IS THE EUROPEAN SOVEREIGN CRISIS SELF-FULFILLING?
EMPIRICAL EVIDENCE ABOUT THE DRIVERS OF MARKET SENTIMENTS

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September 2012
2012-22

Abstract

We assess the nature of the European sovereign crisis in the light of a model borrowed from the second generation of currency crises. We bring the theory to the data to empirically test the presence of self-fulfilling dynamics and to identify what may have driven the market sentiment during this crisis. To do so we estimate the probability of default of five European “peripheral” countries during January 2006 to September 2011 with a panel smooth threshold regression. Our estimation results suggest that 1/ both the fundamentals and “animal spirit” ignited the European sovereign crisis; 2/ the sovereign Credit Default Swap market (CDS), the rating agencies and the CDS of the banking sector have played dominant roles in driving market sentiments.

Key Words: European sovereign crisis, Panel Smooth Threshold Regression Models.

J.E.L Classification: E4, F3, F4, E6, H6, C23.

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We have benefited from important remarks from L. Boone, V. Borgy, J-Messier and J. Creel. We thank Wissal Ayadi and Yves Rannou for research assistance. Any remaining error are our responsibility.
Introduction

The fiscal crisis in Greece that began in the autumn of 2009 has turned into a full-fledged sovereign crisis across Europe. The ten-year process of interest rate convergence has been wiped out and two distinct categories have now emerged, the peripheral and the core European economies. Yet, although the accumulation of excessive public debt was an issue in Greece, most Eurozone countries did not display undisciplined fiscal behaviors before the crisis. In Spain, for example, the public debt amounted to less than 60% of GDP in 2009 (one of the Maastricht criteria). Even invoking a broader set of economic fundamentals seems insufficient to explain the sudden eruption of the crisis. Unemployment and the trade deficit had been increasing progressively; in fact, Ireland’s trade balance had been restored at the time of the crisis. Why, then, did the crisis erupt so abruptly and unexpectedly? Which role did speculation play during the crisis? Did the financial markets play a positive disciplinary role by forcing the governments to adjust their macroeconomic policies? Or was speculation self-fulfilling and did it make sovereign states vulnerable to erratic speculative movements? The answers to these questions are important because they will determine subsequent policy responses to the crisis and establish a new attitude about speculation.

The academic answer to these topical questions is still being debated. On the one hand, several empirical papers have evidenced nonlinearity in the spread determination model. Two different regimes have been described, a crisis and a non-crisis regime with additional fundamental factors important to the crisis regime (Aizenman et al. (2011), Gerlach et al. (2010), Montfort and Renne (2011), Borgy et al. (2011), Favero and Missale (2011)). Investors have apparently priced risk differently since the beginning of the crisis. However, in the absence of a structural model, the reason for a change in the spread determination model remains unclear. A few theoretical papers have argued in favor of the presence of self-fulfilling speculation. In these works, the surge in the spreads is due to a shift from optimistic to pessimistic market sentiments (Arghyrou and Tsoukalas (2010), Conesa and Kehoe (2011), De Grauwe (2011)). Yet, these hypotheses have not been tested empirically; and, if they are confirmed, a more precise idea about what drives market sentiment would be needed.

Similar questions motivated the development of the ”second generation” approach to currency crises1. In the second-generation model, the economic fundamentals are not sufficient to explain the sudden eruption of a crisis.

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The credibility of the government’s commitment to maintaining a fixed-exchange rate regime becomes a subject of speculation by rational investors. The expectation of devaluation increases the cost of maintaining a peg and therefore the policy-maker will move to devalue. Such interaction between investors’ beliefs and the government’s objectives give rise to self-fulfilling dynamics and multiple equilibria. In this paper, we draw on these theoretical elements to give a functional form to the European sovereign crisis. More precisely, we use Jeanne and Masson’s (2000) escape clause model that analyzes the benefits and costs to policymakers of exiting from a peg and specifies the probability of devaluation as applied to the European Monetary System crisis of 1993. We transpose their approach to model the probability of default in the context of the European sovereign crisis. Their framework has the advantage of proposing a linearized reduced form of the self-fulfilling speculation model, which is amenable to the data using econometric techniques. We extend the Jeanne and Masson’s (2000) model to reduce constraints as much as possible. In particular, we obtain a linearized form where not only the constant but also coefficients of the fundamentals are allowed to vary. In sum, we rely on their framework to assess the plausibility of self-fulfilling dynamics and multiple equilibria empirically during the European sovereign crisis.

An important limit of Jeanne and Masson’s approach, however, is that the variable that coordinates investors with optimistic or pessimistic expectations is not observable. In other words, the model is tuned on the dynamics of the beliefs of market participants. Yet, it is key to better understanding the crisis and designing proper regulations.

To address this issue, we estimate the model within a threshold regression model. This specification has the advantage of offering a parametric solution to explain the nonlinearity. Indeed, it allows the parameters to change as a function of a threshold variable. We test different market signals that may have coordinated the expectations of market participants during the crisis and induced nonlinearity. We select six candidates among the financial variables that convey public information both about the economy as well as the mood of the market participants\(^2\). Again, to relax constraints and allow an infinite number of regimes, we adopt a smooth threshold regression model that allows the coefficients to vary smoothly along the threshold variable. In sum, we use the panel smooth threshold regression approach (PSTR), initially proposed by Gonzalez et al. 2005), to estimate the sovereign spreads of five European ”peripheral” countries: Spain, Ireland, Italy, Portugal and

\(^2\)For example, the Euribor-OIS spread, the difference between the Euro Interbank Offered Rate and the overnight indexed swap rate, which reflects both the cost of lending as well as the perception of risk by banks in lending to each other.
Greece during January 2006 to September 2011. This modeling strategy allows us to test the hypothesis that the elasticities in the spread determination model changed smoothly over time according to market signals, a nonlinear pattern that we interpret as evidence of multiple equilibria.

The contributions of this paper are threefold. First, we adapt and extend an existing model of self-fulfilling speculation to obtain a structural approach to assess the nature of the European sovereign crisis. Second, we bring the model to the data. Our estimation results suggest that both the fundamentals and “animal spirit” ignited the European sovereign crisis. Third, we adopt an empirical strategy to explain the dynamics of investors’ beliefs during the crisis. We show that the sovereign Credit Default Swap (CDS) market, the rating agencies and the CDS of the banking sector have played dominant roles in driving market sentiments.

The remainder of this paper is organized as followed. In the first section, we present our theoretical framework. In Section 2, we justify our empirical strategy and, in Section 3, we present the estimation procedure and data. Our empirical results are detailed in Sections 4 and 5. In Section 6 we propose concluding remarks.

1 The Escape Clause Model and Sovereign Crises

The basic logic of self-fulfilling multiple equilibria derives from the circularity between market expectations and the policy-maker’s decision. In the seminal model, the policy-maker’s decision is about maintaining the fixed exchange rate or devaluing. In this Section, we transpose the reasoning to a situation in which the government decides to default or not. We rely on Jeanne and Masson (2000) (JM hereafter) and clarify which modifications we introduce to extend their model with the objective of reducing constraints.

The benefit of defaulting arises from the reduction of the interest burden on the outstanding debt. The authorities’ optimal policy may validate market expectations ex post; that is, default if investors expect a default. This is due to the fact that default expectations increase the policymaker’s benefit from defaulting. In fact, if investors become pessimistic, they sell government bonds, which increases the interest rate and interest rate payments and thus leads to the burden of public debt and the subsequent required austerity efforts. The benefit from defaulting then becomes higher. In sum, whether or not a default occurs depends on market expectations.

Default expectations depress output by rising the interest rate, which
makes fiscal austerity more costly. In consequence on the one hand the benefit function of default \(B(.)\) is higher than the cost (the loss of credibility in the capital market) when fundamentals, \(\phi_t\), fall short of a certain threshold, \(\phi^*\). On the other hand, this threshold results from a strategic complementarity between market expectations and the government’s decision rule. To clarify this circularity, JM’s model defines both investors’ expectations and the government’s benefit.

The expectations of identical rational investors are forward looking. They not only depend on the investors’ beliefs about future fundamentals but also on their own beliefs about the future beliefs of other investors. Rational investors know that the expectations of other investors will influence the benefits of defaulting in the next period as well as the objective probability of defaulting, \(\pi_t\):

\[
\pi_t = \text{Prob}[B(\phi_{t+1}, \pi_{t+1}) > 0 | \phi_t]
\]  

(1)

Denoting \(\phi^{*e}\) as the level of the fundamental under which investors expect the policymaker to default, the default probability is precisely the probability that fundamentals will be lower than \(\phi^{*e}\):

\[
\pi_t = \text{Prob}[\phi_{t+1} < \phi^{*e} | \phi_t] = F(\phi_t, \phi^{*e}),
\]  

(2)

where \(F(., .)\) is supposed to have a negative first partial derivative.\(^4\)\(F_1\).

In turn, the government chooses the optimal triggering level of the fundamental, \(\phi^*\), which makes its net benefit equal to zero, given investors’ expectations:

\[
\phi \mapsto B(\phi, F(\phi, \phi^{*e})).
\]

As we suppose that the benefit function is a strictly decreasing function of the fundamental, \(\phi^*\) is the unique level of the fundamental at which the net benefit is equal to zero. In sum, there is a unique equilibrium for each level\(^3\).\(\phi^{*e}\).\(^4\)This property means that the fundamental process is not negatively autocorrelated, or, in other words, that an increase in the current value of the fundamental shifts the conditional cumulative distribution function of the next period fundamental in the same direction.

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\(3\)Contrary to JM (2000), who consider the benefit from maintaining a peg, we consider the benefit from defaulting.

\(4\)This property means that the fundamental process is not negatively autocorrelated, or, in other words, that an increase in the current value of the fundamental shifts the conditional cumulative distribution function of the next period fundamental in the same direction.
of investors’ expectations\(^5\).

Solutions with multiple equilibria, which are the key feature of JM’s model, are due to shifts in investors’ expectations. More precisely, if expectations shift from being optimistic to pessimistic, investors sell government bonds, which increases the interest rate and thus the benefit to the policymaker from defaulting. The self-fulfilling character of the default expectations comes from the fact that a high default probability tends to validate itself by increasing the net benefit of default.

To formalize this idea, JM (2000) suppose \(n\) different states, \(s = 1, \ldots, n\), each one corresponding to a different level of the fundamental triggering default, in our case, \(\phi^*_s\). If the state at date \(t\) is \(s\), the policymaker defaults if and only if \(\phi_t < \phi^*_s\). At time \(t\), there are as many critical thresholds \(\phi^*_s\) as there are possible states of the economy\(^6\) as perceived by the agents. The selection of the state depends on investors expectations. Therefore, the probability of default is the sum of the default probabilities, \(F(\phi_t, \phi^*_s)\), weighted by the probability to be in one of the \(n\) different states of the economy in the future given the current state:

\[
\pi_t = \sum_{s=1}^{n} \text{Prob}(s_{t+1} = s|s_t)F(\phi_t, \phi^*_s)
\]  

(3)

From here forward, we extend JM’s model (2000) to relax linearity more broadly. We assume that the government refers to a different fundamental process, \(\phi^*_s\), at each state, \(s\). More precisely, at each state, the government refers to a combination of different fundamentals, such as debt to GDP, unemployment, etc. We assume that the weights of the fundamentals in this combination vary with the state. For example, the deeper the recession (bad state \(s\)), the higher the debt-to-GDP ratio and the closer to default. Hence, the government is more sensitive to the level of the debt-to-GDP ratio in a bad state of the economy than in a good state. We therefore have different associated critical thresholds \(\phi^*_s\).

Accordingly, we introduce the probabilities \(F^{(s_t,j)}(\phi^*_t, \phi^*_j)\) that fundamental \(\phi^*_j\) in \(t + 1\) will be lower than the expected critical threshold, \(\phi^*_j\), conditionally on the current fundamental, \(\phi^*_t\), for each couple of states, \((s_t, s_j)\).

\(^5\)See the justification in Appendix.

\(^6\)As in Jeanne and Masson (2001), we suppose a (strict) ordering of the different thresholds. But, in our case, we suppose that \(\phi^*_1 > \ldots > \phi^*_s\) if state \(s = 1\) is better than state \(s = 2\) and so on.
$j$). Equation (3) becomes:

$$\pi_t(s_t) = \sum_{j=1}^{n} \text{Prob}(s_{t+1} = j|s_t) F^{(s_t,j)}(\phi_t^{s_t}, \phi_j^{e})$$  \hspace{1cm} (4)

The circularity between market expectations and the policy-maker’s decision is represented here precisely: at any date, $t$, the government takes into account not only the state, $s_t$, and the corresponding fundamental process, $\phi_t^{s_t}$, but also the expectations of the investors through the probability $\pi_t(s_t)$ specified in Eq.4. Accordingly, at each state $s_t(= 1, \ldots, n)$, the net benefit function of the government becomes a function of $\phi_t^{s_t}$ only, as specified as follows:

$$\phi_t^{s_t} \rightarrow B[\phi_t^{s_1}, \sum_{j=1}^{n} \text{Prob}(s_{t+1} = j|s_t) F^{(s_t,j)}(\phi_t^{s_t}, \phi_j^{e})]$$

As previously, the government chooses the optimal triggering level of fundamental $\phi_t^{s_t}$, which makes its net benefit equal to zero:

$$\phi_t^{s_t} = H(s_t)(\phi_1^{e}, \ldots, \phi_n^{e})$$

In a rational expectations equilibrium, each $\phi_t^{s_t}$ should satisfy the fixed point equations:

$$\forall s = 1, \ldots, n, \phi_t^{s} = H(s)(\phi_1^{s}, \ldots, \phi_n^{s})$$

The vector of solutions ($\phi_1^{s}, \ldots, \phi_n^{s}$) corresponds to the sunspot equilibria. There are at least $n$ equilibria, but JM(2000) prove that this result implies an infinite number of equilibria. In addition, each equilibrium results from self-fulfilling dynamics. In fact, the level of the fundamental under which investors expect the policy maker to default, $\phi_t^{e}$, is validated, $\phi_t^{e} = \phi_t^{s}$. Due to the properties of the different $F$-type functions and of the benefit function, these solutions exist and are unique (see details in the appendix).

In our last step, to bring the model to the data, we need to linearize (Eq 4). We specify the fundamental processes:

\footnote{We suppose that each of these functions has the specific properties required in Jeanne and Masson (2000). See the appendix for details.}
\[ \phi^s_t = \alpha_{0,s} + \alpha'_s X_t + u_{t,s} , \]
where \( \alpha_s \) is the vector of coefficients, \( X_t \) is a vector of relevant economic fundamentals, and \( u_{t,s} \) is an i.i.d. stochastic term reflecting other exogenous determinants of the policy maker’s behavior. As in JM, we suppose that the fluctuations of the fundamentals are of limited magnitude at each state. Thus linearizing the default probability around the mean value \( \phi^{st} \) of \( \phi^s_t \) yields:

\[ \pi_t(s_t) \approx \rho_{0,s_t} + \rho'_{s_t} X_t + u_{t,s_t} \quad (5) \]

with \( \rho_{0,s_t} \) and \( \rho_{s_t} \) given by:

\[ \rho_{0,s_t} = \sum_{j=1}^{2} \text{Prob}(s_{t+1} = j | s_t) [ F^{(s_t,j)}(\bar{\phi}(s_t), \phi^*_j) + F'_1^{(s_t,j)}(\bar{\phi}(s_t), \phi^*_j) (\alpha_{0,s_t} - \bar{\phi}^*)] \]

\[ \rho_{s_t} = \sum_{j=1}^{2} \text{Prob}(s_{t+1} = j | s_t = i) F^{(s_t,j)}(\bar{\phi}^s_t, \phi^*_j) \alpha_{s_t} \]

where \( F_1 \) is the first partial derivative of \( F \) (Details are given in appendix). The probability of default is a nonlinear function of the fundamentals. Note that, unlike in JM (2000), in our model, not only the constant but also the coefficients vary with the state of the economy. The self-fulfilling speculation model to sovereign crises can now be tested empirically by testing the hypothesis of linearity. In the following, we explain our empirical strategy.

2 Empirical strategy: specification and estimation

The theoretical model involves non-linearity, a result that leads us to adopt a regime-switching approach in the estimation. Instead of adopting a Markov Switching Regime (MSR) approach à la Hamilton (1994) as JM (2000) did, we estimate the model using a threshold regression (TR) model. In fact, the MSR does not reveal the sources of nonlinearity, i.e., the reasons for shifts in investors’ expectations. It is still relevant to try to identify the variables that instantaneously coordinate the expectations of all investors. It is precisely the advantage of a TR model that allows us to characterize nonlinearity as a function of an observable variable. More precisely, the default probability can be estimated as follows:
\[
\pi_t = \left[ \rho_{0,1} + \rho_1' X_t \right] \left( 1 - g(q_t, c) \right) + \left[ \rho_{0,2} + \rho_2' X_t \right] g(q_t, c) + u_t, \quad (6)
\]

where \( g(.) \) is an indicator function:

\[
g(q; c) = \begin{cases} 
1 & \text{if } q_t \leq c \\
0 & \text{otherwise}
\end{cases}
\]

At each date, the observable variable, \( q_t \), that coordinates investors’ expectations is compared to an estimated value called the location parameter, \( c \). For illustration, \( q_t \) is the sovereign grade of the country by rating agencies. If the sovereign grade is higher than \( c \), the market is optimistic, which means that the estimated default probability equals \( \tilde{\pi}_t = \rho_{0,1} + \rho_1' X_t \) (regime 1). In turn, if the sovereign grade is downgraded below the location parameter, the market’s expectations shift to pessimistic and the estimated default probability is equal to \( \tilde{\pi}_t = \rho_{0,2} + \rho_2' X_t \) (regime 2). However, this specification allows only a sharp transition, a limit common with the MSR model. To circumvent this limit, our solution is to use a smooth transition function – a logistic function of order 1:

\[
g(q_t; \gamma, c) = \frac{1}{1 + \exp\left(-\gamma(q_t - c)\right)}, \quad \gamma > 0. \quad (7)
\]

This continuous function, bounded between 0 and 1, has an S-shape. The \( \gamma \) parameter determines the smoothness, i.e., the speed of the transition from one regime to the other. The higher the value of the \( \gamma \) parameter, the faster (i.e., sharper) the transition. There is an infinite number of intermediate regimes between regime 1 and regime 2 as defined above.

In sum, our empirical strategy has two enviable advantages over MSR. First, the introduction of an observable variable explaining the nonlinearity sheds light on what may coordinate investors’ beliefs. Second, the infinite number of intermediate regimes allows us to confirm empirically the theoretical result of an infinite number of equilibria.

From now on, we present the STR specification applied to panel data (PSTR model initially proposed by Gonzales et al. (2005)). The choice of panel data is motivated by the low time dimension of macroeconomic data. Indeed in our case, the countries of our panel are supposed to be governed by the same type of economic forces. In addition, the PSTR model is a
solution to account for individual heterogeneity (Fouquau et al., 2007). The PSTR specification of Eq(8) is the following:

\[
\pi_{it} = \mu_i + \rho_1' X_{it} (1 - g(q_{it}; \gamma, c)) + \rho_2' X_{it} g(q_{it}; \gamma, c) + u_{it} \\
= \mu_i + \rho_1' X_{it} + (\rho_2' - \rho_1') X_{it} g(q_{it}; \gamma, c) + u_{it} \\
= \mu_i + \beta_1' X_{it} + \beta_2' X_{it} g(q_{it}; \gamma, c) + u_{it} 
\] (8)

for \( i = 1, \ldots, n \), with \( \beta_1' = \rho_1' \) and \( \beta_2' = (\rho_2' - \rho_1') \). The terms \( u_{it} \) are i.i.d. errors, \( \mu_i \) represent individual fixed effects and \( q_{it} \) are the threshold variables introduced above.

The estimation of the PSTR model consists of several stages. In the first step, a null hypothesis of linearity is tested against the alternative hypothesis of a threshold specification. Then, if the linear specification is rejected, the estimation of the parameters of the PSTR model requires eliminating the individual effects, \( \mu_i \), by removing individual-specific means and then applying nonlinear least squares to the transformed model (see González et al., 2005).

In González et al.’s (2005) procedure, testing the linearity in a PSTR model (equation 8) can be done by testing \( H_0 : \gamma = 0 \) or \( H_0 : \beta_0 = \beta_1 \). In both cases, the test is non-standard since the PSTR model contains unidentified nuisance parameters under \( H_0 \) (Davies, 1987). The solution is to replace the transition function, \( g(q_{it}; \gamma, c) \), with its first-order Taylor expansion around \( \gamma = 0 \) and to test an equivalent hypothesis in an auxiliary regression. We then obtain:

\[
Spread_{it} = \mu_i + \theta_0 X_{it} + \theta_1 X_{it} q_{it} + \epsilon_{it}. 
\] (9)

In these auxiliary regressions, parameter \( \theta_1 \) is proportional to the slope parameter, \( \gamma \), of the transition function. Thus, testing the linearity against the PSTR simply consists of testing \( H_0 : \theta_1 = 0 \) in (9) for a logistic function with an usual LM test.

3 Data

The estimation of the model of Eq. (8) is subject to two major data constraints. On the one hand, the macroeconomic variables included to measure economic fundamentals have a low frequency (quarterly or monthly)
and some are available with a lag of two quarters. On the other hand, the
sovereign crisis started in 2009, representing three years of crisis at the time
of this analysis. Therefore, to obtain a critical number of observations, our
estimation is based on an unbalanced panel of the five peripheral European
countries in which the sovereign yield has been most under pressure, Greece,
Ireland, Italy, Spain and Portugal, between January 2006 and September
2011. The definitions, frequency and sources of the data are presented in
the appendix.

Our dependent variable is an estimate of the default probability, in per-
centage, measured as the sovereign bond spread, which prices the default
risk of a country. It is defined as the difference between the sovereign bond
yield and the risk-free rate of the same maturity. For each country in the
sample, we use the long-term German yield, which is the benchmark risk-
free rate for the Euro area, and the government yield of this country at
the same maturity. We rely on monthly observations of Maastricht criterion
bond yields provided by the Eurostat database.

A key choice is the set of explanatory variables included in $X_t$ in Eq (8).
We test the following variables: debt-to-GDP ratio, unemployment, unit la-
bor cost, risk, liquidity.

First, the country’s credit risk is traditionally related to fiscal sustain-
ability. We therefore include the debt-to-GDP ratio from Eurostat. The
fiscal data are revised data.

Other variables relevant in forming default expectations are those vari-
ables that may appear in the authorities’ objective function. The economic
activity and the country’s competitiveness are potential candidates because
the deterioration of these fundamentals increases the social cost of austerity
efforts and thus the benefit from defaulting. We proxy the economic activity
using the unemployment rate rather than GDP to avoid colinearity issues
with the debt-to-GDP ratio. The unit labor cost is included to proxy the
country’s competitiveness. These data are taken from Eurostat. The trade
balance (a proxy for competitiveness) is excluded from the vector of determi-
nants because of the specific behavior of Ireland, which ran a trade surplus
(the variable is positive), contrary to the other countries in the sample. This
variable was found to be not significant in other studies (De Grauwe, P., Y.
Ji, 2012). An issue with our macroeconomic data is that they are available
only at a quarterly frequency (debt, unemployment and unit labor cost). To

\footnote{We exclude deficit data to avoid collinearity with the rest of the economic variables. The correlation between the primary deficit and unemployment is 0.46 and that between the primary deficit and the unit labor cost is -0.37}
transform them to monthly frequencies, we used a local quadratic with the average matched to the source data.\(^9\)

In line with the literature, we include a variable of liquidity risk and a measure of international risk aversion. Our proxy for liquidity is the size of the government’s bond markets. For each country in the sample, liquidity is measured as the share of total outstanding Euro-denominated long-term government securities issued in the Euro zone. Data are available on a monthly basis from the European Central Bank. Following Borgy et al. (2010), our measure for international risk aversion is computed as the spread between US AAA corporate bonds and US 10-year sovereign bonds. Data are available on a daily basis from Bloomberg. We compute the average of daily data to obtain monthly frequencies. In the following, we proceed to the estimation of Eq(8) in two steps.

4 TV-PSTR Estimation Results

We start the empirical estimation of Eq(8) using a TV-PSTR and then proceed to the PSTR in the next section. In this case, the threshold variable is imposed to be time. The primary objective is to check the rejection of linearity, which will be interpreted as evidence of multiple equilibria. In fact, if the linearity hypothesis in the test presented below is rejected, this will indicate that the determination of default probability (proxied by the spread) was modified during the period of the estimation.

The TV-PSTR equation is the following:

\[
\pi_{it} = \mu_i + \beta_1'X_{it} + \beta_2'X_{it}g(T; \gamma, c) + u_{it}
\]  

(10)

for \(i = 1, ..., n\) and \(t = 1, ..., T\), \(\mu_i\) represent individual fixed effects and \(u_{it}\) are i.i.d. errors. \(X_{it}\) include: debt-to-GDP, squared debt-to-GDP, unemployment, unit labor cost, risk, liquidity. As the effect of debt is usually found to be nonlinear and this effect is captured through the introduction of the squared debt-to-GDP ratio (De Grawe and Ji, 2012), we include it to avoid the rejection of linearity due only to this effect.

\(^9\)We used Eviews software for this transformation. To check the robustness, we compared our results with a transformation based on a cubic spline with the last observation matched to the source data. We present the results in Table 4.
Table reports the estimated parameters of the TV-PSTR and the linearity tests. The result of the parameter constancy test rejects the null hypothesis of a linear relationship at the 1% significance level \((LM = 87.3)\). It confirms the theoretical model according to which the determination of the probability of default changed during the period. Other papers have also shown that the spread determination was not constant during the same period using break models or regime-switching features (Borgy et al. 2011, Mody, 2009). However it is not realistic to consider a sharp transition given the progressive increase in the spreads. Our approach has the advantage of allowing a smooth transition process (see Figure 1). The threshold value, \(c\), representing the inflexion point of the transition process, is located in March 2010. The complete modification of the spread determination occurred within one year between October 2009 and October 2010 (in October 2009, the spread determination was defined at 97% by regime 1 and in October 2010 at 97% by regime 2). Our TV-PSTR model thus correctly captures the increase in market tensions about the European sovereign starting with the announcement of the revision of the fiscal deficit in Greece by Prime Minister Papandreu in November 2009. The determination model of default probability for the European sovereign had radically changed in Fall 2010 in respect to Fall 2009, a result that we interpret as evidence of a shift in investors’ expectations.

In fact, Table 1 indicates that most coefficients increased: debt (from \(\hat{\beta}_1 = 0.05\) to \(\hat{\beta}_1 + \hat{\beta}_2 = 0.26\)), risk (from 0.48 to 1.33), and unemployment (from -0.05 to 0.25). We mention that the increase in the weight of debt is slightly reduced by the negative coefficient of the squared debt in the second regime (from 0 to -0.001). The effect of liquidity also increases significantly. While it has a sign contrary to expectations in the first regime (\(\hat{\beta}_1 = 1.54\)), it becomes highly negative in the second extreme regime (\(\hat{\beta}_1 + \hat{\beta}_2 = -13.15\)), implying that the lack of liquidity increases the probability of default (consistent with the linear findings in Beber et al. (2009)). In addition, the coefficient of our competitiveness indicator (ULC) goes from -0.04 to -0.19, contrary to the expected effect. However, eliminating ULC does not modify the value of the other estimated coefficients\(^{10}\). In total, the estimation reveals the increasingly important constraint on fiscal policy played by financial markets. At the same time, investors also became sensitive to the business cycle, a result that shows the potential counter-effective impact of fiscal austerity. The estimation results illustrate the dilemma faced by European policy makers between fiscal austerity and stimulating growth policies.

This first step confirms the existence of multiple equilibria and identifies precisely the period of transition and its specific dynamics. Now, we would

\(^{10}\)Results available upon request to the authors.
like to go one step forward and identify the drivers that instantaneously coordinated the expectations of all investors. To do so, in the following section, we proceed with the estimation of a PSTR model that allows the nonlinearity to depend on an observable variable.

5 Sunspots or observable drivers of investors expectations?

We test different market signals that may have coordinated the expectations of market participants. We recall that the PSTR specification of the spread is as follows:

$$\pi_{it} = \mu_i + \beta_1' X_{it} + \beta_2' X_{it} g(q_{it}; \gamma, c) + u_{it}$$

for $i = 1, \ldots, n$ and $t = 1, \ldots, T$, $\mu_i$ represent individual fixed effects and $u_{it}$ are i.i.d. errors. In order to estimate the PSTR model, we need threshold variables $q_{it}$. We select six candidates among financial variables that convey public information both about the economy as well as the mood of the market participants. The candidate threshold variables $q_{it}$ are: rating, sovereign CDS, bank CDS, i-traxx Europe, i-traxx Crossover, Euribor-OIS spread.

First, rating is the average of the sovereign grades published by the three main international rating agencies, Standard and Poors, Moodys and Fitch (taken from Reuters). In fact, the sovereign crisis brought credit ratings agencies to the front. Rating agencies help investors overcome their lack of information about the variables that will determine whether a borrower will service debt. These agencies use qualitative letter ratings in descending order\(^{11}\). We use the linear transformation of Afonso, Gomes and Rother (2007) to obtain a continuous numerical scale from the letter ratings.

Second, sovereign CDS is the premium of sovereign credit default swaps, which are bilateral contracts between a buyer and seller under which the seller sells protection against the credit risk of the reference country. The CDS premium, the insurance cost, is used here to measure market assessments of the health of borrowers and the likelihood of default. We select the 5-year maturity, which is the most traded contract in the CDS market, taken from Bloomberg.

\(^{11}\) S & P and Fitch use similar ratings from AAA to CCC-, while Moody’s system goes from Aaa to Caa3. Although they do not use the same qualitative codes, there is a correspondence between each rating level.
Third, bank CDS denotes the premium of CDS on the main banks in the country where the default probability is estimated. The nexus of the financial sector, sovereign credit risk, is a feature of financial crises in general (Reinhart and Rogoff, 2009) and the European sovereign crisis in particular (De Grauwe, 2010, Acharya, Drechsler, Schnabl, 2011). To avoid a credit crunch and loss of real sector output, governments engaged in large-scale financial-sector bailouts. Such bailouts are costly because they require immediate issuance of additional debt by the sovereign. This leads to an increase in the sovereign’s credit risk. We use the average of the CDS premia of major banks weighted by the CDS market volume, taken from Reuters.

Fourth and fifth, we consider two broader indicators of the health of the corporate sector in Europe: i-Traxx Europe and i-Traxx Crossover are credit default swap index products. i-Traxx Europe comprises the most liquid 125 CDS referencing European investment grade credits while Crossover comprises the most risky 40 constituents at the time the index is constructed.

Last, Euribor-OIS spread captures the difference between the Euro Interbank Offered Rate and the overnight indexed swap rate. It reflects the risk banks perceive in lending to each other (the higher the spread, the more reluctant the banks are to lend to each other). The three last variables are taken from Reuters.

For each model, the first step is to test the linear specification of the spread against a specification with threshold effects. The results of these tests are reported in Table 2. The linearity tests clearly reject the null hypothesis of a linear relationship regardless of which threshold variable is included in the specification. The remarkably high level of rejection makes the presence of multiple equilibria a given. This is consistent with our preliminary result from the time-varying specification. The second step consists of selecting the best threshold variables, with the objective of identifying the drivers that mostly coordinate investors expectations. As suggested by González et al. (2005), the "optimal" threshold variable corresponds to the variable that leads to the strongest rejection of the linearity hypothesis. Among the six variables tested, the sovereign CDS is unambiguously the market variable that drives investors’ expectations as it yields the strongest rejection statistics of the null hypothesis (LM= 282). This first result illustrates the crucial role that the sovereign CDS market has played during the crisis. It is consistent with the findings of Delatte, Gex and Lopez (2012) pointing to the amplification role played by the credit derivative market in times of market distress. According to the estimation, the CDS market plays a more important role in coordinating investors’ expectations than do the rating agencies, which rank second, also with very high rejection statistics (LM= 231). This high rejection score indicates that the rating agencies are an important cat-
analyst of market expectations. Bank CDS rank third, also with high rejection statistics (LM = 186). This suggests that the borrowing cost of an economy is highly sensitive to the health of the country’s domestic private financial sector. According to the theoretical model, deterioration of the banking sector’s balance sheet changes the market’s expectations about the country’s default probability. The market shifts to a pessimistic equilibrium and sovereign default becomes more likely. This is precisely what may explain the sudden acceleration of the European crisis during summer 2011 when several major European banks published deteriorated balance sheet figures due to sovereign debt exposure. In comparison, the European corporate CDS indices (i-Traxx Europe and i-Traxx Crossover) and the Euribor-OIS spread have much lower rejection statistics (LM = 51.8, 77.9 and 39.9), which suggests that they are not good candidates for threshold variables. In total, the PSTR specification identifies three market variables that coordinate investors’ expectations, with the sovereign CDS market clearly issuing the leading signal.

We examine more precisely the impact of these variables on the determination of default probabilities by investors. We consider which determinants have their weight changed most when the sovereign CDS premia increase (equivalently, bank CDS premia increase or the rating is downgraded). We also consider which determinants matter most to investors when their expectations based on these indicators become strongly pessimistic.

Table 3 reports the value of the estimated coefficients in the three selected models. The coefficients are defined at each date and for each country as weighted averages of the values obtained in the two extreme regimes. The coefficients in the PSTR model can therefore be different from the estimated parameters defined in the extreme regimes, i.e., the parameters $\hat{\beta}'_1$ and $\hat{\beta}'_1 + \hat{\beta}'_2$ in equation 11. For each model, we first need to interpret the sign of parameter $\hat{\beta}_2$, which indicates an increase ($\hat{\beta}_2 > 0$) or a decrease ($\hat{\beta}_2 < 0$) in the estimator as the threshold variable increases.

The effect of the threshold variables on the determinants of the spread is reported for the three selected models in Table 3. We find similar patterns for a majority of the coefficients, which suggests that our estimations are robust. The estimated coefficient of the determinant variables risk and unemployment unambiguously increase in the second regime. The way investors price the fiscal situation is captured by the interaction of debt and squared debt, which makes a direct interpretation of the coefficients impossible. We plot it below. The coefficient of ULC becomes negative in the second regime, which is contrary to the expected sign. Only the evolution of liquidity is ambiguous as it is not consistent across the three models. Removing ULC and
liquidity does not change our results\textsuperscript{12}.

We would like to examine the variation in the impact of each determinant during the period. However, as mentioned above, the coefficients could be different from the estimated coefficients in the extreme regimes. Therefore, we plot the evolution of each estimator multiplied by the variable using the historical values of the threshold variable (for example, \( \hat{\beta}'_1 \text{risk} + \hat{\beta}'_2 \text{risk} \times g(q_it; \gamma, c) \)). (Figure 2). To interpret the proper evolution of the fiscal situation, we plot the sum of debt and squared debt multiplied by their respective coefficients. For the sake of synthesis and for statistical argument, we do this exercise for the sovereign CDS model only. In fact, this model performs better in rejecting linearity and minimizes the sum of the squared residuals. In sum, this specification best captures the determination model used by investors to price the spread of a country.

We note that sovereign CDS continuously increased during the period. Figure 2 indicates that the fiscal situation has become more and more influential in the determination of European spreads during the period, a finding that confirms our time-varying results and the existing results in the recent literature (Haugh et al. 2009, Borgy et al. 2011). In addition, this influence becomes primary at the end of the period. For example, in September 2011, the estimated fiscal situation alone implied a spread equal to 796 bp in Portugal, while it was 951 bp in reality. Figure 2 also plots the evolution of the coefficients of risk and unemployment. The graphical representation indicates that the influences of unemployment and risk are almost null in the optimistic state and they become very important in the pessimistic state. In particular, the level of unemployment was not priced in the spread before the crisis but it became a significant driver afterwards, which confirms the argument that the business cycle matters to investors. In sum, unemployment adds to the fiscal situation in the macroeconomic variables monitored by investors, a pattern that implies no simple economic resolution of the crisis. We comment on our findings in the following concluding remarks.

6 Concluding remarks

Here, we have assessed the nature of the European sovereign crisis in the light of a model borrowed from the second generation of currency crises. We estimated the probability of default using panel non-linear estimation methods, the TV-PSTR and PSTR models. Two important objectives were to empirically test the presence of self-fulfilling dynamics and to identify what may have driven the market sentiment during this crisis. In total, our PSTR estimation confirms that the determination model of default probability is

\textsuperscript{12}Results available upon request.
not linear, a result that we interpret as evidence of multiple equilibria and self-fulfilling mechanisms during the European crisis. The progressive deterioration of the market sentiment about peripheral sovereigns has been validated by an increase in these countries’ spreads. The contagion from Greece to the rest of the peripheral countries has probably operated through simultaneous shifts in market sentiment. These findings provide evidence that a closer monitoring of market activity is needed. This market perception is influenced by the sovereign CDS market, rating agencies and the market’s perception of risk in the banking sector. The point is that the sovereign CDS market appears to be highly concentrated with 80% of the transactions operated by five big players (DTCC, 2012). The risk is that abrupt movements in the CDS market may generate panic in the underlying sovereign bond market. More transparency in this market is crucial to avoid spoiling the efforts made in most countries to balance their budgets. We hope that the framework presented in this paper opens opportunities for new research. In particular, it would be insightful to relate the volumes traded in the sovereign and banking CDS markets with the nonlinear effects evidenced here. This would constitute a step forward in assessing the plausibility of speculative attacks against sovereigns.
Table 1: Linearity tests and estimation of the probability of default with a Time-Varying PSTR model

<table>
<thead>
<tr>
<th>Determinants</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>0.055$^{***}$</td>
<td>0.209$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(4.74)</td>
<td>(5.43)</td>
</tr>
<tr>
<td>Squared Debt</td>
<td>0.000</td>
<td>-0.001$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.83)</td>
<td>(-3.51)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.048$^{***}$</td>
<td>0.297$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(-3.19)</td>
<td>(7.51)</td>
</tr>
<tr>
<td>Unit Labor Cost</td>
<td>-0.011</td>
<td>-0.167$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(-0.79)</td>
<td>(-6.34)</td>
</tr>
<tr>
<td>Liquidity</td>
<td>1.543$^*$</td>
<td>-14.698$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(1.78)</td>
<td>(-6.14)</td>
</tr>
<tr>
<td>Risk</td>
<td>0.480$^{***}$</td>
<td>0.851$^*$</td>
</tr>
<tr>
<td></td>
<td>(9.85)</td>
<td>(1.77)</td>
</tr>
</tbody>
</table>

Smooth Parameter $\gamma$       0.529
Loc Parameter                  51.5
Linearity Test                 87.26$^{***}$
RSS                             76.28
Information Crit. BIC            -1.22

Notes: The T-stat in parentheses are corrected for heteroskedasticity. ($^*$): significant at the 10% level; ($^{**}$): significant at the 5% level and ($^{***}$): significant at the 1% level. $\beta_1$ and $\beta_2$ correspond to the coefficient in Eq (11). $\beta_1$ is the coefficient in the first extreme regime. The coefficient in the second extreme regime is $\beta_1 + \beta_2$.

Table 2: Linearity Tests with a PSTR model

<table>
<thead>
<tr>
<th>Sovereign CDS CDS Rating ItraX Itrax EURIBOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS Bank Europe OIS</td>
</tr>
<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>LM</td>
</tr>
<tr>
<td>282.2</td>
</tr>
<tr>
<td>p-value</td>
</tr>
<tr>
<td>(0.00)</td>
</tr>
<tr>
<td>RSS</td>
</tr>
<tr>
<td>19.83</td>
</tr>
<tr>
<td>BIC</td>
</tr>
<tr>
<td>-2.57</td>
</tr>
</tbody>
</table>

Notes: The corresponding $LM$ statistic has an asymptotic $\chi^2(p)$ distribution under $H_0$. The corresponding p-values are reported in parentheses.
Table 3: Estimation of the probability of default with a PSTR model (quadratic transformation)

<table>
<thead>
<tr>
<th>Determinants</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sovereign CDS</td>
<td>CDS Bank</td>
<td>Rating</td>
</tr>
<tr>
<td>Debt</td>
<td>$-0.030^{**}$</td>
<td>0.211***</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(-2.61)</td>
<td>(4.51)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>Squared Debt</td>
<td>$0.000^{***}$</td>
<td>$-0.001^{**}$</td>
<td>$-0.001^{**}$</td>
</tr>
<tr>
<td></td>
<td>(3.68)</td>
<td>(-4.48)</td>
<td>(-2.14)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>$-0.099^{***}$</td>
<td>0.335***</td>
<td>$-0.253^{***}$</td>
</tr>
<tr>
<td></td>
<td>(-2.97)</td>
<td>(3.63)</td>
<td>(-3.13)</td>
</tr>
<tr>
<td>Unit Labor Cost</td>
<td>0.045**</td>
<td>$-0.062^{**}$</td>
<td>0.056*</td>
</tr>
<tr>
<td></td>
<td>(2.16)</td>
<td>(-2.02)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>Liquidity</td>
<td>1.694*</td>
<td>$-4.31$</td>
<td>19.314***</td>
</tr>
<tr>
<td></td>
<td>(1.76)</td>
<td>(-0.92)</td>
<td>(5.80)</td>
</tr>
<tr>
<td>Risk</td>
<td>$-0.2$</td>
<td>1.447</td>
<td>$-2.184^{***}$</td>
</tr>
<tr>
<td></td>
<td>(-0.94)</td>
<td>(1.71)</td>
<td>(-4.05)</td>
</tr>
<tr>
<td>Smooth Parameter $\gamma$</td>
<td>0.002</td>
<td>0.003</td>
<td>0.554</td>
</tr>
<tr>
<td>Loc Parameter</td>
<td>466.1</td>
<td>9.06</td>
<td>15.7</td>
</tr>
<tr>
<td>RSS</td>
<td>19.8</td>
<td>50.9</td>
<td>57.8</td>
</tr>
<tr>
<td>Information Crit. BIC</td>
<td>-2.57</td>
<td>-1.63</td>
<td>-1.66</td>
</tr>
</tbody>
</table>

Notes: The $T$-stat in parentheses are corrected for heteroskedasticity. (*) significant at the 10% level; (**) significant at the 5% level and (***) significant at the 1% level. $\beta_1$ and $\beta_2$ correspond to the coefficient in Eq (11). $\beta_1$ is the coefficient in the first extreme regime. The coefficient in the second extreme regime is $\beta_1 + \beta_2$. 
Table 4: Estimation of the probability of default with a PSTR model (cubic transformation)

<table>
<thead>
<tr>
<th>Determinants</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TV-PSTR</td>
<td>Sovereign CDS</td>
</tr>
<tr>
<td>Debt</td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(4.97)</td>
<td>(5.55)</td>
</tr>
<tr>
<td>Squared Debt</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(-3.61)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.05</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(-3.49)</td>
<td>(7.54)</td>
</tr>
<tr>
<td>Unit Labor Cost</td>
<td>-0.01</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(-0.52)</td>
<td>(-6.41)</td>
</tr>
<tr>
<td>Liquidity</td>
<td>1.88</td>
<td>-15.47</td>
</tr>
<tr>
<td></td>
<td>(2.08)</td>
<td>(-6.66)</td>
</tr>
<tr>
<td>Risk</td>
<td>0.52</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(9.63)</td>
<td>(1.15)</td>
</tr>
</tbody>
</table>

| Smooth Parameter $\gamma$ | 0.567 | 0.003 |
| Local Parameter           | 51.7  | 446.1 |
| Linearity Test            | 82.0*** | 275.5**** |
| RSS                        | 74.3  | 19.8  |
| Information Crit. BIC     | -1.25 | -2.57 |

Notes: strut Notes: The T-stat in parentheses are corrected for heteroskedasticity. (*) significant at the 10% level; (**) significant at the 5% level and (***) significant at the 1% level. $\beta_1$ and $\beta_2$ correspond to the coefficient in Eq (11). $\beta_1$ is the coefficient in the first extreme regime. The coefficient in the second extreme regime is $\beta_1 + \beta_2$. The variable debt is with a cubic spline.
Figure 1: Transition function with a TV-PSTR
Figure 2: Impact of the determinant factors with a PSTR model

Note: we plot the evolution of each estimator multiplied by the variable along the historical values of the threshold variable (for example, \( \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 g(q_t; \gamma, c) \)). with \( x_1 \) is an explanatory variable defined in the text.
Appendix 1: Existence of multiple Sunspot equilibria

At each date \( t \), the probability the investors attribute to default for next period is the sum of the (conditional) default probabilities \( F^{(s_{t},j)}(\phi_{t}^{s_{t}}, \phi_{j}^{*e}) \) in the different states \( j \) at date \( t+1 \), weighted by the corresponding transition probabilities, i.e.:

\[
\pi_{t}(s_{t}) = \sum_{j=1}^{n} \text{Prob}(s_{t+1} = j/s_{t}) F^{(s_{t},j)}(\phi_{t}^{s_{t}}, \phi_{j}^{*e})
\]

where \( \phi_{j}^{*e} \) denote the expected value of the critical threshold in state \( j \). As in JM, we suppose that the partial derivative of each functions \( F^{(s_{t},j)} \) with respect to \( \phi_{t}^{s_{t}} \) is negative. This property means that an increase in the current value of the fundamental shifts the conditional cumulative distribution function of the next period fundamental in the same direction. Given these expectations, in each state \( s_{t} \) at date \( t \), the net benefit function of the policymaker is a function of the current value \( \phi \) of \( \phi_{t}^{s_{t}} \):

\[
\phi \rightarrow B(\phi, \pi_{t}(s_{t})) = B(\phi, \sum_{j=1}^{n} \text{Prob}(s_{t+1} = j/s_{t}) F^{(s_{t},j)}(\phi, \phi_{j}^{*e})) \quad (1)
\]

We suppose that the function:

\[
(\phi, \pi) \rightarrow B(\phi, \pi)
\]

is respectively decreasing and increasing with respect to \( \phi \) and \( \pi \). First, the fundamental phi reflects the sustainability level of the country’s economy. If it is high, the state is rather good and the benefit from default is low; second, when the default probability increases, the benefit from default also increases, because the interest rates increase as explained in the text. Thus the function defined in (1) is decreasing in \( \phi \); indeed, its partial derivative with respect to \( \phi \) has for expression:

\[
B_{1}(\phi, \pi_{t}(s_{t})) + \sum_{j=1}^{n} \text{Prob}(s_{t+1} = j/s_{t}) B_{2}(\phi, \pi_{t}(s_{t})) F_{1}^{(s_{t},j)}(\phi, \phi_{j}^{*e})
\]

and is strictly negative because \( B_{1} < 0, B_{2} > 0 \) and \( F_{1}^{(s_{t},j)} < 0 \).

Thus the government chooses the unique level of \( \phi \) for which the net benefit is equal to zero. We denote this value by \( \phi_{s_{t}}^{*} = H_{(s_{t})}(\phi_{1}^{*e}, ..., \phi_{n}^{*e}) \).
In this way we define n values $\phi^*_s$ for the n possible values of $s$. In a rational expectations equilibrium, each $\phi^*_s$ should be equal to the expected corresponding threshold $\phi^*_e$ and the set of these thresholds should therefore satisfy the fixed point equations:

$$\forall s, \phi^*_s = H(s)(\phi^*_1, ..., \phi^*_n)$$

We suppose that:

$$\phi^*_1 > ... > \phi^*_n$$

if state $s = 1$ is better than state $s = 2$ and so on.

Now, the arguments of Jeanne and Masson (2001) apply. The fundamental-based equilibria can be viewed as degenerate cases of the sunspot ones, when the transition probabilities Prob($s_{t+1} = j/s_t$) are equal to 1 if $s_{t+1} = s_t$ and 0 otherwise and the F-type functions $F^{(i,j)}$ reduce to one unique function $F$. In that case, the economy never jumps and always remains in its initial state; thus, JM prove that there exists at least one equilibrium and there may be multiple fundamental-based equilibria associated with different thresholds, provided that the function $F$ and the benefit function $B$ have the good properties mentioned above.

Now, let us turn to the sunspot equilibria and remark that the probability that economy shifts to higher states than state 1 in the next period increases investors’ default expectations and decreases the corresponding fundamental threshold chosen by the policymaker to a level $\phi^*_1 = H_1(\phi^*_1, ..., \phi^*_n) < H(\phi^*_1)$, because the benefit function decreases with the level of the fundamental process. Similarly, the threshold $\phi^*_n = H_n(\phi^*_1, ..., \phi^*_n)$ associated with the worst state $n$ has to be higher than $H(\phi^*_n)$. These inequalities can be consistent with the inequality $\phi^*_1 > \phi^*_n$ if and only if there are multiple solutions in the case of fundamental-based equilibria with the shape of function $H$ as the one depicted in JM (p.334) and with $\phi^*_n \in [0, \phi_I]$ and $\phi^*_1 \in [\phi_{II}, \phi_{III}]$.

So provided that the $F^{(i,j)}$ functions on one hand and the functions $F$ and $B$ on the other hand have the good properties expressed before, one can claim that there exist multiple sunspot equilibria.

**Appendix 2: Linearization of the default probability**

First, we specify the fundamental variable as a linear combination of macroeconomic indicators, depending on the underlying state:

$$\forall t, \phi^*_t = \alpha_{0,s_t} + \alpha'_{s_t} X_t + u_{t,s_t}$$  \hspace{2cm} (2)
with \( X_t \) denoting a vector of different economic indicators.

Moreover, in the lines of Jeanne and Masson (2001), we suppose that the fundamental processes \( \phi_t^s \) don’t deviate too much from their mean values \( \bar{\phi}^s \):

\[
\forall t, \forall s = 1, 2 \; \phi_t^s = \bar{\phi}^s + \delta \phi_t^s
\]

where \( \delta \phi_t^s \) is supposed to be of limited magnitude.

Thus, the default probability specified as previously:

\[
\pi_t(s_t) = \sum_{j=1}^{2} \text{Prob}(s_{t+1} = j/s_t)F(s_{t},j)(\phi_{s_{t}}^{s}, \phi_{j}^s)
\]  

(3)

can be linearized around \( \bar{\phi}^{(s_t)} \) as follows:

\[
\pi_t(s_t) \approx \sum_{j=1}^{2} \text{Prob}(s_{t+1} = j/s_t)[F(s_{t},j)(\bar{\phi}^{s}, \phi_{j}^s) + F_1(s_{t},j)(\bar{\phi}^{s}, \phi_{j}^s)(\alpha_{0,s_t} - \bar{\phi}^s)] + u_{t,s_t}
\]

Accordingly, the previous equation can be rewritten as:

\[
\pi_t(s_t) \approx \rho_{0,s_t} + \rho_{s_t} X_t + u_{t,s_t}
\]  

(4)

with \( c_{s_t} \) and \( \theta_{s_t} \) given by:

\[
\rho_{0,s_t} = \sum_{j=1}^{2} \text{Prob}(s_{t+1} = j/s_t)[F(s_{t},j)(\bar{\phi}^{s}, \phi_{j}^s) + F_1(s_{t},j)(\bar{\phi}^{s}, \phi_{j}^s)(\alpha_{0,s_t} - \bar{\phi}^s)]
\]  

(5)

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