Schäuble versus Tsipras: A New-Keynesian DSGE Model for the Eurozone Debt Crisis*

Daniel Cohen† Mathilde Viennot‡ Sébastien Villemot§

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Abstract

We calibrate a New-Keynesian DSGE and use the comparison of two value functions to compute an implicit default probability. We compare the robustness of a small Eurozone in a fixed exchange rate model to a flexible rate economy. We demonstrate that default thresholds are higher in the Eurozone case but the thresholds are more likely to be reached. We analyze the role of consumption habit persistence in that framework, and show that it plays an important role in determining default probabilities and debt levels. Furthermore, this role is enhanced when we compare three frameworks: a flexible case, a Schäuble case (the country goes out of the monetary union if it defaults) and a Tsipras case (the country stays in the monetary union even if it defaults). We find a "Schäuble theorem" : in a monetary union and if habit formation is sufficiently high, if you give a country the choice between (i) default and leave the zone and (ii) default and stay in the union, it will always choose (ii), default and stay. From a monetary union policy maker's point of view, you should not give the choice and impose (i). This results is reversed in case of low habit persistence. Last, the impact of fiscal policies may change from one framework to the other: a fast speed of consolidation in fiscal rules can help preventing defaults, but only if habit persistence is low and if the country does not stay in the zone.

Keywords Sovereign debt, habit consumption, default risk.

JEL classification F34, F41, F45.

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†Paris School of Economics and CEPR. E-mail: `daniel.cohen@ens.fr`

‡Paris School of Economics. E-mail: `mathilde.viennot@ens.fr`

§OFCE – Sciences Po. E-mail: `sebastien.villemot@sciencespo.fr`

1 Introduction

The Eurozone (EZ) has experienced a major sovereign debt crisis past 2009. Greece, then Ireland and Portugal lost their access to the financial markets and had to request financial assistance from the other EZ countries. Then it was the turn of Spain, and to a lesser extent of Italy (in the summer of 2011) to experience huge spikes in their financing rates. Greece eventually wrote down more than 50% (in face value term) of its public debt.

What happened? Two shocks of a different nature actually hit the EZ countries which came under stress. The Greek shock resulted from the sudden discovery of a major deficit of the public sector in 2009. After many revisions, it reached the almost unprecedented level of 15.5% of GDP. The speed at which such a deficit could be brought down to normal was clearly finite and became the root of Greece's problems. In the case of Ireland, the issue was more straightforward. The banking crisis saddled with debt a country which was viewed as perfectly solvent (respecting all the criteria of the Maastricht treaty with honors). Here a major unexpected shock on debt created the crisis.

Although relatively simple to describe and analyze in retrospect, these two polar cases do not fit well the literature on sovereign debt. For one thing, in most models, the primary surplus is a "control variable", i.e. one that government can monitor at will. Clearly, as the Greek case demonstrated, there are limits to the speed at which the primary deficit can be contracted. Although these costs to adjust the primary surplus can be taken into account in a model à la Arellano (2008) by introducing an adjustment cost on any debt changes, in our model preferred habit will reduce endogenously the speed of adjustment. One contribution of this paper is to model explicitly how these limits can be accounted for.

Another dimension of the EZ crisis is the discontinuous break in the debt-to-GDP ratio. Because of the banking crisis, the Irish government suffered from a huge jump in its public debt. This changed the dynamics of debt accumulation, in ways standard models do not usually account for. Usually the debt build results from a country (willingly) running excessive deficits. The risk of a discrete jump is another feature that we want to embed in our model.

We analyze a simple DSGE model in the spirit of Smets and Wouters (2003). We analyze how the risk of default evolves, in each of the three polar cases: in a flexible exchange rate regime, in a Eurozone case (fixed exchange rate, with full capital mobility) where the country switches to a flexible economy if it defaults, and in a Eurozone case where the country stays in the zone whatever happens. We calibrate how much unexpected debt or deficit a EZ country can take. We discuss the impact of a certain rigidity of the economy, namely the degree of habit consumption, as it increases the persistence of a shock. We then analyze the speed at which the debt can be reduced.

Our main results are the following. The risk of default is larger within the Eurozone than in the pure flexible exchange rate system. Perhaps surprisingly, the key parameter driving this result is consumption habit. As it rises, the benefit, in a fixed exchange rate system, of regaining control of its domestic monetary policy rises, and so does the risk of default. Nonetheless, there is a key difference between the two Eurozone cases: since the fixed rate is preferred to flexible change when habit formation is pretty high, the country that must leave the monetary zone in case of default won't default. We thus can write a "Schäuble" theorem: in a monetary union and in case of large habit formation, if you give a country the choice between (i) default and leave the zone and (ii) default and stay in the union, it will always choose (ii), default and stay. If habit formation is low, the opposite law appears.

The paper is organized as follows: we first present the model framework (I), then present the calibration and benchmark results (II), and then analyze the sensitivity of our results to habit formation and policy tools (III).

2 Quantifying default risk in a DSGE model

The main objective of this model is to bring the literature on sovereign default and DSGE models back together: although models of default à la Eaton and Gersovitz (1981) allow value function comparison and endogenize the default decision, they cannot afford more than two state variables; on the other hand, DSGE models are unable to endogenize the default decision, and are therefore forced to introduce sovereign spreads as a proxy for sovereign risk.

In line with Mendoza and Yue (2012) who solve an RBC model with fully endogenous default, we propose another strategy for filling the gap between these two classes of models by introducing default risk in a more complex New-Keynesian DSGE model: we compute an out-of-model value function corresponding to the one the country must face in case of default and compare it to the one the country faces in the DSGE model without default. In this way, we can compute a default probability, at the cost of an approximation: the risk of default is not internalized by agents before it has materialized.

2.1 A small-open economy model with flexible exchange rates (FLEX)

We first analyze a simple small open economy model in a flexible exchange rate regime, which can be though as representing a European country before the introduction of the euro.

2.1.1 Preferences

There is a continuum of households indexed by i. Every household i maximizes a utility function with goods and labor over an infinite horizon.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u^i(C_t^i, H_t, L_t^i) \tag{1}$$

Where β is the discount factor. Consumption is relative to a time-varying external habit variable:

$$u^i(C_t^i, H_t, L_t^i) = \log(C_t^i - H_t) - \varphi \frac{(L_t^i)^{1+\sigma_L}}{1+\sigma_L} \tag{2}$$

Where $H_t = hC_{t-1}$ is consumption habit, φ disutility of labor, σ_L represents the inverse elasticity of work effort w.r.t. real wage (Frisch elasticity).
Households rent capital to firms and decide how much to invest. They also can buy public bonds in domestic currency. The budget constraint for each household i writes:

$$B_t^i + C_t^i = \frac{R_{t-1} + \Delta_{t-1}}{\pi_t} B_{t-1}^i + Y_t^i - I_t^i - \tau_t C_t \tag{3}$$

Where B_t are the real holdings of government bonds, $\pi_t = 1 + \frac{P_t}{P_{t-1}}$ is the inflation rate, τ_t the tax rate on consumption (which allows to skip the issue of capital taxation), R_t the (gross) nominal interest rate, Δ_t a risk premium, π_t the gross inflation rate, I_t the investment decision. Their revenues write:

$$Y_t^i = (w_t^i L_t^i + A_t^i) + (r_t^k z_t^i - \psi(z_t^i)) K_{t-1}^i + Div_t^i \tag{4}$$

where z_t is the capital utilization rate and $\psi(z_t) = \gamma_1(z_t - 1) + \frac{\gamma_2}{2}(z_t - 1)^2$ a cost-adjustment function.

A_t^i are the net cash inflow from participating in state-contingent securities (Arrow-Debreu) : following Christiano et al. (2001), we assume that there exists domestic state-contingent securities that insure households against variations in household specific labor income. As a result, the first component in the household's income will be equal to aggregate labor income.

In the households' budget constraint, $\Delta_t = \Psi \left(e^{D_t - \bar{D}} - 1 \right)$ is the default risk premium (following Schmitt-Grohé and Uribe (2003)), R_t the gross nominal return on bonds and D_t being the real external debt (see Government section). $\bar{D}$ is the external debt target, equal to zero in case of default.

Consumption and savings behavior Maximization of preferences with respect to consumption and holdings of bonds gives the following first-order condition (Euler condition):

$$\mathbb{E}_t \left[\beta \left(\frac{C_t - H_t}{C_{t+1} - H_{t+1}} \frac{1 - \tau_t}{1 - \tau_{t+1}} \right) \frac{R_t + \Delta_t}{\pi_{t+1}} \right] = 1 \tag{5}$$

Labor supply and wage setting Labor is differentiated across households, so there's a monopoly power over wages that become sticky à la Calvo (1983). Wages can be optimally adjusted after some random "wage-change signal" (see Kollman (1997)): with probability $1-\xi_w$, the household i set a new nominal wage $\tilde{w}_t$.

There is also partial indexation of wages on past inflation:

$$w_t^i = \pi_{t-1}^{\chi_w} w_{t-1}^i \tag{6}$$

where χ_w is the degree of wage indexation (if 0, non-optimized wages, remain constant).

Maximizing preferences with respect to labor, we obtain the demand for labor:

$$L_t^i = \left(\frac{w_t^i}{W_t}\right)^{-\eta} L_t \tag{7}$$

Where $L_t = \left(\int_0^1 (L_t^i)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$ is the aggregate labor demand, $W_t = \left(\int_0^1 (w_t^i)^{1-\eta} di\right)^{\frac{-1}{\eta-1}}$ the aggregate nominal wage and η the elasticity of substitution between labor varieties.

Because of Calvo pricing, we have the following mark-up equations (reallocation of wages):

$$\frac{\tilde{w}_t}{w_t}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \left(\frac{\pi_t^{\chi_w}}{\pi_{t+i}}\right) \frac{\eta-1}{\eta} \frac{L_{t+i}^i}{(C_{t+i}^i - H_{t+i}^i)(1-\tau_{t+i})} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \varphi (L_{t+i}^i)^{(1+\sigma_L)} \tag{8}$$

The nominal wage at time t of a household i that is allowed to change its wage set so that the present value of the marginal return of working is a mark-up over the present value of the marginal cost (of working).

We obtain the law of motion of the aggregate wage index:

$$1 = \xi_w \left(\frac{\pi_{t-1}^{\chi_w}}{\pi_t}\right)^{1-\eta} \left(\frac{W_{t-1}}{W_t}\right)^{1-\eta} + (1-\xi_w)\left(\frac{\tilde{w}_t}{W_t}\right)^{1-\eta} \tag{9}$$

Investment and capital accumulation Households choose the capital stock K_t, investment I_t and the utilization rate z_t in order to maximize their preferences. The capital accumulation equation is given by

$$K_t = (1-\delta)K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{10}$$

Where $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ is an adjustment cost function (equals 0 in steady-state, where there is constant I).

We obtain the following first-order conditions for capital (Tobin's q), investment and capital utilization rate:

$$\mathbb{E}_t\left[\frac{1}{\beta}\left(\frac{C_{t+1}-H_{t+1}}{C_t-H_t}\frac{1-\tau_{t+1}}{1-\tau_t}\right)\right]q_t = q_{t+1}(1-\delta)+z_{t+1}r^k_{t+1}-\psi(z_{t+1}) \tag{11}$$

$$q_t\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]-1 + \beta\mathbb{E}_t q_{t+1}\left(\frac{C_t-H_t}{C_{t+1}-H_{t+1}}\frac{1-\tau_t}{1-\tau_{t+1}}\right)S'\left(\frac{I_{t+1}}{I_t}\right)\frac{I^2_{t+1}}{I^2_t} = q_t S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \tag{12}$$

$$r^k_t = \psi'(z_t) \tag{13}$$

2.1.2 Technologies and firms

The country produces a unique final good Y_t, which is produced using a continuum of intermediate goods $y_{j,t}$. Those intermediate goods are produced using labor L_t, imported materials M_t and capital $z_t K_{j,t-1}$, each in a single monopolistic firm. The final good is consumed by the households.

Final-good sector The final good is produced using the following technology:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}}\,dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{14}$$

The final good is indeed determined by a Dixit-Stiglitz aggregator that combines a continuum of differentiated intermediate inputs $y_{j,t}$ for $j \in [0,1]$.

Intermediate goods producers Each intermediate (domestic) good is produced using the following technology:

$$y_{j,t} = A_t(z_t K_{j,t-1})^{\alpha_K} M_t^{\alpha_M} L_{jt}^{1-\alpha_K-\alpha_M} \tag{15}$$

where A_t is an AR(1) productivity shock following $\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon^A_t$.

Because of perfect competition in the final good market, aggregate prices write

$$P_t = \left(\int_0^1 p_{j,t}^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}} \tag{16}$$

where $p_{j,t}$ is the price in t of the intermediate good y_j. Cost minimization leads to

$$\frac{w_t L_t}{r_t z_t K_{t-1}} = \frac{1-\alpha_K-\alpha_M}{\alpha_K} \tag{17}$$

$$\frac{\varepsilon_t M_t}{r_t z_t K_{t-1}} = \frac{\alpha_M}{\alpha_K} \tag{18}$$

Where $\varepsilon_t = E_t \dfrac{P_t^*}{P_t}$ is the real exchange rate (E_t being the nominal exchange rate) and the price of imported materials. The firms marginal cost is given by

$$MC_t = \frac{1}{A_t} W_t^{1-\alpha_K-\alpha_M} r_t^{\alpha_K} \varepsilon_t^{\alpha_M} [\alpha_K^{-\alpha_K}(1-\alpha_K-\alpha_M)^{-(1-\alpha_K-\alpha_M)} \alpha_M^{-\alpha_M}] \tag{19}$$

Thus, nominal profits can write

$$\Pi_{j,t} = (p_{j,t} - MC_t) y_{j,t} \tag{20}$$

As in Calvo (1983), prices can be optimally adjusted after some random "price-change signal:" with probability $1-\xi_p$, the intermediate firm j sets a new nominal price $\tilde{p}_{j,t}$. Optimal price inflation becomes thus $\tilde{\pi}_{j,t}$.

We allow partial indexation χ_p: $P_t = \pi_{t-1}^{\chi_p} P_{t-1}$

Profit optimization by producers that are allowed to reoptimize their prices at time t results in the following first-order condition:

$$\frac{\tilde{\pi}_{j,t}}{\pi_t} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left(\frac{y_{j,t+i}}{(C_{t+i} - H_{t+i})(1-\tau_{t+i})} \right) \left(\left(\frac{\pi_t^{\chi_p}}{\pi_{t+i}} \right) - MC_{t+i} \frac{\epsilon}{\epsilon - 1} \right) = 0 \tag{21}$$

The price set by the firm j at time t is a function of expected future marginal costs. The price will be a mark-up over these weighted marginal costs. If prices are perfectly flexible ($\xi_p = 0$), the mark-up in period t becomes $\dfrac{\epsilon}{\epsilon - 1}$.

We obtain the law of motion of the aggregate price index:

$$1 = \xi_p \left(\frac{\pi_{t-1}^{\chi_p}}{\pi_t} \right)^{1-\epsilon} + (1-\xi_p) \left(\frac{\tilde{\pi}_{j,t}}{\pi_t} \right)^{1-\epsilon} \tag{22}$$

Exports Exports are given by $X_t = \varepsilon_t^{\iota} Y_t^*$ with Y_t^* an exogenous parameter for foreign demand following $(Y_t^* - 1) = \rho_Y (Y_{t-1}^* - 1) + \varepsilon_t^Y$.

2.1.3 Government

The government raise taxes $T_t = \tau_t C_t$. Public expenditures G_t are exogenous and follow an AR(1) process $G_t - \bar{G} = \rho_G (G_{t-1} - \bar{G}) + \varepsilon_t^G$.

The primary surplus in real terms is given by

$$P_t^{sur} = \tau_t C_t - G_t \tag{23}$$

The government can sell bonds to households (B_t) in domestic currency which return $R_t + \Delta_t$ next period and bonds to foreign investors (D_t) in foreign currency which return $R_t^* + \Delta_t$ next period, where R_t^* is the foreign gross nominal interest rate.

Interests on debt at date t are

$$Int_t = \left(\frac{R_{t-1}+\Delta_{t-1}}{\pi_t} - 1\right) B_{t-1} + \left(\frac{R^*_{t-1}+\Delta_{t-1}}{\pi_t} - 1\right) \frac{E_t}{E_{t-1}} D_{t-1}$$

The government faces the following budget constraint:

$$B_t + D_t + \tau_t C_t = \frac{R_{t-1}+\Delta_{t-1}}{\pi_t} B_{t-1} + \frac{R^*_{t-1}+\Delta_{t-1}}{\pi_t} \frac{E_t}{E_{t-1}} D_{t-1} + G_t \tag{24}$$

All variables are expressed in real terms; the return on D_t being obviously affected by the currency position.
The public deficit and the debt target are determined by the following fiscal rule:

$$P^{sur}_t - Int_t = \alpha_B \left(B_{t-1} + \frac{E_t}{E_{t-1}} D_{t-1} - \overline{BD}_t\right) \tag{25}$$

where $\overline{BD}_t$ is the total debt target and α_B the control force. The balance of payment is given by:

$$D_t = \frac{R^*_{t-1}+\Delta_{t-1}}{\pi_t} \frac{E_t}{E_{t-1}} D_{t-1} + \varepsilon_t M_t - X_t \tag{26}$$

Last, the choice of the real exchange rate is determined by the uncovered interest parity equation:

$$(R_t + \Delta_t) = \mathbb{E}_t \left(R^*_t \frac{E_{t+1}}{E_t}\right) + \vartheta \left(e^{(D_t - \bar{D})} - 1\right) \tag{27}$$

Where $\vartheta \left(e^{(D_t - \bar{D})} - 1\right)$ is a risk premium à la Schmitt-Grohé and Uribe (2003).

2.1.4 Market equilibrium

- The final goods market is in equilibrium if production minus exports equals demand by households for consumption and investment and by the government (note that Y_t measures aggregate production, GDP would be obtained by subtracting imports):

$$Y_t - X_t = C_t + G_t + I_t + \psi(z_t) K_{t-1} \tag{28}$$

- **Capital markets**: the demand for capital by intermediate goods producers equals the supply of capital by households
- **Labor markets**: firms' demand for labor equals labor supply at the wage level set by the households
- **Interest rate**: Monetary policy decisions are made thanks to a Taylor rule. In the capital market, government debt is held by domestic investors and foreign investors at rates $R_t + \Delta_t$ and $R^*_t + \Delta_t$.

  We can write the following Taylor rule, with $\bar{R}$ the long-term (gross) interest rate:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_\pi} \left(\frac{\pi_t}{\bar{\pi}}\right)^{r_\pi (1-\rho_\pi)} \tag{29}$$

- **Default risk**: With the satellite model, we quantify the sovereign risk in the core model. The country defaults on its external debt ; this assumption is motivated by the empirical literature on the original sin, which documents that virtually all of the debt issued by emerging countries is denominated in foreign currency (see for instance Eichengreen et al. (2003)). In the recent Greek case, external debt (bilateral loans from Eurozone countries) gathered 21% of total debt, the remaining being hold by the European Stability Mechanism (56%), the International Monetary Fund (13%) and the European Central Bank (10%).

2.2 A small-open economy with fixed exchange rate: the Schäuble and Tsipras models

Two other versions of the model involve a country which is part of a monetary union. The nominal exchange rate is now fixed. The real exchange rate becomes $\frac{\varepsilon_t}{\varepsilon_{t-1}} = \frac{\pi_t^*}{\pi_t}$ with π_t^* the inflation in the rest of the monetary union.

The framework is almost the same, except that the monetary policy is exogenous:

$$R_t = R_t^*$$

with R_t^* the foreign interest rate.

In a first version of this monetary union model, the country stays in a fixed-exchange rate regime after a default (Tsipras). In a second one, the country is back to a full flexible regime after a default (Schäuble).

2.3 Modelling the implied default risk: the satellite model

For all three models, we consider a satellite model whose purpose is to quantify the risk of default in the core model (*i.e.* before default) and compute a default frequency. Indeed, because of algorithmic and computational limits, it is not possible to introduce endogenous default risk in such a model. Using a satellite model allows us to quantify an implied risk of default delivered by our DSGE model, at the cost of some approximation.

As in the canonical endogenous default model à la Aguiar and Gopinath (2006), we assume that, after a default on its external debt, a penalty is imposed on the country in the form of a proportional cost to production, and that the country remains in financial autarky for eternity; as a consequence, the country forgoes all the benefits, in the form of additional investment finance and consumption smoothing, offered by borrowing abroad.

Thus, post-default production is:

$$Y_t^d = (1 - \lambda_Q) Y_t \tag{30}$$

where λ_Q governs the magnitude of the default cost and the government budget constraint becomes:

$$B_t + T_t = \frac{R_{t-1} + \Delta_{t-1}}{\pi_t} B_{t-1} + G_t \tag{31}$$

In all three cases (Flex, Schäuble, Tsipras), the financial autarky in the satellite model implies that external debt remains zero, which in particular means that the trade balance must be equilibrated at all times (imports must be matched by imports). In the Flex and Schäuble cases, the country has control over its monetary policy (through a Taylor rule), and the nominal exchange rate plays the role of the adjustment device.

In a nutshell, exchange rate and monetary regimes after default are the following:

- Flexible case: the model does not change after default, the country remains in a flexible exchange rate regime and has its own independent monetary policy.
- Schäuble case: the country goes back to a flexible exchange rate regime after default, and hence regain its independent monetary policy.
- Tsipras case: the country remains in the monetary union after default (and hence adjustment through the exchange rate is not possible) and in financial autarky (and hence adjustment through external debt is no longer possible). In the modelization, something has thus to give in, and I choose to make adjustment through the nominal interest rate, which is not fixed by the ECB because of autarky but neither freely adjustable through a Taylor rule. Another possibility (to be explored) is to allow adjustment through debt, and therefore by dropping the fiscal rule.

The core model and the satellite model are self-contained and do not depend on the other one. Default in this model is not endogenous, as incorporating the default risk would raise the dimensionality of the model one step too high.

The comparison of the value function of the core model J^r with that of the satellite model J^d delivers the implicit default probability with

$$\begin{aligned} J^d(K_{t-1}, A_t, B_{t-1}, H_t, R_{t-1}, \pi_{t-1}, \varepsilon_{t-1}, \Delta_{t-1}) &= \max_{C_t, L_t, K_t, B_t} \{u(C_t, L_t, H_t) \\ &+ \beta E_t J^d(K_t, A_{t+1}, B_t, H_{t+1}, R_t, \pi_t, \varepsilon_t, \Delta_t) \end{aligned} \tag{32}$$

$$\begin{aligned} J^r(K_{t-1}, A_t, B_{t-1}, D_{t-1}, H_t, R_{t-1}, \pi_{t-1}, \varepsilon_{t-1}, \Delta_{t-1}) &= \max_{C_t, L_t, K_t, B_t, D_t} \{u(C_t, L_t, H_t) \\ &+ \beta E_t J^r(K_t, A_{t+1}, B_t, D_t, H_{t+1}, R_t, \pi_t, \varepsilon_t, \Delta_t) \end{aligned} \tag{33}$$

The model is solved in the following way:

- The core model is solved and we compute the value function J^r corresponding to the non-default case: this computation gives us a mean debt-to-GDP ratio and a simulation path of 10 000 periods for all the model variables.
- The satellite model is solved and we compute the value function J^d corresponding to the post-default model.

- We compare J^r and J^d on the 10 000 simulation points, which enables us to compute the default probability (percentage of periods in which $J^r - J^d < 0$. The default threshold is the level of external debt for which $J^r = J^d$, for the state variables evaluated at their steady-state value.

The results show how often the country would default ex-post in the model (default frequency).

3 Calibration and benchmark results

We base our calibration on (Smets and Wouters, 2003) for the DSGE inputs, Mendoza and Yue (2012) for the international economics inputs and on Aguiar and Gopinath (2006) for the default specificities. Consequently, the external debt target $\bar{D}$ is calibrated as to match the default threshold obtained by Aguiar and Gopinath (2006), which is approximately 30% quarterly. Table 1 summarizes the calibration.

This calibration is quite standard for both default and New-Keynesian DSGE models. As in Smets and Wouters (2003), we calibrate consumption habit around 0.8 for the Euro area. Our discount factor β must be high in order to keep a targeted inflation around 2% in annual terms. We also calibrate the total debt target $\overline{BD}_t$ and the speed of convergence α_B to match Maastricht criteria: a debt target ratio at 60% annual and 20 years needed to get back to it. The parameters linked to the risk premium directly come from Schmitt-Grohé and Uribe (2003).

Note that $\bar{R}$, β, $\bar{\pi}$ and $\bar{\Delta}$ at steady-state must satisfy the Euler equation:

$$\beta \frac{\bar{R} + \bar{\Delta}}{\bar{\pi}} = 1 \tag{34}$$

In the benchmark calibration, we set $\bar{R} = \bar{\pi}/\beta$ which implies $\Delta = 0$ at steady-state and therefore $D = \bar{D}$.

Table 1: Benchmark calibration of the model (all specifications)

Parameter	Symbol	Value
Consumption habit	h	0.85
Discount factor	β	0.995
Capital utilization, linear term	γ_1	0.035
Capital utilization, quadratic term	γ_2	0.001
Capital share in output	α_K	0.3
Imported materials share in output	α_M	0.15
Capital depreciation rate	δ	0.025
Capital adjustment cost parameter	κ_I	1
Elasticity of substitution between labor varieties	η	3
Elasticity of substitution between good varieties	ϵ	9
Labor disutility parameter	φ	5.89
Inverse Frisch elasticity	σ_L	2.4
Wage indexation parameter	χ_w	0.763
Calvo parameter for wages	ξ_w	0.737
Price indexation parameter	χ_p	0.469
Calvo parameter for prices	ξ_p	0.908
Steady-state inflation	$\bar{\pi}$	1.005
Steady-state gross nominal interest rate	$\bar{R}$	$\bar{\pi}/\beta \simeq 1.01$
Total debt target	$\overline{BD}_t$	$2.4Y_t$
Back to equilibrium debt targets (fiscal rule)	α_B	1/80
Government expenditures target in AR(1) process	$\bar{G}$	$0.18\bar{Y}$
Standard deviation of TFP shock ε_t^A	σ_A	$\exp(-3.97)$
Standard deviation of government expenditures shock ε_t^G	σ_G	$\exp(-2.16)$
Standard deviation of foreign demand shock ε_t^Y	σ_Y	$\exp(-4.12)$
Persistence of TFP process	ρ_A	0.9
Persistence of government expenditures process	ρ_G	0.9
Persistence of foreign demand process	ρ_Y	0.9
Interest smoothing coefficient in Taylor rule	ρ_π	0.85
Feedback coefficient to inflation in Taylor rule	r_π	1.5
Foreign nominal gross interest rate	R_t^*	$\bar{\pi}/\beta \simeq 1.01$
Risk premium in uncovered interest parity	ϑ	0.001
Price elasticity of exports demand	ι	1
Schmitt-Grohé parameter for risk premium Δ_t	Ψ	0.007742
External debt target	$\bar{D}$	$0.3\bar{Y}$
Loss of output in autarky (% of GDP)	λ_Q	0.03

Quarterly frequency

The first results arise from welfare comparisons in the core and post-default models (see Table 2). Post-default seems to be preferred in a monetary union, bringing stability for output. Indeed, in a flexible regime, there is a noise on the exchange rate that the central bank cannot handle in our model by intervening directly on change markets ; the fixed regime is thus more stable, especially after a default.

Table 2: Welfare comparisons and moments of simulated variables

	CORE MODEL			
	Welfare	External debt	Consumption	Output
Flexible regime	$J_r = -800.2$	$\bar{D} = 0.23$	$\bar{C} = 0.19$	$\bar{Y} = 2.70$
		$\sigma(D) = 0.75$	$\sigma(C) = 0.25$	$\sigma(Y) = 1.93$
Monetary union	$J_r = -799.6$	$\bar{D} = 0.23$	$\bar{C} = 0.19$	$\bar{Y} = 2.70$
		$\sigma(D) = 0.61$	$\sigma(C) = 0.25$	$\sigma(Y) = 1.89$

	SATELLITE MODEL			
	Welfare	External debt	Consumption	Output
Flexible regime	$J_d = -838.7$	$\bar{D} = 0$	$\bar{C} = 0.18$	$\bar{Y} = 2.66$
		$\sigma(D) = 0$	$\sigma(C) = 0.24$	$\sigma(Y) = 2.73$
Monetary union	$J_d = -810.5$	$\bar{D} = 0$	$\bar{C} = 0.19$	$\bar{Y} = 2.63$
		$\sigma(D) = 0$	$\sigma(C) = 1.39$	$\sigma(Y) = 0.60$

Regarding default occurrences and debt thresholds, we can see that the default threshold is very high in either the flexible, Tsipras and Schäuble models, and consequently the implicit default probability is almost zero (Table 3). These results are much more realistic than standard default models, even if these results owe to the fact that the country does not internalize the risk of default and its corresponding cost, and should therefore not be taken at face value. Nonetheless, internalizing the risk of default would then change little to the results (except if the country were to deliberately seek to default, which is unlikely).

Table 3: Default probabilities and debt thresholds - Flexible, Schäuble and Tsipras models

		Default probability	Default threshold (at SS)
Baseline	Flexible regime	0.05%	223%
	Schäuble regime	0.0%	369%
	Tsipras regime	0.72%	366%

Quarterly frequency

In the Tsipras model instead, there is a positive risk of default, which is the outcome of the fact that default is not too costly: defaulting while maintaining the fixed exchange rate regime (barring only the ability to borrow) is not as costly as in the other cases. The reason has to do with the fact that the country regain its monetary policy while keeping the stability brought by the fixed regime.

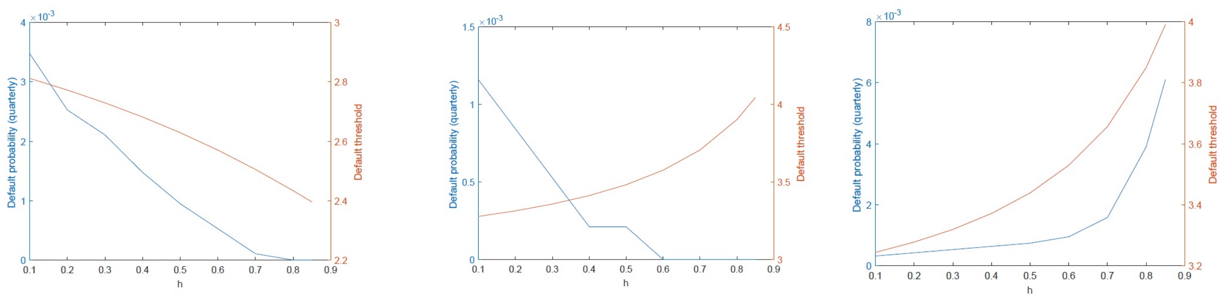
4 Sensitivity analysis

The results obtained on the benchmark calibration are driven by three key parameters: the total debt target in fiscal rule $\overline{BD}_t$, the level of consumption habit h and the speed of convergence in fiscal rule α_B. Let us analyze the sensitivity of our results to these parameters.

4.1 Consumption habits

First, we take a close look to sensitivity with respect to consumption habit in our three benchmarks models. Figure 1 summarizes the results of such a sensitivity exercise when h ranges from 0.1 to 0.85 (its benchmark value), while keeping other parameters constant.

Figure 1: Default probabilities and debt thresholds on baseline calibration - Sensitivity with respect to consumption habit



a) Flexible model

b) Schäuble model

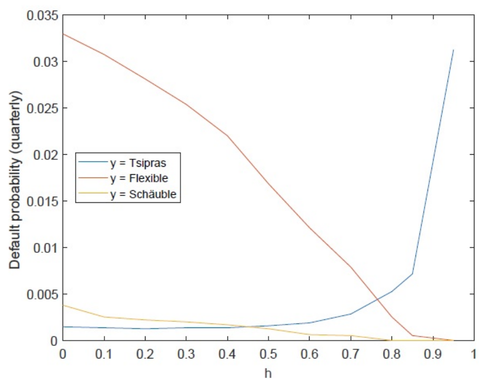
c) Tsipras model

Consumption habit has a remarkable influence on the risk of default. In the FLEX model, a high degree of habit rises the default threshold and lowers the default probability. In the Tsipras model the opposite effect emerges. A higher degree of consumption habit simultaneously raises the debt ceiling and the risk of default. Finally the Schäuble model is a combination of both cases. Higher consumption habit means more debt and less default risk.

The intuition behind these results comes as follows. The higher the consumption habit parameter h, the lower the volatility of consumption (almost three times higher in the low h case than in the high h scenario, for all models). As h rises, two conflicting forces operate. As the desired $\sigma(c)$ falls, the debt is reduced to stabilize consumption. But on the other hand, a higher stock of debt service hampers the ability to respond

to a (large) negative shock on GDP. This is why all combinations are possible. Rising debt threshold *cum* rising default risk, declining debt threshold *cum* declining risk or rising debt and declining risk. See Carré et al. (2015) for further insight on why debt threshold and default risk are not necessarily correlated.
Specifically, *ceteris paribus*, default, when it reduces the number of instruments is less likely for large h values. The reason why this is not the case in the Tsipras case is, as we indicated earlier, that default allows the country to regain full control of its monetary policy without having to pay the consequences of exchange rate volatility. Default then becomes more likely when h rises. Schäuble is the worst of both cases, so that the risk of default does decline as in the Flexible model, but sustainable debt is also higher as the cost of default becomes even higher.

Figure 2: Defaults probabilities and habit formation



Additionally, habit consumption makes wealth cut by hW_{-1} which gives little leeway when h rises. This has mainly two consequences. First, when a negative shock occurs, it has less impact on agents' welfare since h is large and so the shock will hit them in a lagged and smoothed way (more persistence). As the agents feel poorer, debt is less volatile and agents come through the crisis easier. Second, it matters when a default choice has to be made. In the Eurozone after a default, you do not need this leeway to adjust in case of a GDP shock, since stability prevents you from adverse shocks. Whereas after a default in a flexible regime, you want degrees of freedom. This implies that if h is small, you have enough leeway to go out of the zone and regain your monetary independence: you will prefer Schäuble rather than Tsipras (see Figure 2). We can thus derive a theorem :

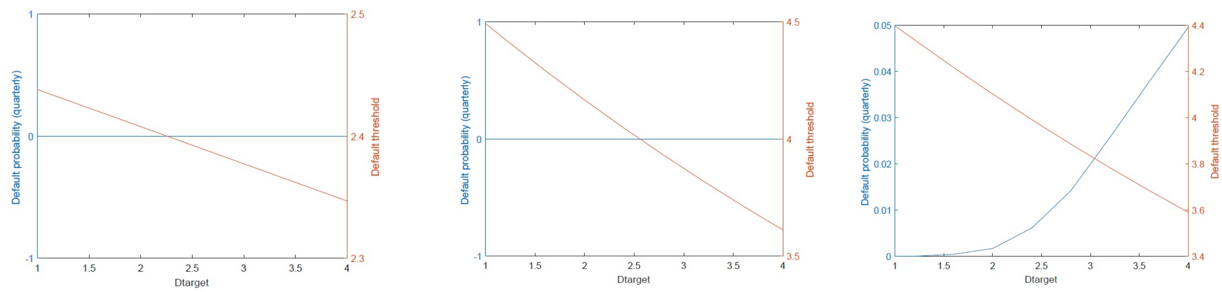
Schäuble Theorem

In a monetary union and if habit formation is sufficiently high ($h > 0.45$), if you give a country the choice between (i) default and leave the zone and (ii) default and stay in the union, it will always choose (ii), default and stay.
This results is reversed in case of low habit persistence($h < 0.45$).

4.2 Maastricht tools

We now analyze the sensitivity of the default risk to the aggregate debt targets (domestic and external together, see Figure 3). We find the same kind of qualitative opposition between the three regimes. Raising the long run debt target does not raise (in the range that is considered) default risk in both Flexible and the Schäuble model, but does so in the Tsipras case. The intuition is the same as in the previous section. With a large habit parameter (0.85 here), the EZ country is more likely to default, as it seeks to regain its monetary instrument. The larger the debt ceiling the more likely it will choose to do so. The flexible and the Schäuble models generate no default, *ceteris paribus*, because of the fear the additional instability brought by the flexible exchange rate regime when it is not compensated by an access to financial markets.

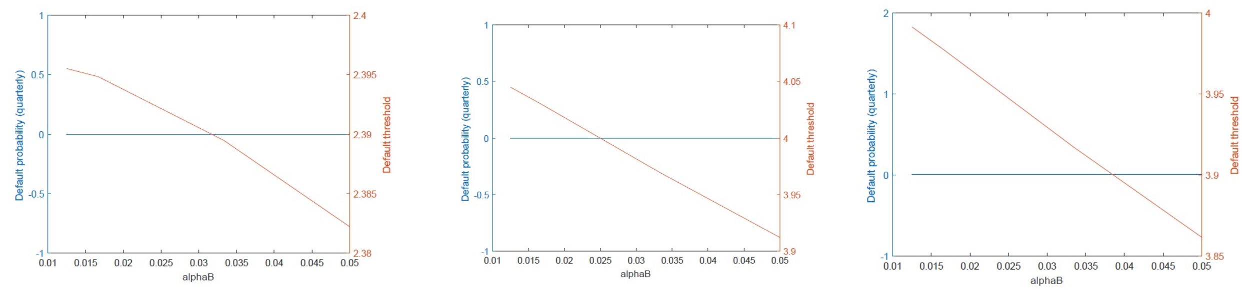
Figure 3: Default probabilities and debt thresholds - Sensitivity with respect to total debt target



a) Flexible model　　b) Schäuble model　　c) Tsipras model

As a last exercise, we present sensitivity results to the speed of convergence in the fiscal rule, α_B. Results are presented in Figures 4 and 5. For large consumption habits ($h = 0.85$), in all cases, a fast speed of convergence does not change the default probability but reduces the debt threshold.

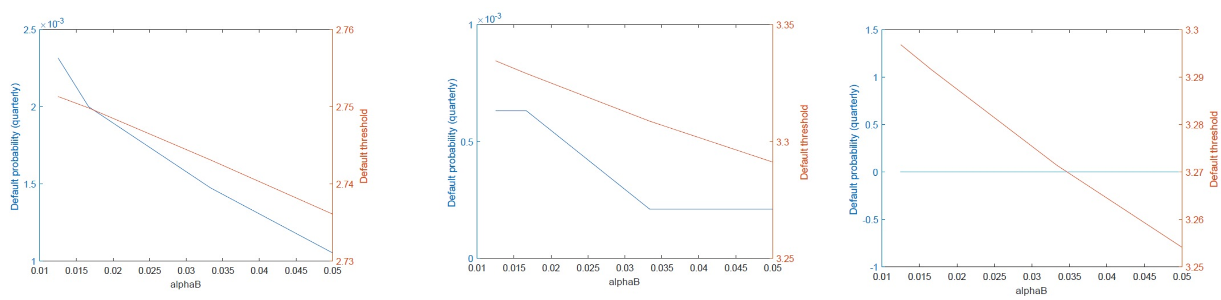
Figure 4: Default probabilities and thresholds with high consumption habits ($h = 0.85$) - Sensitivity with respect to speed of convergence



a) Flexible model b) Schäuble model c) Tsipras model

With low consumption habits (Figure 5), raising the speed of fiscal convergences reduces the default risk in flexible and Schäuble models but does not affect it in the Tsipras case. Indeed, raising up the speed of convergence limits the risk that the country will err in the side of too much debt, as it is very volatile, and hence reduces the risk of default in the Flexible and Schäuble regimes. Nonetheless, we can see that the quantitative effect is very small, so this result has to be qualified; furthermore, with weak fiscal instruments, the risk of default is larger for large habit persistence in Flexible and Schäuble cases and may explain why tougher fiscal rules are here needed.

Figure 5: Default probabilities and thresholds with low consumption habits ($h = 0.25$) - Sensitivity with respect to speed of convergence



a) Flexible model b) Schäuble model c) Tsipras model

5 Conclusion

Calibrating a New-Keynesian DSGE and using the comparison of two value functions to compute an implicit default probability, we have compared the robustness of a small

Eurozone in a fixed exchange rate model to a flexible rate economy. The model that we have presented highlights the critical differences between a small open economy within the Eurozone and a flexible exchange rate economy. Furthermore, analyzing the role of consumption habit persistence in three frameworks (a flexible case, a Schäuble case where the country goes out of the monetary union if it defaults, and a Tsipras case where the country stays in the monetary union even if it defaults), we were able to find a Schäuble theorem : in a monetary union and if habit formation is sufficiently high, if you give a country the choice between (i) default and leave the zone and (ii) default and stay in the union, it will always choose (ii), default and stay. This results is reversed in case of low habit persistence. This can be explained by both stability brought by the Eurozone in terms of currency noise and persistence in negative shocks brought by large habit parameters, which make the country like even more the stability.

For the conventional set of parameters the risk of default is larger in the Eurozone case when the country can maintain its fixed exchange rate regime after defaulting. This is somehow what happened to Greece : leaving the Eurozone and simultaneously losing access to the financial markets, on the other hand, would have been too costly.

Last, we have shown that the impact of fiscal policies may change from one framework to the other: a fast speed of consolidation in fiscal rules can help preventing defaults, but only if habit persistence is low and in flexible and "Grexit" frameworks. In the Greek case (Tsipras), this model thus show that imposing a faster speed of fiscal convergence was not relevant for preventing the country from another default.

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A Impulse response functions

A.1 The Flexible model

Figure 6: Impulse response functions for the FLEX model - Productivity

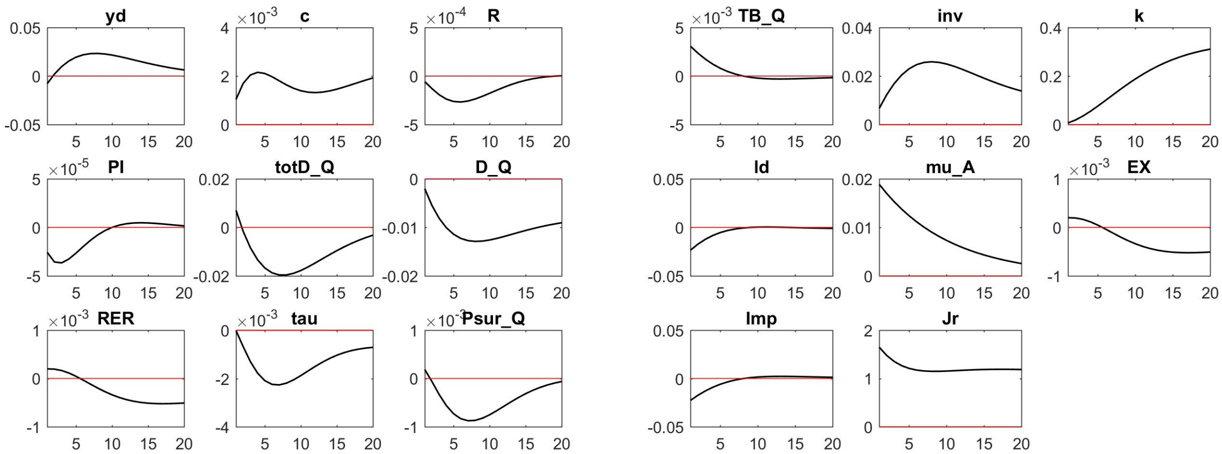


Figure 7: Impulse response functions for the FLEX model - Foreign demand

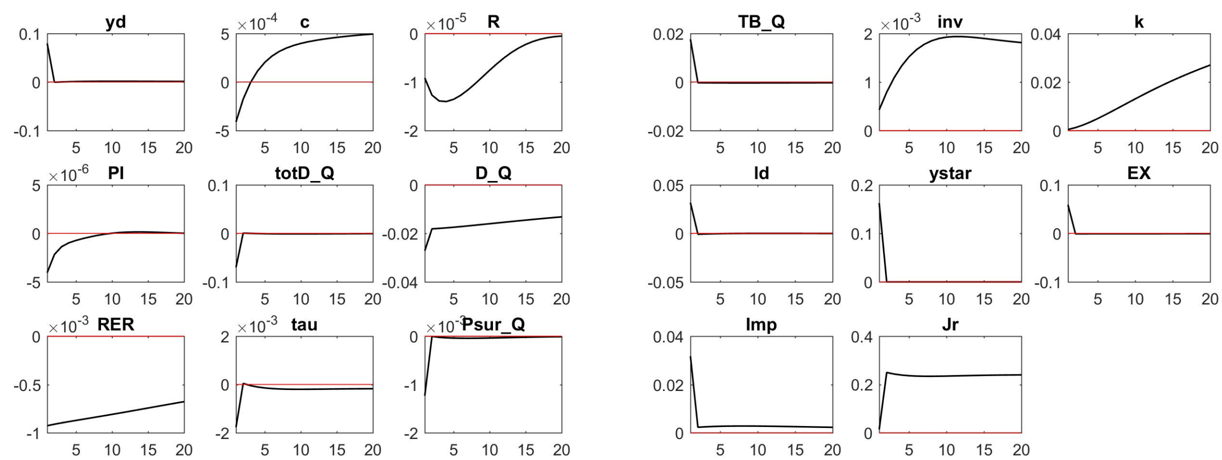
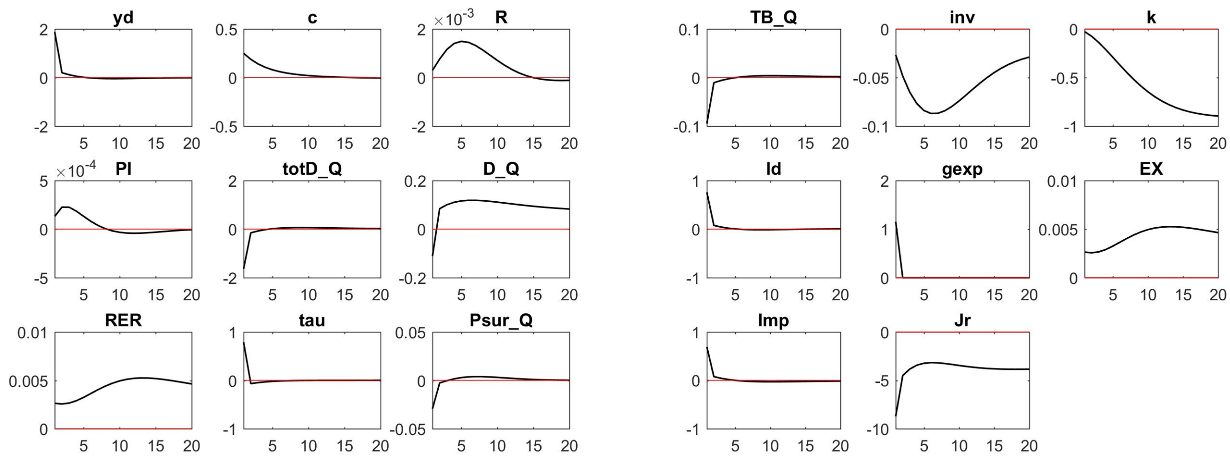


Figure 8: Impulse response functions for the FLEX model - Government expenditures



A.2 The Schäuble model

Figure 9: Impulse response functions for the Schäuble model - Productivity

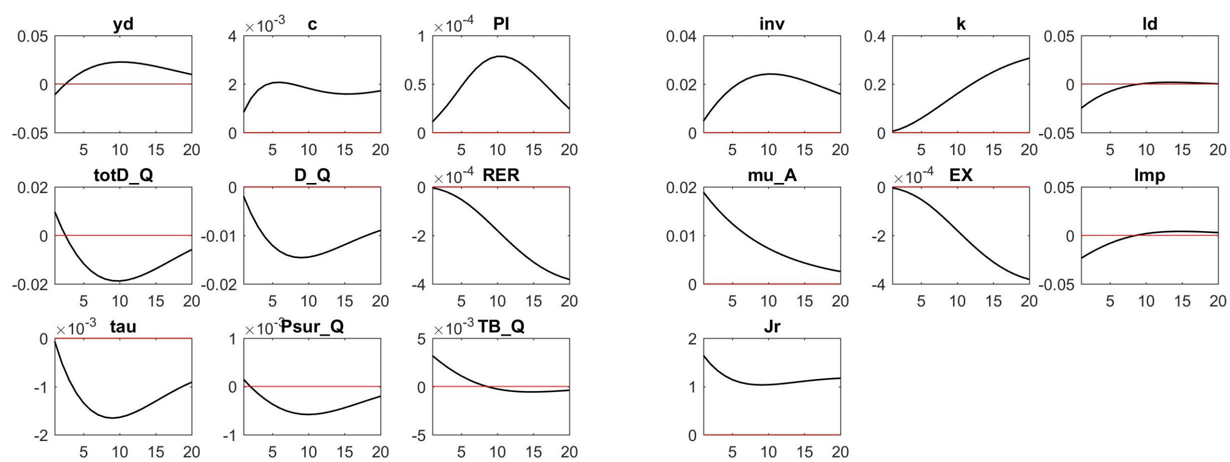


Figure 10: Impulse response functions for the Schäuble model - Foreign demand

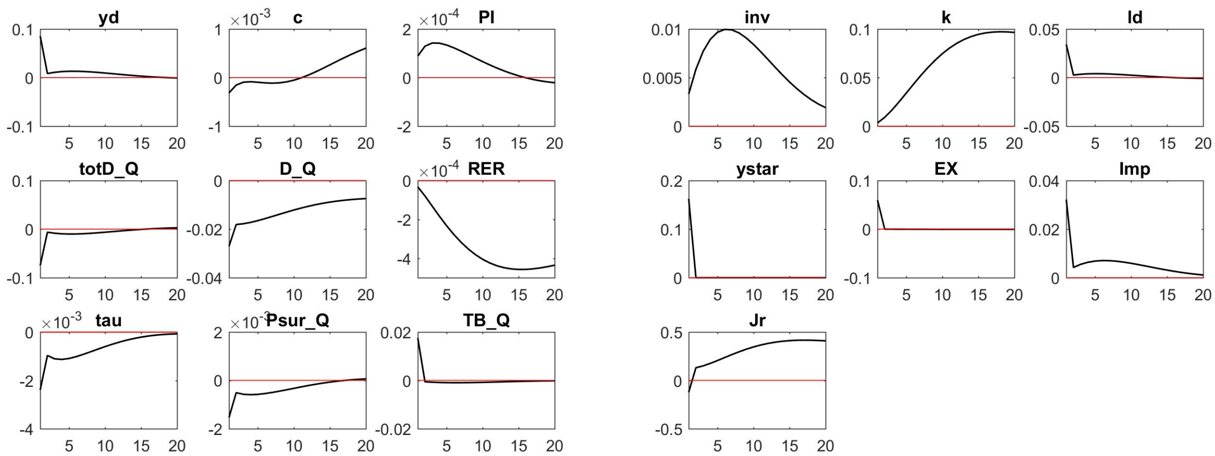
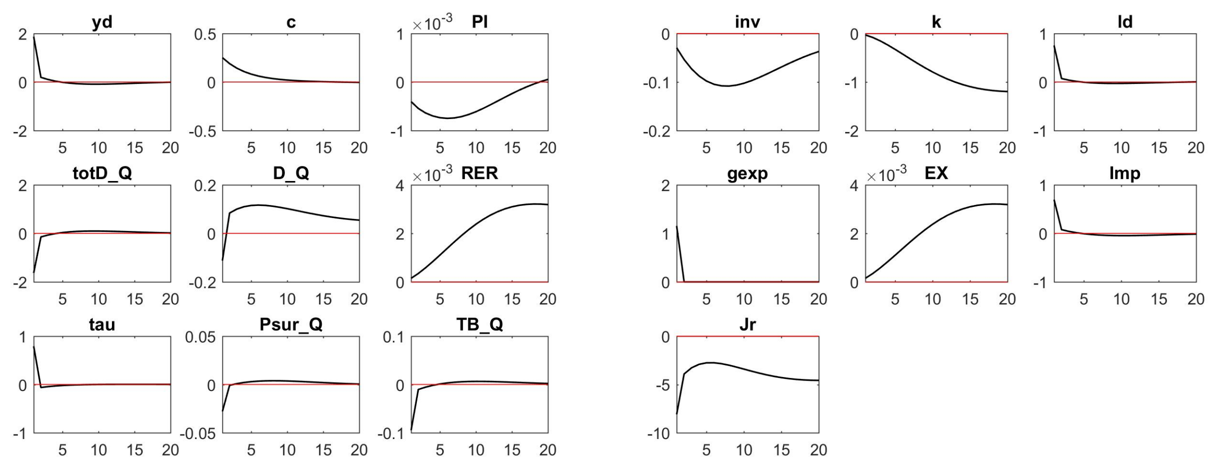


Figure 11: Impulse response functions for the Schäuble model - Government expenditures



A.3 The Tsipras model

Figure 12: Impulse response functions for the Tsipras model - Productivity

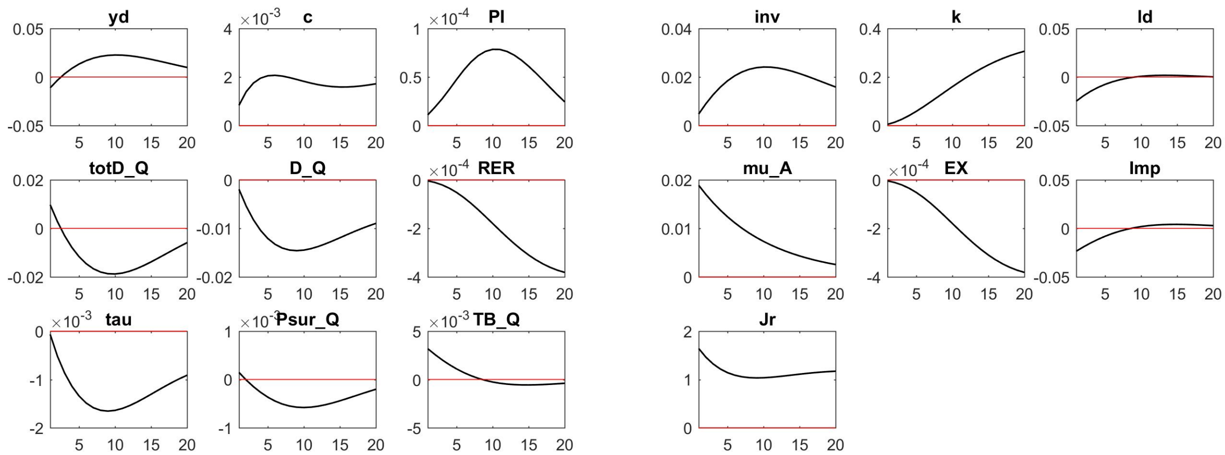


Figure 13: Impulse response functions for the Tsipras model - Foreign demand

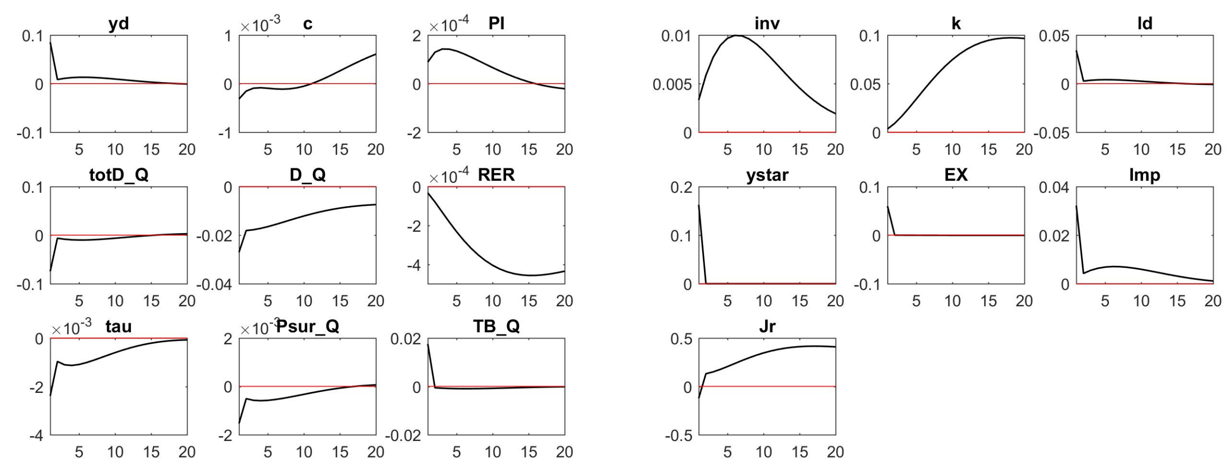


Figure 14: Impulse response functions for the Tsipras model - Government expenditures