( V_{j,t} \) and interest rates, it cannot depend on contemporaneous or future private credit and fiscal policy.

QED.

Proposition 1 clarifies the behavior of savers. Their spending reacts neither to \( G_{j,t} \) nor to \( Z_{j,t} \). This result is related to – but also different from – Ricardian equivalence. To understand it, one needs to focus
on the budget constraint of the patient households. Clearly, shocks to interest rates will affect this budget constraint and also the Euler equation, so we know that they will affect spending $X_{s,j,t}$. What is surprising is that, even though changes in the borrowing constraints of impatient agents or changes in fiscal policy have a direct impact on disposable income $\tilde{Y}_{j,t}$, savers do not react. The reason is that patient agents know that higher spending today – which increases output – means higher interest payments in the future – which decreases spending and output. The key result is that the net present value of disposable income does not change. Changes in $B_{j,t}, G_{j,t}, Z_{j,t}, \tau$ have no effect on the permanent income of patient agents. Shocks to foreign demand, on the other hand, affect consumption expenditures of patient households because they affect their permanent income. Of course, even when expenditures remain constant, this does not mean that real consumption remains constant. In fact real consumption always changes because prices (wages) always react to changes in aggregate spending. These results depend on our using the preferences of Cole and Obstfeld (1991). Farhi and Werning (2013) discuss the implications of these preferences for government multipliers in currency unions.

2.2 Scaling and Spreads

Proposition 1 explains why savers do not react to demand shocks. But savers react to interest rates and to foreign demand shocks. We consider an economy subject to four series of shocks: the borrowing limit of the impatient households $B_{j,t}$, foreign demand $F_{j,t}$, interest rates, and fiscal policy. We assume that the variance of these shocks is small and linearize the Euler equation (5) as $E_t [X_{s,j,t+1}] \approx \beta (1 + r_{j,t}) X_{s,j,t}$.

The equivalent equation for the monetary union as a whole is $E_t [X_{s,t+1}^*] \approx \beta (1 + r_t^*) X_{s,t}^*$, where $r_t^*$ is the interest for the monetary union as a whole. We define the spread shock as:

$$1 + \rho_{j,t} \equiv \frac{1 + r_{j,t}}{1 + r_t^*}$$

We show in the Appendix that if we scale all our variables by $X_{s,t}^*$:

$$x_{s,j,t} \equiv \frac{X_{s,j,t}}{X_{s,t}^*}$$

Then we have

$$E_t [x_{s,j,t+1}] \approx (1 + \rho_{j,t}) x_{s,j,t}$$

(14)
From now on we work with scaled variables (in lower case). For example, the patient budget constraint becomes:

\[ x_{s,j,t} + \frac{\beta}{1 + \rho_{j,t}} s_{j,t+1} = s_{j,t} + \tilde{y}_{j,t}. \]

Finally, we maintain assumption A1 throughout the paper.

**Assumptions A1.**

- \( E_t [f_{j,t+1}] = f_{j,t} \),
- \( E_t [\rho_{j,t+1}] = 0; \)

Assumptions A1 say that the shocks on foreign demand are permanent and spreads are iid. These two conditions are assumed to hold throughout the paper.

### 2.3 Impulse responses to shocks

We now present simple impulse response functions to build intuition about the mechanisms of the model. These intuitions will be useful when we perform our simulations. The details of the assumptions and policy functions used to compute these impulse responses are in the Appendix.

We present in Figures (5), (6), (7) and (8) the simple impulse reaction functions\(^7\) that illustrate the impact of shocks on household debt (\(b_{j,t}^h\)), public spending (\(g_{j,t}\)), interest rates (\(r_{j,t}\)) and foreign demand (\(f_{j,t}\)). An increase in household debt generates a boom in nominal GDP, employment, spending of impatient households (but not of patient ones who increase their saving, as explained in Proposition 1) and imports. Public debt falls but the net foreign asset position deteriorates. A fiscal expansion through an increase in public expenditures has qualitatively similar effects except that in this case public debt increases, although it decreases temporarily in percentage of GDP. The GDP multipliers for household debt and government spending both increase with \(\alpha_i\) and \(\chi_i\). The reason is that a higher share of spending on domestic goods reduces leakage through imports. A higher share of impatient agents in the economy implies that an increase in household debt or disposable income (through higher government spending) has a larger impact on

---

\(^7\)For these impulse response functions, we use the following parameters: \(\alpha = 0.75, \chi = 0.5, r = 0.05, \kappa = 0.2, \tau = 0.4\). Prices, wages and employment are normalized to unity at time \(t = 0\). The debt to income ratio is set at 60% for impatient households at time \(t = 0\), so that the household debt to income ratio is 30%. The government debt to GDP ratio is set at 50% and the net foreign asset position over GDP is set at zero at time \(t = 0\). The shock is a 20% increase of the variable at \(t = 1\).
aggregate expenditures. Remember that patient agents expenditures do not react to private or public debt changes.

An increase in interest rates is very different because it induces patient households to save more so it reduces their expenditures and generates a recession (fall in nominal GDP and in employment) that obliges impatient households to reduce their spending. Imports fall and the net foreign asset position improves. Because of lower tax revenues, the recession increases public debt. Finally, an increase in foreign demand permanently increases nominal GDP which induces patient households to increase their saving. Spending of both patient and impatient households increase. The net foreign asset position improves. Public debt falls because of higher tax revenues.

Figure 5: Private Credit Expansion
Figure 6: Fiscal Expansion

Figure 7: Interest rate shock
3 Calibration and Reduced Form Model

We next analyze a cross-sectional experiment to compare the model predictions and the data. We describe the sources of the cross-sectional and aggregate data we use in the Appendix. We use data for 11 Eurozone countries from 2000 to 2012: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal and calibrate the shocks on the observed data.

3.1 Calibration

In order to map the observed data into the model we rebase the data with aggregate Eurozone spending, as in Equation (13). We compute a benchmark in the same manner for private and public spending, transfers and unit labor costs. The normalized data is the ratio of the observed data to this benchmark level. This enables us therefore to interpret these as deviations from the benchmark levels for each data series we observe. For both the household debt and the government debt, the rebased levels are the ratios of debt to the benchmark levels of GDP. Aggregate variables (employment, nominal GDP, private and public spending, transfers, taxes, exports...) are analyzed either in per capita terms or in ratios of GDP. For employment per capita, we take the deviation with respect to 2002 with an index of 1 for that year. 2002 is also the base year for private
and public spending and unit labor costs. The normalized data for household and public debt are shown in figure (33) in the Appendix. Normalized public spending and transfers are shown (34). Note also that government spending is adjusted for expenditures on bank recapitalization.

The parameters that serve in the simulations are given in Table (1).

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Discount Factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Domestic share of consumption</td>
<td>$\alpha_j$</td>
</tr>
<tr>
<td>Share of credit constrained households</td>
<td>$\chi_j$</td>
</tr>
<tr>
<td>Phillips curve parameter</td>
<td>$\kappa$</td>
</tr>
</tbody>
</table>

For the country specific domestic share of consumption, $\alpha_j$, we rely on Bussiere et al. (2011) who compute the total import content of expenditure components, including the value of indirect imports. For consumption expenditures and for our sample our countries the average implied domestic share in 2005 (the latest date in their study) is 72.7%. The lowest is 66.4% for Belgium and the highest is 78.7% for Italy. For the country specific share of credit constrained borrowers, $\chi_j$, we use the Eurosystem Household Finance and Consumption Survey (HFCS)\(^8\). This survey has been used recently by Kaplan et al. (2014) to quantify the share of hand-to-month households. The later paper defines these as consumers who spend all of their available resources in every pay-period, and hence do not carry any wealth across periods. They argue that measuring this behavior using data on net worth (as consistent with heterogeneous-agent macroeconomic models) is misleading because this misses what they call the wealthy hand-to-mouth households. These are households who hold sizable amounts of wealth in illiquid assets (such as housing or retirement accounts), but very little or no liquid wealth, and therefore consume all of their disposable income every period. They define hand-to-mouth consumers as those households in the survey whose average balances of liquid wealth are positive but equal to or less than half their earnings. We use a related measure by Mendicino (2014), who for each country computes the fraction of household with liquid assets below two months of total households gross income to approximate the share of credit constrained households. The average for our set of countries is 48% with a maximum of 64.8% for Greece and a minimum of 34.7% for Austria. Ireland did not participate in the survey so for this country we use the average of the eurozone. These two parameters, the share of credit constrained households ($\chi_j$) and the domestic share of consumption ($\alpha_j$), for our panel of countries are shown on figure (9).

\(^8\)The survey took place in 2010. In Greece and Spain, the data were collected in 2009 and 2008-09 respectively.
Figure 9: Share of credit constrained households ($\chi_j$) and domestic share of consumption ($\alpha_j$)

To transform observed aggregate household debt in the data into household debt $b_{j,t}$ in the model which is debt per impatient household (with share $\psi_j$), we take the household to benchmark income ratio for each country and then divide it by the share of impatient in the country $\chi_j$. We use $t_0 = 2002$ as our base year.

Given the absence of an intermediate goods sector in our model, we cannot use the value of gross exports as a measure of foreign demand, $F_{j,t}$. The trade linked to international production networks has been well documented (see for example Baldwin and Lopez-Gonzalez (2013)). In the context of our model, we need to measure the domestic value added that is associated with final consumption in the rest of the world, which corresponds to value added based exports. As detailed in the data appendix, we use the data from the OECD-WTO Trade in Value-Added (TiVA) initiative to measure domestic value added embodied in gross exports. The normalized value-added based exports are shown in figure (35) in appendix. Finally, we take into account net EU transfers which are the difference between EU spending in the country and the country contribution to the EU. In our model, they play exactly the same role as foreign demand so we add EU net transfers to exports in the goods market equation.

Finally, we must take a stand on how to measure interest rate spreads across countries. From the perspective of our model, we need to measure the funding cost of agents who are on their Euler equation (equivalently, if we think of firms, their Tobin’s $q$ equation). We start from the yields on long term government debt (annual averages of 10 year government bond rates) and we compute for each country the deviation from the median of the Eurozone. These spreads can be large, and it is clear that they partly reflect credit risk. In the case of Greece, the spread was over 20%, but this was not the expected return. The issue is then: should we remove expected credit losses? If yes, how? The answer is complex. If there are no
deadweight losses from government default, we should remove expected credit losses, since these would be transfers across agents, and would not represent real funding costs. But in reality there are costs of default. There is also the important issue of debt overhang in the banking sector. When its debt becomes risky, the bank has an incentive to take risks. Investing in its own sovereign debt becomes attractive because it is correlated with its own risk. When debt overhang is large and the correlation is high, the bank may end up treating the entire yield as an expected return because it only cares about the non-default state (since in the other states it is bust in any case). Even if the spread reflects credit risk, it would still represent a funding cost for bank-dependent borrowers. The bottom line is that there is no simple way to adjust the spreads. We use the following approach. We gather data on deposit rates and we compare it to government yields. We then apply a function to reduce the yields so that their fluctuations become comparable to the ones of deposit rates. We apply the following filter:

\[
\rho = \Delta \cdot \begin{cases} 
1 \% & \Delta \in [-1\%, 0] \\
3 \% & \Delta \in [0, 2\%] \\
5 \% & \Delta > 2\% 
\end{cases} 
\]

where \(\Delta\) is the deviation of the yield from the Eurozone median and \(\rho\) is our measure of spreads in funding costs. What this means is that, for the first 100 basis points, we treat the spread as a funding cost. This is consistent with estimates of flight to quality towards German assets and liquidity risk. Then we divide the spread by two, etc. Above 500 basis points, we assume that only 1/8th of the spread represent funding costs. This filter creates funding costs that are comparable to the (limited) data we have on deposit rates. Since the filter is rather ad-hoc, we have performed a large number of robustness checks. The results are robust as long as we trim the large spreads (otherwise the drop in consumption by savers is simply too large to be consistent with the data). Both the 10 year government bond spreads (the deviation of the yield from the Eurozone median) and \(\rho\) as measured in the equation above are shown in figure (36) in appendix.

3.2 Reduced Form Simulations

We first consider the predictions of the model taking as given the observed series for private debt \((b_{j,t}^h)\), fiscal policy \((g_{j,t}, z_{j,t}, \tau_j)\) and interest rate spreads \((\rho_{j,t})\). In other words, the reduced form model \(\mathcal{R}\) is a mapping

\[
\mathcal{R} : \left( b_{j,t}^h, g_{j,t}, z_{j,t}, \tau_j, \rho_{j,t} \right) \to \left( y_{j,t}, n_{j,t}, p_{j,t}, c_{j,t}, \ldots \right)
\]
To run the simulations, we first need to set the initial conditions in a particular year. We use 2002 as our base year.

1. Natural employment and prices are normalized to $n^* = 1$ and $p^h_{i,t_0} = 1$ (so nominal GDP is normalized in the base year to: $y_{j,t_0} = 1$)

2. Variables set to their observed values are: $b^h_{j,t_0}$, $z_{j,t_0}$, $g_{j,t_0}$, $b^g_{j,t_0}$, $b^g_{j,t_0-1}$, $r_{j,t_0}$. We then get $f_{j,t_0}$, $\tau_j$ and $\tilde{y}_{j,t_0}$ from market clearing and budget constraints:

   (a) Foreign demand $f_{j,t_0}$ is chosen to match net exports $e_{j,t_0} = \frac{1}{\alpha_j} (f_{j,t_0} - (1 - \alpha_j) (y_{j,t_0} - g_{j,t_0}))$

   (b) Get $\tau_j$ from the government budget constraint $g_{j,t_0} + z_{j,t_0} - \tau_j y_{j,t_0} = \frac{b^g_{j,t_0+1}}{1+\tau_{j,t_0}} - b^g_{j,t_0}$

   (c) Disposable income at time $t_0$ is $\tilde{y}_{j,t_0} = (1 - \tau_j) y_{j,t_0} + z_{j,t_0}$

3. Savers’ assets $s_{j,t_0}$ and $s_{j,t_0-1}$ are chosen to solve the equilibrium conditions

   (a) $s_{j,t} = (1 + \rho_{j,t}) (s_{j,t-1} + \tilde{y}_{j,t}) - \mathbb{E}_t [\tilde{y}_{j,t+1}]$

   (b) $(1 - \alpha_j) \mathbb{E}_t [\tilde{y}_{j,t+1}] = \frac{\alpha_j}{1+\tau_j} (1 - \chi_j) s_{j,t+1} - \alpha_j \chi_j b^h_{j,t+1} - b^g_{j,t+1} + f_{j,t}$

   (c) $(1 - \alpha_j) \tilde{y}_{j,t} = \alpha_j \chi_j \left( \frac{b^h_{j,t+1}}{1+\tau_{j,t}} - b^h_{j,t} \right) + \alpha_j (1 - \chi_j) \left( s_{j,t} - \frac{s_{j,t+1}}{1+\tau_{j,t}} \right) + \frac{b^g_{j,t+1}}{1+\tau_{j,t}} - b^g_{j,t} + f_{j,t}$

We then feed exogenous processes for the different shocks (using scales values) for observed household debt ($b^h_{j,t}$), fiscal policy ($\tau_j$, $z_{j,t}$,$g_{j,t}$), and interest rate spreads $\rho_{j,t}$. For the sake of completeness, we also feed exogenous foreign nominal demand shocks $f_{j,t}$, even though they are not crucial for our analysis. For each country, we simulate the path between 2001 and 2012 of nominal GDP $y_{j,t}$, nominal spending $x_{j,t}$, employment $n_{j,t}$, net exports $e_{j,t}$ and public debt $b^g_{j,t}$.

The normalized data on observed shocks that serve to feed the model for each country are shown in figures (33), (34), (36) and (35) in appendix.

Just to be clear, there is no degree of freedom in our simulations of nominal variables. There is no parameter which is set to match any moment in the data. The model is entirely constrained by observable micro estimates and by equilibrium conditions. The only parameter that we can adjust is the slope of the Phillips curve $\kappa$ but it does not affect the nominal GDP in euros, it only pins down the allocation of nominal GDP between prices (unit labor cost) and quantities (employment).

Figures (10), (11), (12) show the simulated and observed nominal GDP, net exports and employment. The reduced form model reproduces very well the cross sectional dynamics in the euro zone for nominal GDP and net exports. In particular, it replicates well the boom and bust dynamics on nominal GDP and
the current account reversal for the crisis hit countries. For employment, the model does also well for the crisis countries and for countries that were hit less severely.

Figure 10: Simulated and observed nominal GDP
Figure 11: Simulated and observed net exports

Figure 12: Simulated and observed employment
4 Structural Model

In order to run counterfactuals we need to take into account that private leverage, spreads and fiscal policy may be interrelated. Hence, we need to identify the structural relations between the three variables. We think of country dynamics as being driven by three structural shocks. The first one is a boom/bust cycle in private debt, which we call credit bubble for short. The second one is a political economy bias in government spending that creates fiscal imbalances. The third is a (potentially self-fulfilling) sudden stop that threatens the stability of the eurozone. Formally, we think of the structural model $\mathcal{S}$ as a mapping

$$\mathcal{S} : \text{(Credit Bubble, Political Economy, Sudden Stop)} \rightarrow \left( b_{j,t}^h, g_{j,t}, z_{j,t}, \rho_{j,t}, b_{j,t}^g, y_{j,t}, n_{j,t}, p_{j,t}, e_{j,t}, \ldots \right)$$ (16)

The key point of the structural model is to explain the variables that we took as exogenous in the reduced form model (15). The challenge is to identify these shocks in the data.

We now present our identification strategy. This strategy is based on a mix of theoretical modeling and empirical identification using instrumental variables. When we talk about the “structural” model, we do not mean that we have provided micro-foundations for every detail of the model. Given the range of data and economic forces that we need to capture, this is not even remotely possible. But we mean that, either there is an explicit theoretical equation, or there is an identified empirical equation that allows us to capture the influence of one variable on the others.

4.1 Using the US to identify private leverage dynamics

We use the US as a control group to estimate leverage dynamics without sudden stops. More precisely, we estimate the following model for deleveraging in a panel of U.S. states

$$b_{j,t}^{h,US} = \sum_{k=1}^{3} \alpha_{k}^{US} b_{j,t-k}^{h,US} + \epsilon_{j,t}$$

for $t = 2008, \ldots, 2012$, $j = 1, \ldots, 52$, and $b_{j,t}^{h}$ is household debt detrended exactly as in the Eurozone. The idea is that these private leverage bubbles reflected various global and financial factors: low real rates, financial innovations, regulatory arbitrage of the Basel rules by banks, real estate bubbles, etc. To a large extent these forces were present both in Europe and in the US. The difference of course is that there was no sudden stops within the US. Hence, we interpret the US experience as representative of a deleveraging outcome in a monetary union without sudden stops.
Another issue is whether fiscal policy was not also active in the US. Perhaps private debt bubbles were associated with large fiscal revenues and large spending. This probably happened to some extent, but compared to the Eurozone, these effects are small (of course we are only talking about cross-sectional variation in government spending). Figure 13 shows this for two states and two countries. A regression for all the states and all the countries shows that the link between private debt and government spending was at least four times smaller in the US than in Europe.

Figure 13: Government Spending

![Figure 13: Government Spending](image)

We therefore argue that the US provides a benchmark for private deleveraging without sudden stops, and with relatively neutral (cross-sectional) fiscal policy. We then take the estimated coefficients $\alpha_k^{US}$ and use them to construct predicted deleveraging in Eurozone countries:

$$\hat{b}_{j,t}^h = \sum_{k=1}^{K} \alpha_k^{US} b_{j,t-k}^h$$

for $t = 2008, \ldots, 2012$ and $j = 1 \ldots 11$. Figure 14 illustrates the results for California and Ireland. The model predicts a somewhat slower deleveraging in Ireland than actually happened. This is also the case for other countries that experienced a sudden stop.

We now posit the following structural equations. Private leverage in euro zone countries is given by:

$$b_{j,t+1}^h = \lambda_0^h + \hat{b}_{j,t}^h + \lambda^p h \rho_{j,t}.$$

Private leverage is equal to the prediction from the US experience plus the impact of the spread. The first
element of the private leverage, $\hat{b}_j^{h,t}$, is interpreted as an exogenous shock. The second element, the impact of the spread, is endogenous and is meant to capture various transmission channels. An obvious one is the funding costs of banks: an increase in $\rho_j,t$ would decrease bank lending. Another channel is that debt demand by impatient agents is not entirely interest inelastic as we have assumed in our simplified model. It will turn out, however, that $\lambda^{bh}$ is not large and that these channels do not appear to be very important.

4.2 Bond pricing

The second equation of the structural model captures bond pricing:

$$\rho_{j,t} = \sigma_t \times \left[ \lambda_0^g + \lambda^g_{\rho} b_{j,t}^g + \lambda^{g,high}_{\rho} b_{j,t}^{g,high} + \lambda^{b,h} b_{j,t}^h + \lambda^{rec} recap_{j,t} \right]$$

(18)

where $\sigma_t$ captures the size of the sudden stop. This equation says that funding costs start to diverge when there is a sudden stop in the eurozone and that the extent to which this happens in different countries depends on public debt $b_{j,t}^g$, private debt $b_{j,t}^h$, but also recapitalization of the financial sector ($recap_{j,t}$). This specification captures financial frictions associated with high leverage (debt overhang, risk shifting, adverse selection, runs, etc.). We allow the impact of public debt on spreads to be non-linear to take into account the case of Greece. More specifically, we allow the coefficient to be higher for levels of debt above 100% of GDP so we define $b_{j,t}^{g,high} \equiv (b_{j,t}^g - 1) (b_{j,t}^g > 1)$. In our data, this non-linear effect is only significant for Greece. Finally, we have assumed so far that spreads are iid (even though the risk-free rate can be persistent). Hence, we assume that $E_t [\sigma_{t+1}] = 0$. This means that agents in our economy anticipate that sudden stops, if any, last for a year.
We estimate the coefficients $\lambda$’s on the period 2008-2012 for the 11 eurozone countries. We do so by running instrumental variable regressions. For $\rho_{j,t}$ in equation (17) we use government debt lagged two and three years as instruments. For government and private debt $b_{j,t}^g$ and $b_{j,t}^h$, in equation (18), we use as instruments government debt levels ($b_{j,t}^g$ and $b_{j,t}^{g,high}$) lagged three years, the exogenous component of private debt (and its lag) predicted by the US experience $\hat{b}_{j,t}^h$. To capture the possibility and the size of a sudden stop in the eurozone, we measure the coefficient $\sigma_t$ as the divergence in spreads across the eurozone countries. More precisely, for each year, $\sigma_t$ equals the mean of the absolute value of spreads in the eurozone.

As expected, it is low and close to zero up to 2007 and starts increasing in 2008 with a maximum in 2012.

The estimated coefficients, which we will use in our simulations, are shown in Table (2):

<table>
<thead>
<tr>
<th>$\lambda_0^h$</th>
<th>$\lambda_{\rho^h}$</th>
<th>$\lambda_0^g$</th>
<th>$\lambda_{\rho^g}$</th>
<th>$\lambda_{\rho^h}$</th>
<th>$\lambda_{\rho^h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.005</td>
<td>-1.2</td>
<td>-2.3</td>
<td>2</td>
<td>1.2</td>
<td>15</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.184)</td>
<td>(0.16)</td>
<td>(0.4)</td>
<td>(0.3)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Note: standard errors in parenthesis.

For the non linear effect of public debt on spreads, we set $\lambda_{g,high,\rho} = 1$ since it gives the best fit for Greece.

### 4.3 Policy Function

The last task is to specify the fiscal policy function of the different governments. Fiscal policy is implemented via government spending $g_{j,t}$ and transfers $z_{j,t}$, while linear taxes are left constant. We assume that the government has two main macroeconomic objectives but is constrained by its costs of funds. His two objectives are to target a level of expenditure $\tilde{g}_{j,t}$ and transfers $\tilde{z}_{j,t}$ (that may be drifting over time), and to stabilize employment around a natural rate $\bar{n}_j$. We choose $\bar{n}_j$ for all countries to be the normalized employment level in 2002 so we set $\bar{n}_j = 1$. The funding constraint is measured by the (lagged) spread $\rho_{j,t-1}$ and is only binding when the spread is positive.\footnote{We use the t-1 spread simply because it fits better, which probably reflects implementation lags in fiscal policy. This is not related to the identification of the model.}

Hence, the policy rule for government spending, with parameters $\gamma_n$ and $\gamma_\rho$, is given by:

\[
g_{j,t} = \tilde{g}_{j,t} + \gamma_n (n_{j,t} - \bar{n}) + \gamma_\rho \rho_{j,t-1} \text{ if } \rho_{j,t-1} > 0, \tag{19}
\]

\[
g_{j,t} = \tilde{g}_{j,t} + \gamma_n (n_{j,t} - \bar{n}) \text{ if } \rho_{j,t-1} < 0,
\]
where $\tilde{g}_{j,t}$ is a country specific drift which is linearly increasing until 2008 and flat afterwards:

$$\tilde{g}_{j,t} = g_{j,0} + \delta g_{j} (t - t_0) (t < 2008) + \delta g_{j} (2008 - t_0) (t > 2008)$$

(20)

with $t_0 = 2002$. Hence, $\delta g_{j}$ represents the average annual spending growth rate during the boom years. We interpret this drift as a political bias in spending decisions. We want to analyze to what extent this spending drift during the boom years may have contributed to the euro crisis. The same structural equations apply to transfers $z_{j,t}$ with specific values for $z_{j,0}$ and $\delta z_{j}$.

We now focus on the four countries that were most harshly hit by the crisis, namely Spain, Greece, Ireland and Portugal. We choose our parameters in the policy rule $\gamma^n$ and $\gamma^p$ and the spending and transfer drift coefficients $\delta g_{j}$ and $\delta z_{j}$ such that the model reproduces as best as possible the dynamics of observed nominal GDP, public debt, employment and spreads.

This leads us to choose the parameters given in Table (3) and (4):

<table>
<thead>
<tr>
<th>$\gamma^n$</th>
<th>$\gamma^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.9</td>
<td>-7</td>
</tr>
</tbody>
</table>

Table 3: Fiscal policy coefficients

<table>
<thead>
<tr>
<th>$\delta g_{j}$</th>
<th>$\delta z_{j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>Greece</td>
</tr>
<tr>
<td>1%</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 4: Fiscal policy drifts: government spending and transfers

Note that the spending and transfers drift necessary to reproduce the debt dynamics is (not surprisingly) much larger in Greece than in the other periphery countries. It is intermediate and similar in Ireland and Spain and almost non existent in Portugal.

We also need to take into account the specific case of Greece which benefited in 2012 from a debt relief that reduced its public debt by around 50% of GDP and in 2011, from a reduction in interest rates and an extension in the repayment period for the EU and IMF rescue package. We do this by adding 10% of GDP and 40% of GDP to government revenues in 2011 and 2012. This allows to better replicate the Greek debt in 2012.
4.4 Fit of the structural model

The structural model is a constrained version of the reduced form model presented earlier. We can now formally write equation (16) as

\[ \Im : \left( \hat{b}^h_{j,t}, \tilde{g}_{j,t}, \tilde{z}_{j,t}; \sigma_t \right) \longrightarrow \left( b^h_{j,t}, g_{j,t}, z_{j,t}, \rho_{j,t}; b^g_{j,t}, y_{j,t}, n_{j,t}, p_{j,t}, e_{j,t}, \ldots \right) \]  

subject to the equilibrium condition of the model and the structural equations (17), (18) and (19). There are three sets of exogenous shocks: the fiscal drifts \( \delta^g_j \) and \( \delta^z_j \) on government expenditures \( \tilde{g}_{j,t} \) and transfers \( \tilde{z}_{j,t} \), the private leverage that would have been predicted by the US experience \( \hat{b}^h_{j,t} \), the sudden stop shock \( \sigma_t \).

We show in figures (15), (16), (17), (18) and (19), how our structural model with these benchmark parameters performs in reproducing the main macro dynamics (nominal GDP, employment, net exports, public debt and spreads) of the four periphery countries. Overall, the structural model does very well in reproducing the boom and bust episodes. The model and observed data depart in a few dimensions: 1) the Irish nominal GDP (but not employment) bust comes too late in the model; 2) the employment boom is under-predicted in Spain and over-predicted in Greece; 3) Spreads are under-predicted in Portugal.
Figure 15: Nominal GDP, Structural Model

Figure 16: Employment, Structural Model
Figure 17: Net Exports, Structural Model

Figure 18: Government Debt, Structural Model
5 Counterfactual experiments

We now present our main structural experiments. The goal is to provide counter-factual simulations of what would have happened to the four countries (Greece, Spain, Ireland and Portugal) that were most affected by the crisis had they followed a different set of policies. We consider four counterfactuals:

- fiscal policy: what would have happened if those countries had pursued a more conservative fiscal policy before 2008?
- macro-prudential policies: what would have happened if they had limited the growth of household debt?
- monetary policy: what would have happened if the “whatever it takes” commitment by the ECB had been announced in 2008 rather than in 2012?
- competitiveness: what would have happened if they had been able to recoup during the bust the competitiveness they had lost in the boom?
For the counterfactual experiments, we use the structural equations (17) and (18) for private debt and spreads respectively and the fiscal policy rule (19). We use the same coefficient estimates from the instrumental variables regression (shown in Table 2) for the structural equations for private debt and spreads and the policy rule (19). For all counterfactual experiments, we compare on the same graph the data and the counterfactual experiment which we define as: structural model with counterfactual parameters + (data - structural model with benchmark parameters) so that we take into account that the structural model with benchmark parameters does not perfectly reproduce the data although we have seen that it does so very well. The simulation generates cross sectional time series for public debt, private debt, employment, nominal GDP, net exports and spreads on the period 2001-2012, using debt in 2000 as an initial point.

5.1 Counterfactual with a more conservative fiscal policy in the boom

How would countries have fared if they had followed more conservative fiscal policies during the boom? We answer this question by removing the fiscal drift bias in the fiscal rule (19). Hence, we set $\delta^g_j$ and $\delta^z_j$ equal to zero for the four periphery countries. For Spain, Ireland and Portugal all the other benchmark parameters are left unchanged. For Greece, we need to deal with the debt relief issue. Given that the counterfactual conservative fiscal policy generates debt to GDP ratios below 110%, we assume that debt relief would not have taken place. Hence, for Greece the counterfactual is the combination of a more conservative fiscal policy but also the elimination of a transfer of around 50% of nominal GDP.

The elimination of the fiscal drift dramatically changes the public debt accumulation for Greece, Ireland and Spain but very little in Portugal which is not surprising given that our exercise suggests that the fiscal drift was very large in Greece, large in Ireland and Spain but small in Portugal. This can be seen in figure (20) where counterfactual public debt in Greece is stabilized close to its level in 2001 and falls in Ireland and Spain to a minimum of 5% and 26% of GDP respectively just before the bust. Hence, fiscal policy, once the fiscal drifts are removed, becomes more conservative but also more countercyclical. This large change in the public debt in turn reduces spreads during the sudden stop in Greece, Ireland and Spain but very little in Portugal as shown in figure (21). Lower spreads allow fiscal policy to be less constrained during the bust which explains part of the increase in debt in the periphery countries in the latest years. The counterfactual conservative fiscal policy in Greece is successful in stabilizing employment as shown in figure (22). Remember that this more conservative fiscal policy in Greece also means that this country does not benefit from the debt relief at the end of the period. The counterfactual conservative fiscal policies in the boom - which allow for less fiscal austerity in the bust - in Spain and Ireland are less successful in stabilizing
employment but allow for an earlier exit from the recession induced by private deleveraging, especially in Ireland. The counterfactual employment dynamics in Ireland now look very similar to Nevada as illustrated in figure (1) in the introduction. In both Ireland and Spain, the boom-bust cycle is attenuated, but one should remember that this entails a large and probably implausible decrease in public debt during the boom, especially in Ireland. In Portugal, the conservative fiscal policy in the boom has little effect, except at the end of the cycle where the fall in spreads reduces fiscal austerity and improves employment.

Figure 20: Government Debt under Conservative Fiscal Policy
Figure 21: Spreads under Conservative Fiscal Policy

Figure 22: Employment under Conservative Fiscal Policy
5.2 Counterfactual with macro-prudential policies in the boom

In this counterfactual we imagine that countries were able to implement policies that eliminated the household leverage boom. We assume that the growth rate of the exogenous part of the private leverage, the part predicted by the US experience, $\hat{b}^{h}_{j,t}$, is set to zero in all four countries. This means that private debt is essentially flat during the whole cycle. We take into account the impact of such policy on the recapitalization of financial institutions during the bust. Using a cross-sectional regression, we estimate that a reduction of private debt by one euro during the boom reduces recapitalization needs by 0.25 euro during the bust.

Figure (23) shows that this macro-prudential policy partially stabilizes employment in all four countries. This is especially so in Ireland: whereas in the data employment falls from peak to trough by 18%, in the counterfactual it falls by only 7%. Not surprisingly given that there was little private leverage boom in Portugal the impact of a counterfactual macro-prudential policy is small in this country. The comparison of the conservative fiscal counterfactual and of the macro-prudential counterfactual shows that the private leverage boom was the key igniting element of the crisis in Ireland and that fiscal policies during the boom played a secondary role. The opposite is true for Greece. Given that the fiscal drifts are not affected in this counterfactual, the fiscal rule (that contains both a spending and transfer drift and a countercyclical component) induces a larger buildup of public debt than in a situation without macro-prudential policy.

The public debt buildup in the boom can be seen in figure (24). It is especially stark in Spain, Greece and Ireland. With the spending and transfer drifts unchanged, larger public debt is substituted to private debt to achieve the employment target. The public debt dynamics appear unsustainable in this counterfactual. There are two opposite effects on spreads: on the one hand the reduction in private leverage and in predicted recapitalization reduces spreads. On the other hand, the larger buildup of public debt pushes spreads upward. In Ireland, the first effect dominates and explains the successful stabilization of employment. In the other three countries, the two effects cancel each other and spreads are not affected (see figure 26). This counterfactual suggests that macro-prudential policies that do not come with a more prudent fiscal rule may not have helped much to generate a fiscally sustainable stabilization of employment. In this sense, macro-prudential policies to contain private leverage and prudent fiscal policies to contain public debt appear to be complementary. Indeed, we checked that only a counterfactual that combines prudent fiscal policies and macro-prudential policies succeeds in stabilizing total debt and employment.
Figure 23: Employment with Macro-Prudential Policies

Figure 24: Public debt with Macro-Prudential Policies
5.3 Counterfactual with “whatever it takes” in 2008

In this counterfactual we ask the following question: what would have happened if the announcements of July 2012 (Mario Draghi’s declaration “Whatever it takes”) and September 2012 (the OMT program) that were successful in reducing the risk of a euro breakup, the sudden stop and the spreads of the periphery countries had come earlier. The experiment is to imagine that these actions were implemented and successful in 2008 rather than 2012. We assume that, starting in 2008, $\sigma_t$, the mean of the absolute value of spreads in the eurozone, that enters the spread equation (18), is kept at its low-2007 level. This effectively eliminates the sudden stop and the dramatic rise in spreads as illustrated in figure (26). This allows to reduce the cost of borrowing constraint in the fiscal rule so that the countries can avoid fiscal austerity and stabilize employment (see figure 27). The employment dynamics of Ireland and Nevada (see figure 1 in the introduction) look very similar. Both Nevada and Ireland suffer a large private leverage induced boom and bust but then recover starting in 2011 because both belong to a monetary union that is successful in eliminating the risk of exiting the dollar and eurozone respectively. Note also that Greece in 2012 still benefits in this counterfactual from the debt relief, which explains part of the boom in 2012. Finally, because the fiscal drifts remain intact in this counterfactual, the public debt is not stabilized as illustrated in figure (26) but the larger fiscal receipts
mean the public debt is not worse than in the data.

There are three channels through which spreads affect aggregate spending in the economy: 1) the savers spending through their Euler equation (14); 2) the leverage of borrowers through equation (17) which in turn affects their spending; 3) public spending and transfers through the borrowing constraint of governments in the fiscal rule (19). Transfers in turn affect the spending of the borrowers. We can use the counterfactual of spreads to quantify the role of each channel. We take the example of Spain and ask how the difference in spreads between the data and the counterfactual affects the different components of spending. Taking into account the leakage of imports, the increase in expenditures by savers is around 0.5 percentage point of GDP. The increase in private leverage for the borrowers has about the same impact on their spending. Public spending increases by around 3.6 percentage points of GDP. Transfers also increase by the same amount but because of import leakage and because only the spending of borrowers reacts to transfers, their impact on total spending is lower, around 1.5 percentage point of GDP. Hence, around 84% of the increase in aggregate spending that comes with lower spreads (around 6 percentage point of GDP) is due to the relaxation of the fiscal constraint by governments. The rest is equally due to the increase in spending by lenders (lower saving) and borrowers (more borrowing). The preeminence of the fiscal channel is robust across countries.

Figure 26: Spreads with Early ECB Intervention
Figure 27: Employment with Early ECB Intervention

Figure 28: Public Debt with Early ECB Intervention
5.4 Counterfactual with competitiveness measures

In this counterfactual we ask the following hypothetical question: what would have happened if periphery countries had been able to recoup during the bust the competitiveness they had lost during the boom years. This is close to a “flexible” exchange rate counterfactual, but it is not identical because it does not impact the net foreign asset position. It captures the idea of a fiscal devaluation.\textsuperscript{11}

More precisely, we assume in this counterfactual that export prices fall in the bust years (2008 to 2012) by the amount prices relative to the euro zone increased in the boom years (2002 to 2007).\textsuperscript{12} Because of unit elasticity of demand this means that exports increase by the amount of the cumulative increase in prices relative to the euro zone on the period 2002 to 2007. In the case of Spain and Greece, the competitiveness shock generates a 7% increase in nominal exports. In the case of Ireland the shock is larger at almost 11%. It is almost nil for Portugal so that in this country the counterfactual has no effect. The fall in employment is attenuated in the other three countries as shown in figure (29). This is especially the case in Ireland where employment rebounds in 2012. Relative to the data, employment is higher in the counterfactual by 7% in Ireland, and around 4% in Spain and Greece. There are several positive effects of the competitiveness shock. There is a direct effect on aggregate demand. This effect is the most important and is larger in more open economies such as Ireland. Savers and borrowers also increase their spending. The later because of increased disposable income and the former because increased foreign demand increases their permanent nominal income. Income taxes improve and governments react early on to the positive competitiveness shock by cutting spending and transfers. The trajectory in public debt is therefore improved as shown in figure (28). This improvement in the debt dynamics is - maybe surprisingly - the quantitatively largest consequence of the competitiveness shock: in 2012 public debt is lower by 28 percentage points of GDP in Ireland, 16 percentage points of GDP in Spain and and 12 percentage points of GDP in Greece. It also allows for a fall in spreads especially in Ireland and Spain as shown in figure (30).

\textsuperscript{11}See Franco (2013) and Farhi et al. (2014) for conditions under which a fiscal devaluation is equivalent to an exchange rate adjustment.

\textsuperscript{12}For Greece, contrary to the other countries, the price increase during the 2002-2007 period is not the observed data on unit labor costs relative to the euro zone but the price increase predicted by the model. The reason is that the data we have on unit labor costs in Greece is extremely volatile and clearly contains mistakes.
Figure 29: Employment with Competitiveness Shock

Figure 30: Public Debt with Competitiveness Shock
6 Conclusion

Understanding the dynamics of the Eurozone is a major challenge for macroeconomics. Eurozone countries have experienced extraordinary levels of real and financial volatility. Unemployment rates have diverged to an extent that nobody had anticipated. Financial flows have been extremely volatile and for the first time in history a major sudden stop materialized in a common currency area. While most observers recognize that private leverage, fiscal policy and sudden stops all played a role, it has proven challenging to analyze them jointly, and even more difficult to disentangle them.

Our paper makes three contributions. First, we present a model that accounts at the same time for domestic credit, fiscal policy, and current account dynamics. Second, we create a data set for 11 countries over 13 years that covers the variables of interest and deal with the various accounting issues. Third, and most importantly, we propose a new identification strategy that allows us to run counter-factual experiments on fiscal policy, macro-prudential policy, ECB policy averting the sudden stop episode and on regaining competitiveness.

Our experiments enable us to quantify the role of the different mechanisms in the euro crisis for Ireland, Spain, Greece and Portugal. The private leverage boom was the key igniting element of the crisis especially in Spain and Ireland. Fiscal policy (essentially pro-cyclical) during the boom worsened the situation by
increasing spreads and constraining governments after the private deleveraging started in 2008 and even more so with the sudden stop episode. This is especially true in Greece and to a lower extent in Spain and Ireland. In these two countries, a more conservative fiscal policy during the boom would have helped but would have entailed an implausibly large fall of public debt. Ireland would have had to enter the crisis with almost no public debt to be able to partially stabilize the private leverage bust after 2008.

A macro-prudential policy to limit private leverage during the boom would have stabilized employment in all countries, and especially in Ireland. However, in the absence of a more prudent fiscal policy, this would have induced a larger buildup in public debt. One lesson we take from our exercise is that fiscal and macro-prudential policies are complements not substitutes in order to stabilize the economy. A prudent fiscal policy, although helpful, cannot by itself undo the consequences of a private leverage boom and the reverse is true also. Both prudent fiscal policies and macro-prudential policies are required to stabilize the economy.

The sudden stop episode worsened the crisis by further constraining the fiscal reaction of governments during the bust. If the ECB words and actions (Mario Draghi’s declaration “Whatever it takes” and the OMT program) had come earlier and had been successful in reducing the spreads, the four countries would have been able to avoid the latest part of the slump but would not have avoided the large buildup in public debt. Finally, the impossibility to devalue and the difficulty to regain competitiveness during the bust was a significant drag on employment in Spain and even more so in Ireland.
References


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Appendix

A. Model

A.1 Scaling

We have already defined the Euler equations for country $j$ and the monetary union (with an asterisk) as a whole:

$$E_t [X_{s,j,t+1}] \approx \beta (1 + r_{j,t}) X_{s,j,t}; \ E_t [X_{s,t+1}^*] \approx \beta (1 + r_{s,t}^*) X_{s,t}^*. $$

and the spread as

$$1 + \rho_{j,t} \equiv \frac{1 + r_{j,t}}{1 + r_{s,t}^*}$$

If we scale the budget constraint (assuming perfect foresight, or equivalently, neglecting the conditional variance of the aggregate shocks), we get

$$\frac{X_{s,j,t}}{X_{s,t}^*} + \frac{S_{j,t+1}}{1 + r_{j,t}} \frac{X_{s,j,t}^*}{X_{s,t}^*} = \frac{S_{j,t}}{X_{s,t}^*} + \frac{\tilde{Y}_{j,t}}{X_{s,t}^*}$$

and up to the usual approximation we have

$$x_{s,j,t} + \frac{\beta}{1 + \rho_{j,t}} s_{j,t+1} = s_{j,t} + \tilde{y}_{j,t}$$

A.2 Phillips Curve

$$\frac{(p_{j,t})^2}{p_{j,t-1}} + (\kappa N^* - 1) p_{j,t} - \kappa Y_{j,t} = 0$$

Defining $\Delta \equiv (\kappa N^* - 1)^2 + 4\kappa \frac{Y_{j,t}}{p_{j,t-1}}$, we find that

$$\frac{p_{j,t}}{p_{j,t-1}} = \frac{1 - \kappa N^* + \sqrt{\Delta}}{2}$$

Note that if $\frac{Y_{j,t}}{p_{j,t-1}} = n^*$, then $\Delta = (\kappa N^* + 1)^2$, and $\frac{p_{j,t}}{p_{j,t-1}} = 1$.

A.3 Budget constraints.

Let us first rewrite the budget constraints and market clearing conditions. Using the market clearing condition, and competition $p_{j,t} = w_{j,t}$, we get

$$y_{j,t} = \alpha_j (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + f_{j,t} + g_{j,t}.$$
Nominal exports are \( f_{j,t} \), nominal imports are \((1 - \chi_j) p^{f}_{j,t} c^{f}_{j,t} + \chi_j p^{f}_{j,t} c^{d}_{j,t} \) since the government does not buy imported goods. So net exports are \( e_{j,t} = f_{j,t} - (1 - \alpha_j) (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) \). We define disposable (after-tax) income as \( \bar{y}_{j,t} = (1 - \tau_j) y_{j,t} + z_{j,t} \).

We can then write the system for nominal variables

- \( x_{b,j,t} = \frac{b^{h}_{j,t+1}}{1 + r_{j,t}} + \bar{y}_{j,t} - b^{h}_{j,t} \), budget constraint of impatient agents
- \( x_{s,j,t} = s_{j,t} + \bar{y}_{j,t} - \frac{s_{j,t+1}}{1 + r_{j,t}} \), budget constraint of patient agents
- \( y_{j,t} = \alpha_j (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + f_{j,t} + g_{j,t} \), market clearing
- \( g_{j,t} + z_{j,t} - \tau_{j,t} y_{j,t} = \frac{b^{g}_{j,t+1}}{1 + r_{j,t}} - b^{g}_{j,t} \), budget constraint of the government
- \( e_{j,t} = \frac{1}{\alpha_j} (f_{j,t} - (1 - \alpha_j) (y_{j,t} - g_{j,t})) \), definition of net exports

Combining the first four equations, we get market clearing at time \( t \):

\[
(1 - \alpha_j) \bar{y}_{j,t} = \alpha_j \chi_j \left( \frac{b^{h}_{j,t+1}}{1 + r_{j,t}} - b^{h}_{j,t} \right) + \alpha_j (1 - \chi_j) \left( s_{j,t} - \frac{s_{j,t+1}}{1 + r_{j,t}} \right) + \frac{b^{g}_{j,t+1}}{1 + r_{j,t}} - b^{g}_{j,t} + f_{j,t}. \tag{22}
\]

It will often be useful to obtain a recursive equation for nominal GDP. Taking the first difference of (22) we get

\[
(1 - \alpha_j) \Delta \bar{y}_{j,t} = \chi_j \alpha_j \left( \beta \Delta \left[ \frac{b^{h}_{j,t+1}}{1 + \rho_{j,t}} \right] - \Delta \left[ b^{h}_{j,t} \right] \right) - (1 - \chi_j) \alpha_j \left( \beta \Delta \left[ \frac{s_{j,t+1}}{1 + \rho_{j,t}} \right] - \Delta [s_{j,t}] \right) \\
+ \beta \Delta \left[ \frac{b^{g}_{j,t+1}}{1 + \rho_{j,t}} \right] - \Delta [b^{g}_{j,t}] + \Delta [f_{j,t}]
\]

### A.4 Pseudo-Steady State

We consider a steady state with constant interest rates equal to the rate of time preference of savers, i.e. \( \beta (1 + r) = 1 \) and the spread is zero: \( \rho = 0 \). The borrowing limit \( b^{h}_{j} \) is exogenous and we consider equilibria where the borrowing constraint (3) binds. Our notion of steady state is complicated by the fact that savings \( s_{j,t} \) are history-dependent. We define a steady state as the long run equilibrium of an economy with initial savings \( s_{j} \) and government debt \( b^{g}_{j} \), subject to no further shocks, constant government spending and constant government debt. All nominal quantities are constant and employment is at its natural rate \( n^* \).\(^{13}\) The long-run equilibrium conditions are

\[
\begin{align*} \\
p^{h}_{j} n^* &= \chi_j \alpha_j x_{b,j} + (1 - \chi_j) \alpha_j x_{s,j} + f + g \\
x_{b,j} &= \bar{y}_{j} - \frac{r}{1 + r} b^{h}_{j} \\
x_{s,j} &= \bar{y}_{j} + \frac{r}{1 + r} s_{j} \\
\tau_{j} p^{h}_{j} n^* &= g_{j} + z_{j} + \frac{r}{1 + r} b^{g}_{j}
\end{align*}
\]

\(^{13}\)We consider here the case where labor supply is inelastic, so \( n^* \) is effectively exogenous.
Nominal output (the price of home goods) is pinned down by

\[ p_j^h n^* = \alpha_j \left( (1 - \tau_j) p_j^h n^* + z_j \right) + \alpha_j \frac{r}{1 + r} \left( (1 - \chi_j) s_j - \chi_j \bar{b}_j \right) + f_j + g_j. \]

There are several ways to specify government policy. Here we assume that the policy is to keep government debt and nominal spending \( g_j \) constant. Long run nominal output is then given by

\[ p_j^h n^* = \frac{\alpha_j}{1 - \alpha_j} r a_j + \frac{f_j}{1 - \alpha_j} + g_j \tag{23} \]

where recall that we have defined \( a \) as net foreign assets. This equation shows the determinants of the long run price level. The long run price level depends on the exogenous components of spending: net asset income, foreign demand, and government spending. All these are inflationary. For a given tax rate \( \tau_j \), transfers are then chosen to satisfy the government’s budget constraint:

\[ z_j = \tau_j p_j^h n^* - g_j - \frac{r}{1 + r} b_j^g. \]

### A.5 Euler Equation and Expected Income

The Euler equation of savers is

\[ \frac{1}{x_{s,j,t}} = (1 + \rho_{j,t}) \mathbb{E}_t \left[ \frac{1}{x_{s,j,t+1}} \right]. \]

and we use the linear approximation

\[ \mathbb{E}_t \left[ x_{s,j,t+1} \right] = (1 + \rho_{j,t}) x_{s,j,t}. \]

Consider the following experiment. Savers enter the period with a given level of savings. Then there is a shock to interest rates. For instance, starting from a steady state where \( \rho = 0 \), if the new rate is such that \( \rho > 0 \), savings jumps up, and if the new rate is such that \( \rho < 0 \), spending jumps up. The budget constraint at time \( t \) is

\[ x_{s,j,t} = s_{j,t} + \bar{y}_{j,t} - \frac{s_{j,t+1}}{1 + \rho_{j,t}} \]

and the expected budget constraint at time \( t+1 \) is

\[ \mathbb{E}_t \left[ \frac{s_{j,t+2}}{1 + \rho_{j,t+1}} \right] = s_{j,t+1} + \mathbb{E}_t \left[ \bar{y}_{j,t+1} - x_{s,j,t+1} \right]. \]

Combining the budget constraints and the linearized Euler equation, we get

\[ (1 + \beta)s_{j,t+1} + \mathbb{E}_t \left[ \bar{y}_{j,t+1} \right] - \beta \mathbb{E}_t \left[ \frac{s_{j,t+2}}{1 + \rho_{j,t+1}} \right] = (1 + \rho_{j,t}) (s_{j,t} + \bar{y}_{j,t}). \tag{24} \]

since on average \( \beta (1 + r) = 1 \).
A.6 Equilibrium Conditions of the Model

Detrending and assuming that rates are the same within a country, we have:

- \( \frac{1}{x_{s,j,t}} = (1 + \rho_{j,t}) \mathbb{E}_t \left[ \frac{1}{x_{s,j,t+1}} \right] \), Euler equation

- \( x_{b,j,t} = \frac{b_{b,j,t+1}}{1+r_{j,t}} + \hat{y}_{j,t} - b_{b,j,t} \), budget constraint of impatient agents

- \( x_{s,j,t} = s_{j,t} + \hat{y}_{j,t} - \beta \frac{s_{j,t+1}}{1+r_{j,t}} \), budget constraint of patient agents

- \( y_{j,t} = \alpha_j (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + f_{j,t} + g_{j,t} \), market clearing

- \( g_{j,t} + z_{j,t} - \tau_{j,t} y_{j,t} = \beta \frac{g_{j,t+1}}{1+r_{j,t}} - b_{g,j,t} \), budget constraint of the government

- \( e_{j,t} = \frac{1}{\alpha_j} (f_{j,t} - (1 - \alpha_j) (y_{j,t} - g_{j,t})) \), net exports

- \( \hat{y}_{j,t} = (1 - \tau_{j,t}) y_{j,t} + z_{j,t} \), disposable income

- \( (1 - \alpha_j) \hat{y}_{j,t} = \alpha_j \chi_j \left( \beta \frac{b_{b,j,t+1}}{1+r_{j,t}} - b_{b,j,t} \right) + \alpha_j (1 - \chi_j) \left( s_{j,t} - \beta \frac{s_{j,t+1}}{1+r_{j,t}} \right) + \beta \frac{g_{j,t+1}}{1+r_{j,t}} - f_{j,t} \), market clearing

- \( n_{j,t} = \frac{y_{j,t}}{p_{j,t}} \), labor market

- \( \frac{P_{j,t-k} - P_{j,t-k}}{P_{j,t-1}} = \kappa (n_{j,t} - n^*) \), inflation

A.7 Proof of Lemma 1

Savers solve

\[
\max \sum_{t \geq 0} \beta^t \log (X_{s,j,t}) \\
X_{s,j,t} + \frac{s_{j,t+1}}{1+r_{j,t}} = S_{j,t} + \tilde{Y}_{j,t}
\]

Let us integrate forward the budget constraint:

\[
\sum_{k=0}^{K} X_{s,j,t+k} \frac{R_{j,t-k}}{R_{j,t,K+1}} + \frac{S_{j,t+K+1}}{R_{j,t,K+1}} = S_{j,t} + \sum_{k=0}^{K} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}}.
\]

where the k-period ahead discount rate for \( k \geq 1 \) from the savers’ perspective

\[ R_{j,t,k} \equiv (1 + r_{j,t}) \cdots (1 + r_{s,j,t+k-1}) \]

and the convention \( R_{j,t,0} = 1 \). The next step is to use the resource constraint

\[
(1 - \alpha_j) \tilde{Y}_{j,t} = \alpha_j \chi_j \left( \frac{B_{j,t+1}}{1+r_{j,t}} - B_{j,t} \right) - \alpha_j (1 - \chi_j) \left( \frac{s_{j,t+1}}{1+r_{j,t}} - S_{j,t} \right) + F_{j,t} + \frac{B_{g,j,t+1}}{1+r_{j,t}} - B_{g,j,t}
\]

Summing and rearranging the terms, we get
\[(1 - \alpha_j) \left( \tilde{Y}_{j,t} + \frac{\tilde{Y}_{j,t+1}}{R_{j,t,1}} \right) = \alpha_j \chi_j \left( \frac{B_{j,t+2}^h}{R_{j,t,1} (1 + r_{j,t+1})} - B_{j,t}^h \right) - \alpha_j (1 - \chi_j) \left( -S_{j,t} + \frac{1}{R_{j,t,1} (1 + r_{j,t+1})} S_{j,t+2} \right) + F_{j,t} + \frac{F_{j,t+1}}{R_{j,t,1}} + \frac{1}{R_{j,t,1} (1 + r_{j,t+1})} - B_{j,t}^q \]

to write:

\[(1 - \alpha_j) \left( \tilde{Y}_{j,t} + \frac{\tilde{Y}_{j,t+1}}{R_{j,t,1}} + \frac{\tilde{Y}_{j,t+2}}{R_{j,t,2}} \right) = -\alpha_j \chi_j \left( B_{j,t}^h - \frac{1}{R_{j,t,2} (1 + r_{j,t+2})} B_{j,t+3}^h \right) + \alpha_j (1 - \chi_j) \left( S_{j,t} - \frac{S_{j,t+3}}{R_{j,t,3}} \right) + F_{j,t} + \frac{F_{j,t+1}}{R_{j,t,1}} + \frac{F_{j,t+2}}{R_{j,t,2}} - B_{j,t}^q + \frac{1}{R_{j,t,2} (1 + r_{j,t+2})} \]

Therefore for a generic horizon \(K\)

\[\sum_{k=0}^{K} \frac{(1 - \alpha_j) \tilde{Y}_{j,t+k}}{R_{j,t,k-1}} = \alpha_j \left( (1 - \chi_j) S_{j,t} - \chi_j B_{j,t}^h \right) - B_{j,t}^q + \sum_{k=0}^{K} \frac{F_{j,t+k}}{R_{j,t,k}} \]

\[-(1 - \chi_j) \alpha_j S_{j,t+K+1} + \frac{1}{R_{j,t,K}} \left( \frac{\alpha_j \chi_j B_{j,t+K+1}^h}{1 + r_{j,t+K}} + \frac{B_{j,t+K+1}^q}{1 + r_{j,t+K}} \right)\]

We take the limit and we impose a No-Ponzi condition

\[\lim_{K \to \infty} \mathbb{E}_t \left[ \frac{S_{j,t+K+1}}{R_{j,t,K+1}} \right] = 0 \]

\[\lim_{K \to \infty} \mathbb{E}_t \left[ \frac{B_{j,t+K+1}^h}{R_{j,t,K+1} (1 + r_{j,t+K})} \right] = 0 \]

\[\lim_{K \to \infty} \mathbb{E}_t \left[ \frac{B_{j,t+K+1}^q}{R_{j,t,K+1} (1 + r_{j,t+K})} \right] = 0 \]

The inter-temporal current account condition is

\[(1 - \alpha_j) \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{Y}_{j,t+k}}{R_{j,t,k}} = \alpha_j (1 - \chi_j) S_{j,t} - \alpha_j \chi_j B_{j,t}^h - B_{j,t}^q + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{j,t+k}}{R_{j,t,k}} \]
The inter-temporal budget constraint is then

\[(1 - \alpha_j) \mathbb{E}_t \sum_{k=0}^{\infty} \frac{X_{s,j,t+k}}{R_{j,t,k}} = (1 - \alpha_j \chi_j) S_{j,t} - \chi_j \alpha_j B_{j,t}^h - B_{j,t}^g + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{F_{j,t+k}}{R_{j,t,k}}\]

The net present value of savers’ spending depends on beginning of period net foreign assets \((1 - \alpha_j \chi_j) S_{j,t} - \chi_j \alpha_j B_{j,t}^h - B_{j,t}^g\) and the net present value of exports.

### A.8 Impulse Response Function

To compute the impulse response functions of the model we need to make an assumption about the path of household debt and fiscal policy

#### Assumptions A2.

- **Shocks on borrowing constraints** are such that \(\mathbb{E}_t [b_{j,t+2}^h] = b_{j,t+1}^h\),
- **Fiscal policy** is such that \(\mathbb{E}_t [b_{j,t+2}^g] = b_{j,t+1}^g\) and the tax rate \(\tau_j\) is constant.
- **The variance of interest rates and foreign demand is small**, and \(\beta_b\) is small enough that \(b_{j,t} = b_{j,t}^h\) at all times;

Assumptions A2 is only necessary to solve for the entire path of macroeconomic variables shown in the impulse response functions. The first condition says that shocks on borrowing are permanent. The second defines a class of fiscal policies. The last point is purely technical. It allows us to linearize Euler equations. We assume that the shocks are small enough that impatient households find it optimal to borrow up to the constraint (this is a joint restriction on the discount factor and the size of the shocks). Note that A2 is not necessary for the reduced form and structural simulations.

We now look for decision rules for the savers \(\{s_{j,t}\}_{t=1,2,...}\) and the other variables of the model, \(y_{j,t}, p_{j,t}^h, \) etc. We get the following Lemma:

**Lemma 2.** Under A1 and A2, savings dynamics satisfy \(\mathbb{E}_t [s_{j,t+2}] = s_{j,t+1}\).

**Proof.** With assumptions A1 (\(\mathbb{E}_t [f_{j,t+1}] = f_{j,t} \) and \(\mathbb{E}_t [\rho_{j,t+1}] = 0\)), we guess and verify that the Lemma is correct. Suppose the Lemma is true, then we obtain two important equations. Equation (24) becomes

\[s_{j,t+1} = (1 + \rho_{j,t}) \left( s_{j,t} + \tilde{y}_{j,t} \right) - \mathbb{E}_t \left[ \tilde{y}_{j,t+1} \right], \tag{25}\]

and expected market clearing at \(t + 1\) is

\[(1 - \alpha_j) \mathbb{E}_t [\tilde{y}_{j,t+1}] = (1 - \beta) \left( \alpha_j (1 - \chi_j) s_{j,t+1} - \alpha_j \chi_j b_{j,t+1}^h - b_{j,t+1}^g \right) + f_{j,t}. \tag{26}\]

Therefore

\[(1 - \alpha_j) s_{j,t+1} = (1 - \alpha_j) (1 + \rho_{j,t}) (s_{j,t} + \tilde{y}_{j,t}) - (1 - \beta) \left( \alpha_j (1 - \chi_j) s_{j,t+1} - \alpha_j \chi_j b_{j,t+1}^h - b_{j,t+1}^g \right) - \frac{f_{j,t}}{1 - \alpha_j}. \]

Using market clearing at time \(t\) we get

\[
\frac{s_{j,t+1} - s_{j,t}}{1 + \rho_{j,t}} = \frac{\alpha_j \chi_j}{1 - \alpha_j \chi_j} \left( \frac{b_{j,t+1}^h}{1 + \rho_{j,t}} - b_{j,t}^h \right) + \frac{1}{1 - \alpha_j \chi_j} \left( \frac{b_{j,t+1}^g}{1 + \rho_{j,t}} - b_{j,t}^g + \frac{\rho_{j,t}}{1 + \rho_{j,t} - \alpha_j} \right) \tag{27}\]
We are interested in simple rules that deliver the property that all economic data for Eurozone countries (employment, population, GDP, consumption, government debt, expenditures...) from 2000 to 2012. Austria, Germany, Italy, Spain, France, Sweden, the Netherlands, Greece, Portugal, Ireland, and Luxembourg are included. All countries are assumed to have small shocks and to maintain fiscal stability. We assume that modern fiscal systems in all OECD countries (see Fatas and Mihov (2012)). We assume that spending and transfers are predetermined. The state space of predetermined endogenous variables is \((s_{j,t+1}, b_{j,t})\). We need to specify the policy function of the government in order to compute the impulse responses. We are interested in simple rules that deliver the property that the public debt follows a random walk: \(\mathbb{E}_t [b_{j,t+2}^g] = b_{j,t+1}^g\) and deliver some automatic stabilization, characteristic of modern fiscal systems in all OECD countries (see Fatas and Mihov (2012)). We assume that spending and transfers are predetermined. From the government budget constraint, this means that a recession at time \(t\) automatically increases government debt at time \(t\). To maintain fiscal stability, we specify that transfers adjust from \(t\) to \(t+1\) to keep public debt constant thereafter: \(\mathbb{E}_t [b_{j,t+2}^g] = b_{j,t+1}^g\). More precisely, we specify the general policy rule as follows

1. Fiscal variables \(p_{j,t}^h, g_{j,t}\) and \(s_{j,t}\) are pre-determined. Government debt \(b_{j,t}^g\) is determined in equilibrium at time \(t\).
2. Set transfers \(z_{j,t+1}\) for next period so that \(\mathbb{E}_t [b_{j,t+2}^g] = b_{j,t+1}^g\) assuming a martingale for \(p_{j,t}^h, g_{j,t}\).
   
   (a) The expected budget constraint is
   
   \[\tau_j \mathbb{E}_t [y_{j,t+1}] = g_{j,t} + z_{j,t+1} + \frac{r}{1+r} b_{j,t+1}^g\]

   (b) By definition we have
   
   \[\mathbb{E}_t [\tilde{y}_{j,t+1}] = (1 - \tau_j) \mathbb{E}_t [y_{j,t+1}] + z_{j,t+1}\]

   (c) Therefore (recall that \(\mathbb{E}_t [\tilde{y}_{j,t+1}]\) is part of our solution at time \(t\)), \(z_{j,t+1}\) is given by
   
   \[z_{j,t+1} = \tau_j \mathbb{E}_t [\tilde{y}_{j,t+1}] - (1 - \tau_j) g_{j,t} - (1 - \tau_j) \frac{r}{1+r} b_{j,t+1}^g\]

B Data & Calibration

B.1 Data sources

B.1.1 Europe

All economic data for Eurozone countries (employment, population, GDP, consumption, government debt, expenditures...) comes from Eurostat. We use data for 11 eurozone countries from 2000 to 2012. Austria,
Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal. We excluded Luxembourg for which household debt data is available only starting in 2005 and other countries that joined in 2007 and later.

The data on household debt comes from the BIS which itself compiled the data from national central banks. This is debt of household and non-profit institutions serving households. Credit covers all loans and debt securities and comes from both domestic and foreign lenders. The series have quarterly frequency and capture the outstanding amount of credit at the end of the reference quarter.

We call government expenditures total government expenditures net of transfers, interest payments and bank recapitalization. The data on spending on bank recapitalization comes from Eurostat. It includes interest payable, capital injections recorded as deficit-increasing (capital transfer) and calls on guarantees and is net of revenues generated by bank recapitalization (guarantee fees, interest and dividends). Transfers is the addition of direct social benefits and of social transfers in kind.

The data on interest rates (10 year government bonds) come from the ECB. The spread is defined as the difference of the 10 year interest rate on government bonds with the median of the euro zone.

For exports we measure the domestic value added that is associated with final consumption in the rest of the world, which corresponds to value added based exports. We use the data from the OECD-WTO Trade in Value-Added (TIVA) initiative to measure domestic value added embodied in gross exports. Data is available only in 2000, 2005, 2008 and 2009. For missing years, we use the ratio of gross exports (from Eurostat) to value added gross exports of the nearest year and multiply this ratio by the gross exports of the missing year to obtain an approximation of value added exports of the missing years.

We use annual averages of 10 year government bond rates as long term rates. The source is OECD.

B.1.2 United States

Data for the United States comes from the BEA, the Flow of Funds (FoF), and from the FRBNY Consumer Credit Panel. BEA and FoF data are standard and widely used so we do not discuss them.

The FRBNY Consumer Credit Panel is described in Lee and van der Klaauw (2012). It is a new longitudinal database with detailed information on consumer debt and credit. This panel is a random sample from consumer credit reports. It is available from 1999 onwards. Credit reporting agencies compile and maintain credit histories for all U.S. residents who have applied for or taken out a loan. Credit bureaus continuously collect information on individual consumers’ debt and credit from lenders and creditors. Most individuals begin building a credit history when they first obtain and use a credit or retail card or take out a student loan, usually when they are at least 18 years of age. New immigrants with little or no credit history from their home country are often older when a credit file is first created for them. The sample design implies that the target population consists of all US residents with a credit history. In addition to most individuals younger than 18, who had little need or opportunity for credit activity, the target population excludes individuals who have never applied for or qualified for a loan.

The data at the State level is available in three data sets on the FRBNY web site:

- State level data for all States from 1999 to 2012, annual data for Q4 only.
- Selected states from 1999 to 2003, quarterly data.
- Selected states from 2003 to 2014, quarterly data.

Lee and van der Klaauw (2012) argue that household debt estimates based on the FRBNY Consumer Credit Panel are similar to estimates reported in the Board of Governors’ Flow of Funds Accounts. There are differences, however. First, the household debt measures in the Flow of Funds are not based on direct data but instead are derived as residual amounts. Total mortgage debt and non-mortgage debt in the second quarter of 2010 were respectively $9.4 and $2.3 trillion, the comparable amounts in the FoF for the same quarter were $10.2 and $2.4 trillion, respectively.

Second, the FoF measure of household mortgage debt includes some mortgage debt held by nonprofit organizations (churches, universities, etc.). On the other hand, FRBNY estimates exclude some debt held by individuals without social security numbers. There may also be differences in the speed at which changes in various types of debt are recorded, where new mortgage accounts usually appear on credit reports with some delay, making some direct comparisons difficult. The comparison is shown in figure (32).

B.2 Scaled data

In order to map the observed data into the model we scale the data in a manner consistent with equation (13) in the model. We construct a benchmark level of nominal GDP for each country and year: it is the nominal
GDP the country would have if it had the same per-capita growth rate as the whole eurozone aggregate expenditure level and its actual population growth. Define

- $Y_{j,t}$: nominal GDP in euros of country $j$,
- $X_t^*$: aggregate expenditures for the Eurozone
- $N_{j,t}$: population of country $j$, and $\bar{N}_t$ for the Eurozone
- Benchmark nominal GDP for country $j$ at time $t$:

$$\hat{Y}_{j,t} \equiv \frac{Y_{j,0}X_t^*}{\bar{X}_0} \frac{N_{j,t}/\bar{N}_t}{N_{j,0}/\bar{N}_0}$$

We have experimented with two definitions of $\bar{Y}_t/Y_0$: one is actual nominal GDP, the other is the trend nominal growth before and after the crisis (starting in 2008 this benchmark growth rate is the average of the eurozone crisis, around 1%). Then we define the scaled nominal GDP as

$$y_{j,t} \equiv \frac{Y_{j,t}}{\hat{Y}_{j,t}}$$

We compute scaled series for private and public spending by scaling by the benchmark nominal GDP. For unit labor costs, we scale by the average unit labor cost in the eurozone. For employment per capita, we take the deviation with respect to 2002 with an index of 1 for that year. We define the scaled level as the ratio of debt to the benchmark levels of nominal GDP:

$$b_{j,t}^g \equiv \frac{B_{j,t}^g}{\hat{Y}_{j,t}}.$$

Finally, when we map to our model, we must remember that we define $b_{j,t}$ as per capita debt, so in fact we have

$$\chi_j b_{j,t} = \frac{B_{j,t}}{Y_{j,t}}.$$
The scaled data for the reduced form shocks for household and public debt, government expenditures and transfers, spreads and foreign demand are given in figures (33), (34), (36) and (35).

Figure 33: Household and public debt
Figure 34: Government expenditures and transfers

Graphs by country

Figure 35: Value added based exports

Graphs by country
Figure 36: Spreads (10 year government bonds)

Graphs by country

B.3 Simulation of the reduced form and structural models

To simulate the reduced form model and compare model outputs to realized data, we use as inputs to the model annual data on household debt, government spending and transfers, government debt\(^{14}\) and interest rates for the simulation period 2001-2012. The level of output, the price level and the level of net foreign assets is set to their 2002 levels in the data. Foreign demand is set using data on exports in value added terms for 2001-2012, normalized so that the level of foreign demand in 2002 satisfies goods market clearing in the model. Finally, taxes are set so that the path for government debt implied by the model coincides exactly with the data.

The structural model is simulated in the same way but the exogenous variables are replaced by structural equations (17) and (18) for private debt and spreads respectively and the fiscal policy rule (19). The tax rate is constant at its 2002 level and equal to total government revenues as a percentage of GDP. The structural shocks are now \(\delta_{j,t}, \sigma_t\) and \(\tilde{g}_{j,t}\).

\(^{14}\)Instead of using government debt directly from data, we construct a simulated government debt series in order to avoid including factors that affect government debt in the data but are not in the model, such as bank recapitalizations, default, revenues from privatizations, etc. The simulated debt series is constructed by adding to \(t - 1\) period debt government expenditures including interest payments and subtracting tax revenues.