SUSTAINABILITY OF PENSION SCHEMES
Building a smooth automatic balance mechanism with an application to the US Social Security[1]

Frédéric Gannon
Université Le Havre, Sciences Po – OFCE
Florence Legros
ICN, Business School
Vincent Touzé
Sciences Po, OFCE

We build a "smooth" automatic balancing mecanism (S-ABM) which would result from an optimal tradeoff between increasing the receipts and reducing the expenditures of a pension scheme. The S-ABM obtains from minimizing a sum of discounted quadratic loss function under the constraint of an intertemporal budget balance. One advantage of this model of "optimal" adjustment is its ability to analyse various configurations in terms of ABMs by controlling the adjustment pace. Notably, this S-ABM permits to specify two particular cases we respectively denote "flat Swedish-type ABM" and "fiscal-cliff US- type ABM". They are obtained by assuming very high adjustment costs on revenue (implying only pension benefit adjustment) and by choosing specific sequences of social time preference rate. We apply this ABM to the case of the United States Social Security to evaluate the potential adjustments necessary to ensure financial sustainability. These assessments are made under various assumptions about forecast time horizon, social time preference and weighting of social costs associated with increased receipts and/or lower expenditures.

Keywords: Pension scheme, financial sustainability, automatic balance mechanisms, dynamic programming.

1. Without implication for any responsibility, we thank Maria del Carmen Boado-Penas, Pierre Devolder, Gilles Le Garrec and the referees for stimulating suggestions.

Most governments are reluctant to reform their pension systems for fear this might induce too high a political cost. In fact, the political debate about the pension issues may often be a source of conflicts (Blanchet and Legros, 2002; Marier, 2008; Weaver and Willén, 2014; Wisensale, 2013). As a consequence, governments tend to procrastinate and to postpone the adoption of measures that would guarantee solvency. Of course, faced with an effective insolvency of their pension systems, they all have introduced reforms – some of them drastic – but without imposing restoring forces. The problem with *ad hoc* reforms is that, quoting Turner (2009), "(they) have a high degree of political risk because their timing and magnitude are unknown".

To avoid pension systems to depend upon choices that politicians would not take willingly, governments can introduce specific and mandatory rules to allow for automatic adjustment mechanisms (AAMs). These AAMs contribute to improve solvency at any date without politicians stepping in, thus avoiding the "need for large program changes made in crisis mode" (Turner, 2009). Implementing AAMs requires not only straightforward and clear choices about transfers between generations, but also strong social acceptance. Automatic Balance Mechanisms (ABMs) may be viewed as stronger and efficient AAMs securing long-run solvency. Several countries (Sweden, Canada, Germany and Japan) have set up different country-specific ABMs (Vidal-Meliá *et al.*, 2009; Boado-Penas and Vidal-Meliá, 2012).

In this paper, we develop a general model of Automatic Balance Mechanism based on the intertemporal minimization of a discounted quadratic loss function. Building an ABM requires defining a measure of the intertemporal budget balance, to choose the time horizon and to adopt a criteria to be optimized. Our "smooth" ABM (hereafter denoted S-ABM) relies on the use of distortion indices, which makes it easy to be implemented in a realistic and practical prospect. Smooth, gradual adjustments replace immediate and abrupt changes, enhancing their short-term political acceptance.

The paper organizes as follows. First, we address the issue of AAMs: what part do they play in adjusting, stabilizing and balancing? Second, we build a "smooth" ABM, assuming a trade-off between present and future receipts and expenditures. Third, we apply this ABM to the U.S. Social Security. The last section concludes.

1. Automatic rules: adjusting, stabilizing and balancing

1.1. The intertemporal pension budget constraint

At the current period ($t = 0$), the forecast expenditures and receipts at time t are respectively denoted EXP_t and REC_t. Assuming negligible administrative costs, EXP_t can be computed as follows:

$$EXP_t = E_0\left(\sum_{j\in\Omega_t^R} p_{j,t}\right) \tag{1}$$

where Z_t^R is the set of retirees for period t and $p_{j,t}$ is the pension paid to each retiree j. REC_t is given by:

$$REC_t = E_0\left(\sum_{k\in\Omega_t^E} \tau_t \times w_{k,t}\right) \tag{2}$$

where Z_t^E is the set of employees at period t, $w_{k,t}$ is the annual sum of monthly taxable wages paid to each employee k and w_t is the payroll tax rate for period t. E_0 denotes the best-estimate operator to forecast future expenditures and receipts.

The intertemporal budget balance of the pension system writes:

$$R_t \,.\, F_{t-1} + REC_t = EXP_t + F_t \tag{3}$$

where $R_t = (1 + r_t)$ is the riskless interest factor with r_t the risk-free interest rate[2] and F_t the value of the financial asset (reserve fund) at the end of period t. Difference $REC_t - EXP_t$ is the primary balance.

What about the financial equilibrium? From an accounting point of view, the ability to pay all promised pensions can be estimated through different methods. Two approaches can be considered.

The first one is an evaluation of the discounted sum of receipts and expenditures for a given time-horizon. This approach is adopted in the United States to assess the present value of the system's underfunding, called "Unfunded Obligation", which is an estimation of the financial sustainability. The US Social Security Administration defines it as: "the excess of the present value of the projected cost of the program through a specified date over the sum of: (1) the value of trust fund reserves at the beginning of the valuation period; and (2) the present value of the projected non-interest income of the program through a specified date, assuming scheduled tax rates and benefit levels". At the

2. We opt here for a certain equivalent (CE) approach to evacuate the explicit consideration of financial risk.

current period, the value of the Unfunded Obligation, denoted UO_0 computes as follows:

$$UO_0 = \sum_{t=1}^{T} \frac{EXP_t - REC_t}{\prod_{i=1}^{t} R_i} - F_0 \tag{4}$$

Sweden has opted for another method: the asset-liability approach (Settergren, 2001). It defines its pension plan as solvent when: contribution assets (computed as the present value of the contributions due to be paid by the current workers) + Value of the reserve fund = Value of pension commitments towards current generations (living pensioners and workers). Hence, the Swedish actuarial balance sheet ratio writes as an asset/liability ratio which gives a measure of solvency i.e the ability of the system to fulfill current commitments. It is important to stress that if the US unfunded obligations are positive, no automatic balance mechanism is legislated – although Board of Trustees annual reports calculate the increase of the contribution rate necessary to guarantee financial sustainability.

Solvency issues have been investigated by Vidal-Meliá and Boado-Penas (2013). They specify the connection between the contribution asset and the hidden asset (similar to the equivalent concepts of "implicit tax on pensions" or "PAYG asset" used in the literature) to evaluate whether using either of these to compile the actuarial balance in PAYG pension systems would provide a reliable solvency indicator. The contribution asset can be interpreted as the maximum level of liabilities that can be financed by the existing contribution rate without periodic supplements from the sponsor, *ceteris paribus*.

The tax gap ratio is another interesting concept and a straightforward evaluation of pension scheme's insolvency. It can be measured in two ways: the excess of net-of-reserve expenditures (sum of present values) with respect to receipts (sum of present values) or the excess of expenditures with respect to net-of-reserve receipts. These ratios can be interpreted as implicit debt/notional asset ratios. An interesting illustration of two polar balancing adjustments can be computed from these tax gap ratios: a full adjustment operated through either receipts or expenditures.[3]

3. Gannon *et al.* (2014) extend the tax gap approach to mixed adjustments with a possible trade-off between full adjustment by tax or by pension.

1.2. How the standard Automatic Adjustment Mechanisms (AAMs) contribute to stabilizing pension schemes

The general problem social planners or governments are facing is how to adjust parameters (payroll tax rate, retirement age, pension benefit calculus, pension index, etc.) with time. Adopting automatic adjusment rules implies choosing a law of motion for parameters defined as a function of economic, financial or demographic variables.

With the Automatic Adjustment Mechanisms (AAMs), the institutional parameters are adjusted according to predefined rules. Otherwise, the changes are considered as discretionary decisions and are likely to depend on the hazards of political choices.

Choosing a specific AAM requires the following actions to be taken (see Bosworth and Weaver, 2011): legitimate the adjustment rule (equity, social justice or solvency), stick to "one tool – one objective" rule, set the frequency of review/assessment, define the elements on which the adjustment is made and fix the degree of automaticity (up to which level the adjustment is mandatory – no questioning –, which warrants credibility to the process).

To control for pension level, four main parameters are available :

(1) Benefit index: its main objective is to maintain the level of pension purchasing power.

(2) Contributory period: eligibility for full pension requires to validate a sufficient number of quarters. The contributory period can be connected to life expectancy.

(3) Retirement age: with a given frequency, this age could be revised with new informations about each cohort's changes in life expectancy.

(4) Pension-earnings links: index rule of past contributions – purchase value of the point in a point DC[4] pension scheme or return rate of "savings" in a NDC[5] pension scheme – or past wages (defined benefit), link between the amount of the pension (net replacement rate in a DB scheme or rent value of the point in a point DC scheme or accumulated-contributions-to-rent coefficient in a NDC scheme) and life expectancy at retirement, etc.

4. Defined contribution (DC).
5. Notional defined contribution (NDC).

As to the pension-earnings link, it can be established according to two approaches: defined contribution (DC) or defined benefit (DB). In a DC pension scheme, as in Sweden, the coefficient of conversion of capital into an annuity can depend on age and birth year. This coefficient can be revised to reflect the evolution of generation mortality tables and life expectancy (Life Expectancy Index). In the case of DB pension scheme (for exemple, as in the US or France), a replacement rate is used to convert average life-cycle wage into a pension. To control this replacement rate, the main adjusment parameter is the number of years contributed to be validated to be eligible for full pension (maximal value of the replacement rate). Additionally, the legislator can reward (bonus) long careers or penalize (malus) short ones. The index used to give a current value to the past wages in a DB pension scheme or the value of the notional accounts in a DC pension scheme plays a role in the link between wages and pensions. According to Settergren (2001), indexing notional pension savings on economic growth is stabilizing, since pensions "will grow (decline) at the same pace as average earnings".

The parametric changes induced by the AAMs can be determined either ex ante or ex post.

In the former case, demo-economic shocks are anticipated and parametric changes in law are planned. For example, as early as 1983, U.S. government launched a clear-cut long-run ex ante adjustment device by progressively increasing the payroll taxes and raising the full pension age. This reform prevented a pending Social Security crisis; moreover, it still potentially guarantees an intertemporal balanced budget for about half a century. Nethertheless, as stressed by Aaron (2011), the weakness of this reform was that it "virtually guaranteed the return of deficits and a funding gap, and the need for further legislation to close it".

In the latter case (ex post adjustments), the parametric values set by national legislation evolve with the knowledge of the states of nature. Changes alter pension formula parameters and contribution rate. Sweden is considered as a major pioneer in adopting automatic stabilizing devices relying on Notional Defined Contributions (NDC) plans in 1994.

1.3. Towards stronger AAMs: Automatic Balance Mechanisms (ABMs)

What happens if the adjusments brought by the standard AAMs do not lead to enough stability? One solution could consist in adopting a clear obligation of financial sustainability in a given time horizon: this is precisely what Automatic Balance Mechanisms (ABMs) are devised for. The choice of an ABM raises four major questions:

— How pension budget balance is defined?

— What are the criteria for choosing changes in current pension law?

— What room is left for optimization?

— What revising frequency and what planning time horizon for full balancing?

At each period of revision, the ideal pension scheme's timing ought to be:

— First step (standard AAMs): setting the values of the pension parameters;

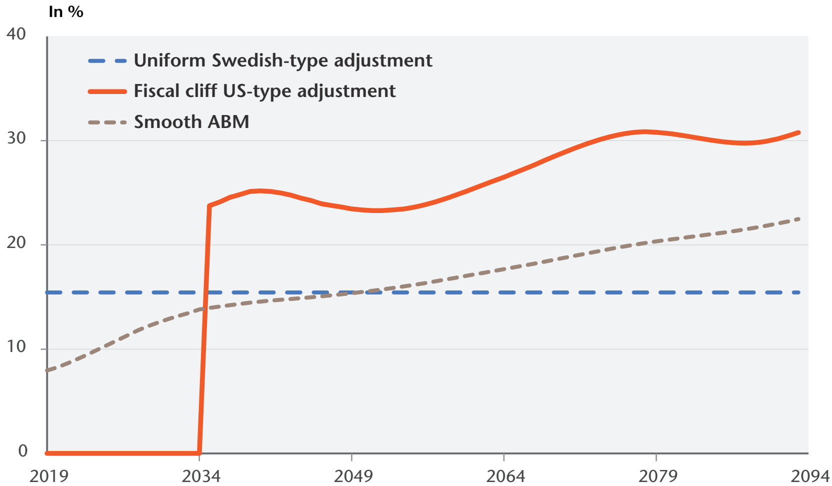
— Second step (intertemporal sustainability): checking the solvency of the pension schemes;

— Third step (ABM): triggering adjustments by resetting targeted parameters.

For example, to reinforce the solvency robustness of its pension system, Sweden set up an ABM in 2001: a uniform and permanent adjustment of present and future pension benefits given the "balance ratio" secures solvency (Settergren, 2001). The return of the "savings" invested in the NDC crucially depends upon this indexing (Settergren and Mikula, 2005).

Other countries (Sakamoto, 2013) have followed Sweden by adding specific indexing of pensions to strengthen solvency. Japan and Germany adopted ABMs based on a demographic but not financial criterion. In 2004, Japan opted for an automatic adjustment of benefit levels to changes in demographic structures measured by the decreasing rate of the number of workers and the increasing rate of life expectancy at 65 (Sakamoto, 2005; Kashiwase *et al.*, 2012). In Germany, the 2001 Riester introduced a new adjustment formula to index pensions which depends on the dependency ratio (Börsch-Supan and Wilke, 2004). In contrast, Canada has opted for a more binding ABM with an obligation to satisfy a financial sustainability criterion for its second pillar pension schemes[6] (Ménard and Billig, 2013; Gannon *et al.*, 2018).

Figure 1. US Social Security (OASDI): expected adjustments by a pension reduction from 2019 to 2093



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario)

In the U.S. Social Security, *a contrario,* there is no ABM. However, it must be reminded that the U.S. Social Security trust funds are not allowed to borrow (Diamond, 2018). This financial and legal constraint is a strong incentive to plan surpluses to compensate anticipated deficits, acting as a credible restoring force. The 75-year annual forecast of Board of trustees (2019) allows a thorough analysis of solvency.[7] Notably, it gives prudential recommandations[8] and an estimation of the year when the system reaches bankruptcy: 2034. After this critical year, if no corrective governmental measures have been taken, the so-called "fiscal cliff" adjustment – obligation to reduce pensions to achieve a financial balance between pension payments and social contributions – is automatic and brutal. This prospect is supposed to raise political awareness and to induce preventive corrective measures.

6. The second pillar is made up of two mandatory partially funded plans: Canadian Pension Plan (CPP) and Quebec Pension Plan.
7. The annual report considers three alternative scenarios. The "intermediate assumptions" reproduce a central scenario reflecting the "best estimates of future experience". The low-cost and high-cost scenario offer alternative future experiences respectively more pessimistic and more optimistic.

Figure 1 compares these two contrasting balancing adjustments based on the 75 years' Social Security administration forecast published in April 2019: the "uniform Swedish-type adjustment" vs. the "fiscal cliff US-type adjustment." Here, we call "Swedish-type adjusment" a permanent and constant reduction of pension[9] which garantees the financial sustainability (UO_0). Figure 1 underlines the difficulty for lawmakers to restore solvency. The Swedish-type adjusment implies an immediate 15.5% pension decrease whereas a strong procrastination results in a fiscal cliff adjustment jump with a 23.7% pension decrease in 2035 and a 30% decrease in 2089.

None of these two potential adjusments is realistic both from a social and political point of view. However, the perspective of the fiscal cliff is a credible threat to adopting a progressive adjustment. Hence the idea to devise a general framework for smooth automatic balance mechanisms.

The same figure 1 presents a simulation based on the model we develop hereafter, assuming a single pension adjustment. This result obtains from an intertemporal tradeoff aiming at smoothing the whole adjustment process. As a matter of fact, integrating a social preference for the present tends to reduce high initial adjustments. Supposing a public choice of a 1.5% annual rate of social preference for the present, the adjustment would require a 8% initial decline in 2019 and about 22.5% reduction in the long run (2093). If adjusting only by reducing pensions seems too rough, adjusting by increasing payroll taxes should also be considered.

8. In the 2013 report, the prudential objective of the Board of Trustees was warranting a minimal reserve fund to smooth out the adjustments:
(i) "The Trustees consider the trust funds to be fully solvent if the funds can pay scheduled benefits in full on a timely basis. A standard method of assessing solvency is the 'trust fund ratio', which is the reserves in a fund at the beginning of a year (which do not include advance tax transfers) expressed as a percentage of the cost during the year. The trust fund ratio represents the proportion of a year's cost which the reserves available at the beginning of that year can cover. The Trustees assume that a trust fund ratio of 100 percent of annual program cost provides a reasonable 'contingency reserve'";
(ii) "Maintaining a reasonable contingency reserve is important because the trust funds do not have borrowing authority. After reserves are depleted, the trust funds would be unable to pay benefits in full on a timely basis if annual revenue were less than annual cost. Unexpected events, such as severe economic recessions or large changes in other trends, can quickly deplete reserves. In such cases, a reasonable contingency reserve can maintain the ability to pay scheduled benefits while giving lawmakers time to address possible changes to the program."
9. In practice, the Swedish ABM is based on the actuarial balance sheet ratio and is computed every year. Here, our computation is not based on asset/liabilities ratio but on a tax gap ratio consisting in comparing the present values of expenditures to receipts net of reserve fund.

2. In search of a smooth ABM (S-ABM)

2.1. Minimizing a discounted quadratic loss function

Let us turn now to the building of a simple model based on intertemporal optimization called “smooth automatic balance mechanism” (S-ABM).

Haberman and Zimbidis (2002) were the first to use optimal control techniques to deal with the issue of pay-as-you-go financing. They consider a discrete time stochastic model with a quadratic loss function and two adjusment parameters (contribution rate and retirement age). Inserting the duration of activity is interesting. However, it could be more tractable, instead, to adjust with a standard AAM that would simply seek to satisfy an objective of actuarial fairness (for example, a matching of retirement and activity durations). There is no explicit social time preference rate (implicity supposed to be equal to zero), which discards the possibility to monitor the adjustment lag.

A similar approach applied to retirement has been adopted by Berger and Lavigne (2007). The adjustment they propose relates solely to the contribution rate.

Pantelous and Zimbidis (2008) enrich the approach of Haberman and Zimbidis (2002) by proposing to estimate an optimal control model based on several parameters: the contribution and replacement rates, the duration of activity and different investment strategies. Godínez-Olivares *et al.* (2015) develop similar approaches by minimizing a logarithmic loss function.

Godínez-Olivares *et al.* (2016) and Boado-Penas *et al.* (2020) explore another approach by minimising a sustainability indicator (subject to be higher than 0) and calculate the optimal path of contribution rate, retirement age and pension indexing.

In our approach, the ABM is the “ultimate” AAM. In contrast, the dynamic optimization problem we tackle relies only on two adjustment modes, respectively through costs and receipts by using respectely a contingent pension indexing (on behalf of solvency) and a possible adjustment of the contribution rate. We do not consider retirement age as a possible adjustment variable taking into account by the ABM because it is assumed to be managed by a specific AAM. Moreover, it accounts for time preference which permits to design the pace of adjustment and to control the magnitude of the initial correction.

The objective function is defined as a quadratic loss function. This analytical approach expresses in a straightforward and simple way the idea of "smoothing out" the changes in the current legislation.

For sake of simplicity, we present a non stochastic approach of our ABM. Our computations are based upon given forecast values of receipts (REC_t) and expenditures (EXP_t). Also, as the first order conditions are linear, the estimated adjustment variables could be considered as forecast values for the current period. In practice, these variables should be revised as the observed values and forecasts would adjust with time. The forecast uncertainty could be considered by using stochastic simulations. For example, Fujisawa and Li (2012) examine how the Japanese automatic balancing mechanism will affect the income of the extreme elderly (people who live beyond 100).

The value of the loss associated to each period is measured by:

$$LF(A_t, B_t) = \alpha \cdot (A_t - 1)^2 + (1 - \alpha) \cdot (B_t - 1)^2 \qquad (5)$$

where A_t and B_t are two deformation coefficients modifying respectively the future payroll tax rates (receipts) and pension benefits (expenditures) relatively to those established by the current pension law. Note that the coefficient B_t can be interpreted as a pension index which is, de facto, a component of the pension rule. α (respectively $(1 - \alpha)$) is the social weight given to the adjustment through receipts (respectively expenditures). $(A_t - 1)$ and $(B_t - 1)$ measure the relative gap with respect to the current legislation. This loss function captures the fact that changing parameters is costly (both socially and politically) and that, by minimizing it, the social planner seeks to limit the amplitude of changes.[10] To achieve this goal, the social planner sets a time horizon T to match discounted receipts with discounted expenditures:

$$\sum_{t=1}^{T} \frac{A_t \cdot REC_t}{\prod_{i=1}^{t} R_i} + F_0 = \sum_{t=1}^{T} \frac{B_t \cdot EXP_t}{\prod_{i=1}^{t} R_i} \qquad (6)$$

10. This choice is rather standard but it penalizes symmetrically good news and bad news. For instance, in case of negative unfunded obligation, the reduction of contributions induces the same cost as an increase in the contribution when the unfunded obligation is positive. This property does not matter here because we study only unbalanced pension schemes.

The optimizing program is based on a sum of discounted losses during T periods:[11]

$$\begin{cases} \min\limits_{\{(A_t,B_t)\}_{t=1,\dots,T}} \sum_{t=1}^{T} \frac{1}{\prod_{i=1}^{t}(1+\delta_i)} LF(A_t,B_t) \\ s.t.\ (6) \end{cases}$$

where δ_t is the social rate of time preference in period t.

Our approach only requires a public choice of three social parameters (α, δ_t and T) and a long-term forecast of expenditures and receipts. It can be applied to any initial structure of pension plans: defined benefit, defined contribution or hybrid (Alonso-Garcia *et al.*, 2018; Devolder and de Valeriola, 2019; Schokkaert *et al.*, 2020).

The first order conditions are:

$$\begin{cases} A_t: \frac{1}{\prod_{i=1}^{t}(1+\delta_i)} \cdot 2 \cdot \alpha \cdot (A_t - 1) = \psi \cdot \frac{REC_t}{\prod_{i=1}^{t} R_i} \\ B_t: \frac{1}{\prod_{i=1}^{t}(1+\delta_i)} \cdot 2 \cdot (1-\alpha) \cdot (B_t - 1) = \psi \cdot \frac{EXP_t}{\prod_{i=1}^{t} R_i} \end{cases} \quad (7)$$

where Lagrange multiplier ψ measures the social value of the marginal slacking of the budget constraint. The problem is well behaved and the second order conditions are checked by strict quasi-concavity.

Proposition: *A smooth-ABM can be implemented by applying the two following rules:*

(i) Estimation of the expected final adjustment target at time T

$$\begin{cases} A_T = 1 + UO_0 \cdot \frac{REC_T}{\prod_{i=1}^{T} \frac{R_i}{1+\delta_i}} \Big/ \sum_{t=1}^{T} \frac{REC_t^2 + \frac{\alpha}{1-\alpha} EXP_t^2}{\prod_{i=1}^{t} \frac{R_i^2}{1+\delta_i}} \\ B_T = 1 - \frac{\alpha}{1-\alpha} \cdot \frac{EXP_T}{REC_T} \cdot (1 - A_T) \end{cases} \quad (8)$$

(ii) Convergence rule to the expected final adjustment target for $1 \le t < T$*:*

$$\begin{cases} (A_t - 1) = \frac{REC_t}{REC_T} \cdot \prod_{i=t+1}^{T} \frac{R_i}{1+\delta_i} \cdot (A_T - 1) \\ (B_t - 1) = \frac{EXP_t}{EXP_T} \cdot \prod_{i=t+1}^{T} \frac{R_i}{1+\delta_i} \cdot (B_T - 1) \end{cases} \quad (9)$$

11. In this paper, we do not question the rationale or the political issues about the choice of T. But it would be worth dealing with some of them: do we wait until T to start a new period (T, $2T$) again (at the risk of having a discontinuity at time T)? Or do we extend progressively the time horizon by successive periods of T years $(0, T)$, $(1, T+1)$,...?

Proof: see appendix.

Our model gives a temporal key to finance the unfunded obligation UO_0. From these optimal adjustment processes[12] (A_t and B_t), we deduce the new forecast dynamics of the reserve funds:

$$F_t^* = A_t \cdot REC_t - B_t \cdot EXP_t - R_t \cdot F_{t-1}^* \quad (10)$$

The expected revision of the current levels of receipts and expenditures evolves approximately as follows with a backward representation:

$$\begin{cases} \frac{\Delta(A_t - 1)}{A_{t-1} - 1} \simeq g_t^{REC} - (r_t - \delta) \\ \frac{\Delta(1 - B_t)}{1 - B_{t-1}} \simeq g_t^{EXP} - (r_t - \delta) \end{cases}$$

where g_t^{REC} and g_t^{EXP} are respectively the expected receipts and expenditures growth rates. We present here expected solutions. In a stochastic version, the adjustment would include the revision of the expected final adjustment target.

This adjustment rule is characterized by the following property: when $A_{t-1} > 1$ (resp. $B_{t-1} < 1$), then $A_t > A_{t-1}$, i.e. a payroll tax increasing, (resp. $B_t < B_{t-1}$, i.e. a decreasing pension index) if receipts (resp. expenditures) growth rate is greater than the interest rate net of time preference. The increase in the contribution rate (resp. decrease in pension) is even stronger than the revenue growth (resp. spending). Notice that if $g_t^{REC} = g_t^{EXP} = (r_t - \delta)$ for all period t, the adjustment is flat and stationnary: $A_t = A_{t-1}$ and $B_t = B_{t-1}$.

This maximizing problem may be completed by adding constraints to the reserve fund level ($F_T > 0$ for a terminal constraint or $F_t \geq 0 \ \forall t$ otherwise) or to the adjustment of the contribution rate ($\tau_t \leq \tau_{max}$ as in Germany, for instance).

2.2. Interpretations

In addition to identifying rules of pension indexing and tax rate increase, our results can be interpreted in two other ways:

(i) Measuring A_t and B_t would allow to show how much the pension schemes are unbalanced in the long run;

12. A_t and B_t will depend on factors such as the evolution of population structure, retirement age, etc. Studying this property in a theoretical or simulated (Auerbach and Lee, 2011) framework sounds promising.

(ii) Revealed preferences: reforms imply changes in legislation. The levels of expenditures and receipts are modified with respect to a previous scenario without reform. Assuming A_t and B_t to be measured with accuracy would associate public decisions with an implicit function of social preferences.

For example, assuming that the measure of financial sustainability we use here is equivalent to the asset/liability ratio, then the previously studied "flat Swedish-type adjustment" can be interpreted as the result of the following parameter choices:

$$\begin{cases} \alpha \to 1 \text{ (no adjustment through receipts)} \\ \delta_t = r_t - g_t^{EXP} \text{ (flat adjustment)} \end{cases}$$

The equality between rate of preference and risk free rate net of the growth rate results in a flat adjustment. These parameter values imply:

$$\begin{cases} A_T = \cdots = A_t = A_1 = 1 \\ B_T = \cdots = B_t = B_1 < 1 \end{cases}$$

About the "fiscal cliff US-type adjustment", the implicit values of δ_t must satisfy:

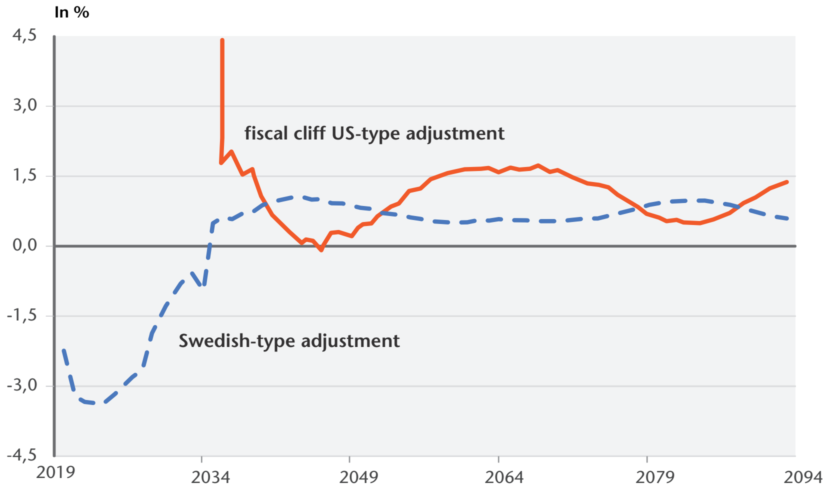
$$\begin{cases} \alpha \to 1 \text{ (no adjustment through receipts)} \\ \delta_t \begin{cases} \to +\infty \text{ for } F_t \geq 0, \\ = \frac{EXP_t - REC_t}{EXP_{t-1} - REC_{t-1}} \cdot \left(\frac{EXP_{t-1}}{EXP_t}\right)^2 \cdot (1 + r_t) - 1 \text{ for } F_t = 0. \end{cases} \end{cases}$$

No adjustment before the depletion of the trust fund requires, literaly, ignoring the future (infinite social time preference). These parameter values imply:

$$\begin{cases} A_T = \cdots = A_t = A_1 = 1 \\ B_t \begin{cases} = 1 \text{ for } F_t \geq 0, \\ = \frac{REC_t}{EXP_t} \text{ for } F_t = 0. \end{cases} \end{cases}$$

Figure 2 gives the evolution of the implicit values of the time prefence rate to obtain a Swedish flat adjustment or a US fiscal cliff adjustment. These values are computed with forecast data from the Board of Trustees of the U.S. federal OASDI (2019). Before 2035, implicit negative (respectively infinite) time preference rates would justify a Swedish-type (respectively fiscal cliff US-type) adjustment. After this critical year, the implicit rates evolve around 1% for the Swedish-flat-type adjustment and between 0 and 2% for the US-type adjustment.

Figure 2. Implicit social time preference rate (δ_t)



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

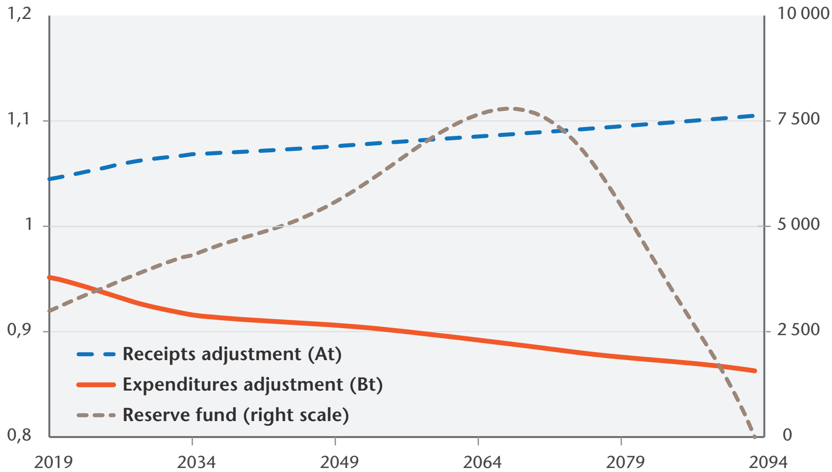
3. Illustrative simulation applied to the U.S. Social Security

3.1. Global analysis of a benchmark set of parameters

As mentioned earlier, the Board of trustees of the U.S. federal OASDI trust funds publishes annual forecasts with a 75-year horizon, contemplating three scenarios: pessimistic ("high-cost"), optimistic ("low-cost") and middle ("intermediate"). This publication plays an important part to contribute to the public debate, by giving a clear idea of the likely survival duration of the pension system. In this section, we look at what the use of ABM requires in terms of increased revenues and spending cuts. In our computations, we rely on the forecast based upon the intermediate scenario. Our data rely on "Table VI.G10. – OASDI and HI Annual Non-interest Income, Cost, and Balance in Current Dollars, Calendar Years 1970-2095". The amount of receipts and expenditures correspond exactly and respectively to the two columns: "Non-interest income" and "Cost". The interest rate is deduced from an implicit return rate of the trust fund (Table VI.G8. – Operations of the Combined OASI and DI Trust Funds) until 2034. Afterward, we use the long-run assumption given by Table IV.B2. – Components of Annual Income Rates.

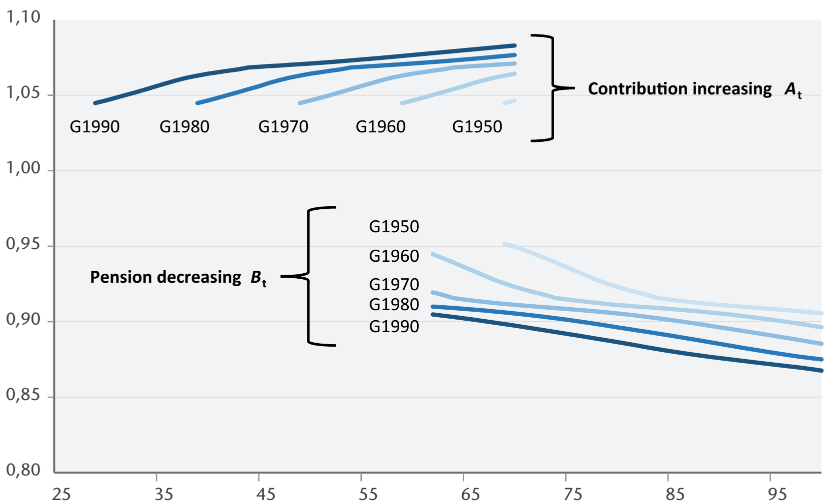
For the following set of parameters, $\alpha = 0.5$, $\delta_t = 1.5\%$ for any t and $T = 75$ years, we compute the evolution of the adjustment coefficients.

Figure 3. Automatic adjustments (A_t and B_t) and reserve trust fund (Current billion $, right scale)



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

Figure 4. Generational impact per age: contribution increasing (A_t) and pension decreasing (B_t)



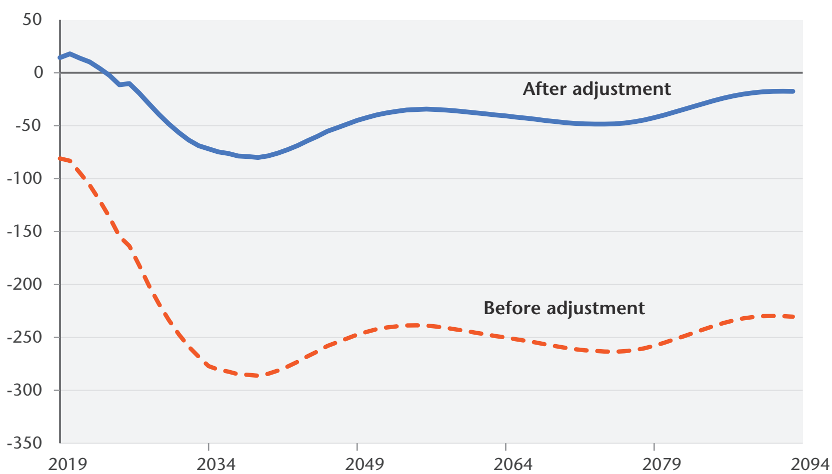
Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

The ABM implies an immediate adjustment consisting in both a 4.5% increase in tax rate and a 4.9% decrease in pension. The adjustment gradually settles in and finally reaches a 10.5% increase in tax rate and a 13.7% decrease in pension.

Figure 3 tracks the relative evolution of payroll tax rate and pensions and the amount of the reserve fund. During the first part of the period, the adjustment generates a surplus (primary balance plus interest income). Then, the reserve fund increases and reaches its maximum in 2067 when the pension scheme becomes unbalanced. From this period on, the reserve fund is used in order to finance pensions and decreases until the end of the period. The fund is depleted in 2093.

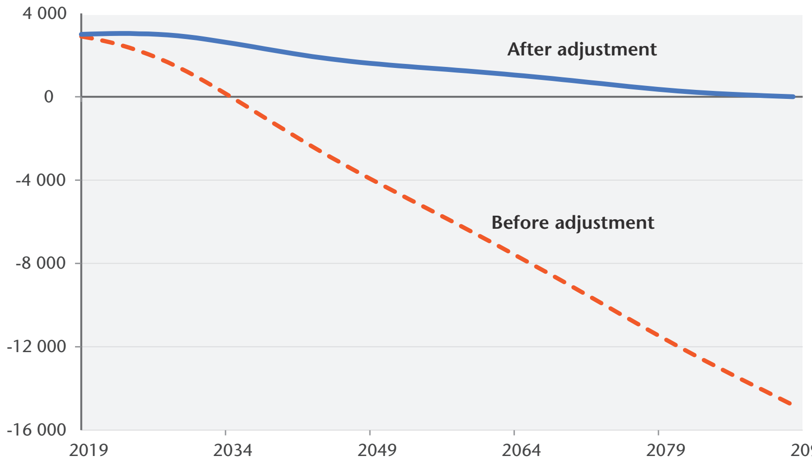
Figure 4 provides the corresponding intergenerational analysis. The upper part of the chart represents the increase in contributions for various generations. Of course, the older the generation, the shorter the period of contributions rising. In other words, the generation born in 1950 (G1950) potentially "suffers" a short period of increased contributions (only for people working after age 69) while the youngest one – born in 1990 (G1990) – "suffers" an increase in its contributions over its whole working period and a stronger pension decrease.

Figure 5. Primary balance (2019 present value in billion US $)



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

Figure 6. Reserve fund (2019 present value in billion US $)



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

In contrast, all generations are affected by a decrease in their pensions. In terms of pension yields, this means the oldest generation will have a higher return from its pension scheme than the youngest one. We also observe that the reserve fund being depleted at the end of the simulation period (figures 5 and 6), other adjustments will have to be made that will undoubtedly decrease the younger and future generations' pension yields after 2093.

3.2. Sensitivity analysis

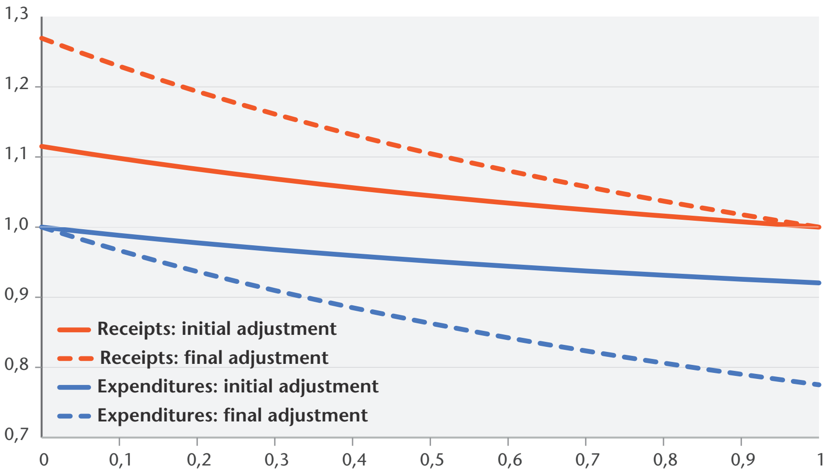
We consider several parametric variants in, respectively, forecast horizon, time preference, weight of social adjustment through receipts (versus expenditures). Figures 7, 8 and 9 respectively show parametric variants.

Figure 7 shows the profile of initial and final adjusments (A_1 and A_T for receipts; B_1 and B_T for expenditures) for variants in the social weight with $\delta = 1.5\%$ and $T = 75$. Choosing α is a crucial political decision because it determines the share of the fiscal burden between employees and pensioners. Not surprisingly, the adjustment through expenditures is more demanding for high values of α. Conversely, the adjustment through receipts is more demanding for low values of α.

For example, if α tends to 0, B_1 and B_T tend to 1 while A_1 tends to 1.12 and A_T to 1.27. That means a 12% increase in tax rate in the short run and a 27% increase in the long run.

In contrast, if α tends to 1, A_1 and A_T tend to 1 while B_1 tends to 0.92 and B_T to 0.75. That means a 8% decrease in pensions in the short run ($t = 1$) and a 22.5% decrease in the long run.

Figure 7. Receipts and expenditures adjustments: sensitivity to social weighting (α)

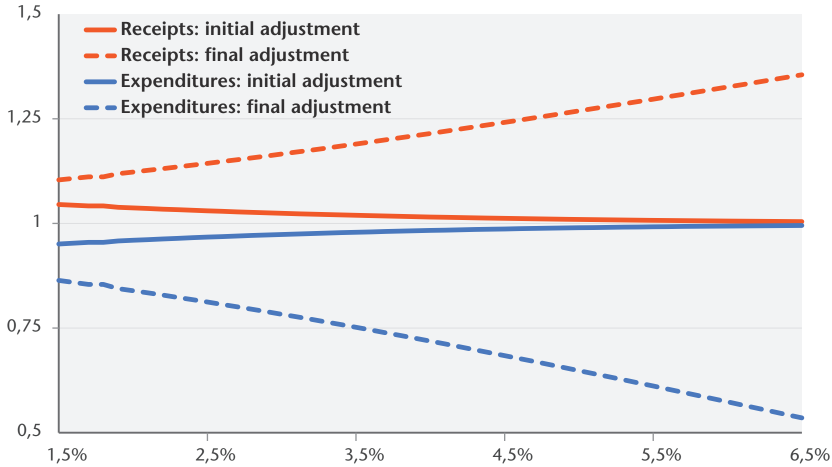


Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

Variations in social time preference (δ) clearly show the consequences of postponing adjustment mechanisms (Figure 8). Doing so induces very high adjustment costs in the future. The gap between short-run (initial) and long-run (final) adjustments ($A_T - A_1$ or $B_T - B_1$) increases exponentially with δ. For example, if $\delta > 5\%$ the gap exceeds 34% for B and 25% for A. Conversely, if $\delta < 2\%$, the gap is less than 13% for B and 8% for A. Note that for weak δ (<0.75%), adjustment is stronger in the short run than in the long run.

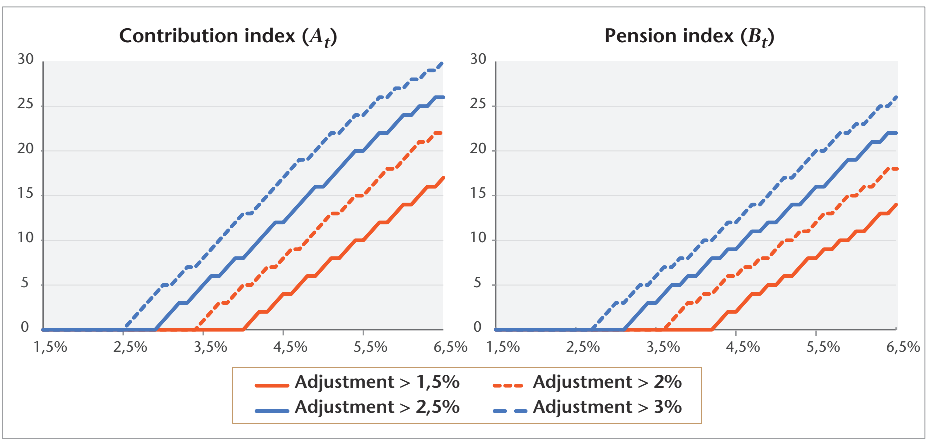
This coefficient induces procrastination since it is a component of the adjustment of the growth rate. Indeed, when this coefficient becomes sufficiently high, it takes several years before significant adjustments. As an illustration (Figs. 9a and 9b), values of δ greater than 5.5% require more than 10 (respectively 8) years for adjustments through A (resp. B) above 1.5% as compared to 15 (resp. 8), 18 (resp. 15) and 24 (resp. 20) years for adjustments above 2, 2.5 and 3%, respectively.

Figure 8. Receipts and expenditures adjustments: sensitivity to time preference (δ)



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

Figure 9. Sensitivity to social time preference (δ): Time lag (number of years) – or procrastination duration – before a significant adjustment



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

The U.S. pension system can pay promised pensions until 2034 (intermediate scenario forecasting). Afterward, the U.S. government will be forced to reform (tax increase or decrease in pensions). The longer the time horizon, the more the planner integrates imbalance. This means the adjustments are very sensitive to time horizon. For a

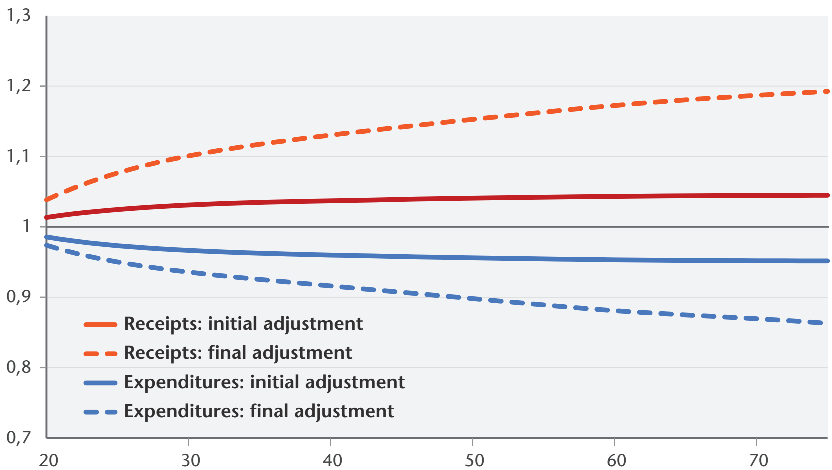
25-year time horizon, the present value of the unfunded fraction of the liabilities is low. It increases with the forecast horizon.

Increasing T has two cumulated effects (Figure 10):

— taking into account a larger period of deficit (A_T and B_T are larger);

— discounting more the value of the last period (A_T and B_T are larger).

Figure 10. Receipts and expenditures adjustments: sensitivity to time horizon (T)



Source: authors' computations based on Social Security Administration data (2019 OASDI Trustees report; intermediate scenario).

4. Conclusion

In this article, we model an ABM starting from a dynamic optimization setting. For a given planning horizon, we obtain formulas that determine how receipts and expenditures should be adjusted at each period. First, we use this ABM-model to identify the implicit social preferences associated to two particular cases: the "flat Swedish-type ABM" inducing a constant and permanent pension ajustment and the "fiscal-cliff US-type ABM" which can be obtained by assuming very high adjustment costs on revenue (implying only pension benefit adjustment) and choosing particular sequences of social time preference rate. Second, we apply these formulas to the financial balance of the US Social Security (OASDI program). Using dynamic optimization avoids brutal adjustments and thus moderates or smooths out the marginal adjustments necessary for financial stability.

The balancing adjustment should result in incremental changes. Indeed, standard AAMs are hoped to lead to sufficient adjustments and to contribute to a better financial balance. The ABM is an ultimate safeguard setting that should be expected to be marginal when the other parameters are well calibrated. Too large adjustments, as those obtained in our application to the US Social Security, suggest that a fundamental reform should recalibrate all parameters and include more significant and efficient AAMs.

Though simple and tractable, giving clearcut indications on the piloting of the pension system, this model raises social justice and political economy issues. First, we suppose a fixed social time preference rate. For example, the choice of this discount rate triggers an ethical problem of dictatorship of the present or the future (Chichilnisky, 1996 and 1997). Second, the acceptance of an ABM by the affiliates is important in terms of public legitimacy: the ability of reform promoters to explain the logic of ABM and people to understand or to accept it is a core issue. Finally, adopting an ABM must be credible and the automaticity must not be weakened by time inconsistency. Future research should examine carefully these issues.

References

Aaron, H. J., 2011, "Social Security reconsidered", *National Tax Journal*, 64 (2, Part 1): 385–414.

Alonso-Garcia J., M. C. Boado-Penas and P. Devolder, 2018, "Adequacy, fairness and sustainability of pay-as-you-go pension systems: defined benefit versus defined contribution", *European Journal of Finance*, 24:13, 1100–1122.

Auerbach, A. J., and R. Lee, 2011, "Welfare and generational equity in sustainable unfunded pension systems", *Journal of Public Economics*, 95(1-2), 16–27.

Berger, C. and A. Lavigne, 2007, "A model of the French pension reserve fund: what could be the optimal contribution path rate?", *Journal of Pensions Economics and Finance*, 6 (3): 233–250.

Blanchet, D. and F. Legros, 2002, "France: The difficult path to consensual reforms", in *Social Security Pension Reform in Europe* (ed. M. Feldstein and H. Siebert), University of Chicago Press.

Boado-Penas, M. C. and C. Vidal-Melia, 2012, "The actuarial balance of the pay-as-you-go pension system: the Swedish NDC model versus the US DB model", in *NDC Pension Schemes in a Changing Pension World*

(Holzmann, R., E. Palmer and D. Robalino, Eds), World Bank, Chapter 23, 443–489.

Boado-Penas M. C., H. Godínez-Olivares, S. Haberman and Pedro Serrano, 2020, "Automatic balancing mechanisms for mixed pension systems under different investment strategies", *European Journal of Finance*, 26 (2-3), 277–294.

Board of Trustees, 2013, *The 2013 Annual Report*, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds.

Board of Trustees, 2019, *The 2019 annual report*, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds.

Börsch-Supan A. and C. B. Wilke, 2004, "The German public pension system: How it was, how it will be", *NBER Working Paper,* No. 10525.

Bosworth B. and K. Weaver, 2011, "Social Security on auto-pilot: International experience with automatic mechanisms", *Working Paper* 2011-18, Center for Retirement Research (Boston College).

Chichilnisky, C., 1996, "An axiomatic approach to sustainable development", *Social Choice and Welfare*, 13(2): 231–257.

Chichilnisky, C., 1997, "What is sustainable development", *Land Economics*, 73(4): 467–491.

Devolder, P. and S. de Valeriola, 2019, "Between DB and DC: optimal hybrid PAYG pension schemes", *European Actuarial Journal*, 9: 463–482.

Diamond, P., 2018, "The future of Social Security", *Economic Inquiry*, 56(2): 661–681.

Fujisawa, Y., and J. Li, 2012, "The impact of the automatic balancing mechanism for the public pension in Japan on the extreme elderly", *North American Actuarial Journal*, 16(2): 207–239.

Gannon, F., S. Hamayon, F. Legros and V. Touzé, 2014, "Sustainability of the French first pillar pension scheme (CNAV): Assessing automatic balance mechanisms", *Australian Journal of Actuarial Practice*, 1(2): 33–45.

Gannon, F., F. Legros, V. Touzé, 2018, "Automatic Balancing Mechanisms in Practice: What Lessons for Pension Policy Makers?", in *Mathematical and Statistical Methods for Actuarial Sciences and Finance* (Corazza M., Durbán M., Grané A., Perna C., Sibillo M. Eds), Springer.

Godínez-Olivares, H., M. C. Boado-Penas and A. Pantelous, 2015, "How to finance pensions: Optimal strategies for pay-as-you-go pension systems", *Journal of Forecasting*, 35(1): 13–33.

Godínez -Olivares, H., M. C. Boado-Penas and S. Haberman, 2016, "Optimal strategies for pay-as-you-go pension finance: A sustainability framework", *Insurance: Mathematics and Economics*, 69: 117–126.

Haberman S. and A. Zimbidis, 2002, "An investigation of the pay-as-you-go financing method using a contingency fund and optimal control techniques", *North American Actuarial Journal*, 6:2, 60–75.

Kashiwase K., M. Nozaki and K. Tokuoka, 2012, "Pension reforms in Japan", *IMF Working Paper*, n° 285.

Marier, P., 2008, *Pension politics: Consensus and social conflict in ageing societies*, Routledge.

Ménard, J.-C. and A. Billig, 2013, *Intergenerational balance of the Canadian retirement income system*, report prepared for the International Social Security Association Technical Seminar on "Proactive and preventive approaches in Social Security supporting sustainability", Office of the Superintendent of Financial Institutions (Canada).

Pantelous, A. and A. Zimbidis, 2008, "Dynamic reforming of a quasi pay-as-you-go social security system within a discrete stochastic multidimensional framework using optimal control methods", *Applicationes Mathematicae*, 35: 121– 144.

Sakamoto, J., 2005, "Japan's Pension Reform", Social Protection *Discussion paper*, World Bank, n° 0541

Sakamoto, J., 2013, "Automatic balancing mechanisms", *PBSS colloquium* in Lyon 24-27 June.

Settergren, O., 2001, "The automatic balance mechanism of the Swedish pension system", *Working Paper*, National Social Insurance Board.

Settergren, O. and B. D. Mikula, 2005, "The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest", *Journal of Pension Economics and Finance*, 4 (2): 115–138.

Schokkaert, E., P. Devolder, J., Hindriks and F. Vandenbrouke, 2020, "Towards an equitable and sustainable points system. A proposal for pension reform in Belgium", *Journal of Pension Economics and Finance*, 19(1), 49–79.

Turner, J. A., 2009, *Social security financing: Automatic adjustments to restore solvency*, AARP Public policy institute Research report, February.

Vidal-Meliá, C. and M. C. Boado-Penas, 2013, "Compiling the actuarial balance for pay-as-you-go pension systems: Is it better to use the hidden asset or the contribution asset?", *Applied Economics*, 45(10), 1303–1320.

Vidal-Meliá, C., M. C. Boado-Penas and O. Settergren, 2009, "Automatic balance mechanisms in pay-as-you-go pension systems", *Geneva Papers on Risk and Insurance – Issues and Practice*, 34(2), 287–317.

Weaver, K. and A. Willén, 2014, "The Swedish pension system after twenty years: Mid-course corrections and lessons", *OECD Journal on Budgeting*, 13(3), 1–26.

Wisensale, S. K., 2013, "Austerity vs. solidarity: Intergenerational conflict in the European Union", *International Journal of Humanities and Social Science*, 3 (1), 21–30.

APPENDIX 1. PROOF OF THE PROPOSITION

The two F.O.C express a tradeoff between increasing the social cost of adjustment and reducing the deficit. At each period, for a given loss level, the tradeoff between A and B implies the following Marginal Substitution Rate (MRS):

$$\left(\frac{\Delta A}{\Delta B}\right)_{\text{given loss}} = -\frac{\Delta LF}{\Delta B}\bigg/\frac{\Delta LF}{\Delta A} = -\frac{(1-\alpha)\cdot(B_t-1)}{\alpha\cdot(A_t-1)} \tag{11}$$

By comparison, the slope of the given budget constraint for a given t is such that:

$$\left(\frac{\Delta A}{\Delta B}\right)_{\text{given budget constraint}} = \frac{EXP_t}{REC_t} \tag{12}$$

where $EXP_t\,/\,REC_t$ is the current balance ratio. In case of global insolvency, this ratio is always greater than 1. At the optimum, the tangency of the two curves implies:

$$-\frac{1-\alpha}{\alpha}\cdot\frac{B_t-1}{A_t-1} = \frac{EXP_t}{REC_t} \tag{13}$$

From the FOC, we deduce that:

$$\begin{cases}(A_t-1) = \frac{REC_t}{REC_{t+1}}\cdot\frac{R_{t+1}}{1+\delta_{t+1}}\cdot(A_{t+1}-1) = \frac{REC_t}{REC_T}\cdot\prod_{i=t+1}^{T}\frac{R_i}{1+\delta_i}\cdot(A_T-1)\\ (B_t-1) = \frac{EXP_t}{EXP_{t+1}}\cdot\frac{R_{t+1}}{1+\delta_{t+1}}\cdot(B_{t+1}-1) = \frac{EXP_t}{EXP_T}\cdot\prod_{i=t+1}^{T}\frac{R_i}{1+\delta_i}\cdot(B_T-1)\end{cases} \tag{14}$$

The intertemporal budget constraint can be rewritten:

$$\begin{aligned}&\sum_{t=1}^{T}\frac{(A_t-1)\cdot REC_t}{\prod_{i=1}^{t}R_i} - \sum_{t=1}^{T}\frac{(B_t-1)\cdot EXP_t}{\prod_{i=1}^{t}R_i}\\ &= \sum_{t=1}^{T}\frac{EXP_t}{\prod_{i=1}^{t}R_i} - \sum_{t=1}^{T}\frac{REC_t}{\prod_{i=1}^{t}R_i} - F_0 = UO_0\end{aligned} \tag{15}$$

By inserting the two expressions (14) in the intertemporal budget constraint (15) and considering (13), we find the final adjustment:

$$\begin{cases}A_T = 1 + \dfrac{UO_0}{\frac{\prod_{i=1}^{T}\frac{R_i}{1+\delta_i}}{REC_T}}\Bigg/\sum_{t=1}^{T}\dfrac{REC_t^2+\frac{\alpha}{1-\alpha}EXP_t^2}{\prod_{i=1}^{t}\frac{R_i^2}{1+\delta_i}}\\ B_T = 1 - \frac{\alpha}{1-\alpha}\cdot\frac{EXP_T}{REC_T}\cdot(1-A_T)\end{cases} \tag{16}$$