Capital Depreciation Allowances and Redistribution in a Neoclassical Growth Model

Günther Rehme*
Technische Universität Darmstadt†

April 30, 2010

Abstract

In this paper it is analyzed whether capital depreciation allowances when coupled with capital income taxes are good instruments for redistribution in the long run. The analysis is placed in a simple two-agent-economy with capital owners and workers, and suggests that in the short run and absent optimizing behaviour accelerated depreciation is good for growth and may stabilize investment in a recession but is generally bad for redistribution. The opposite holds for capital income taxes. However, when the private sector and the government act optimally the optimal depreciation allowance is maximal in the long run. This removes the accumulation distortion of capital income taxes. Furthermore, the latter and so redistribution may be nonzero, depending i.a. on the social weight of those who receive redistributive transfers, the distribution of pre-tax factor incomes, and the intertemporal elasticity of substitution. It is argued that accelerated depreciation allowances, extensively used in the current economic crisis, are an important indirect tool for redistribution.

KEYWORDS: Capital Depreciation Allowances, Capital Income Taxes, Redistribution, Economic Growth

JEL classification: O41, H21, D33

* This paper was written in 2008/9 while I was a visiting professor at Humboldt University Berlin. I thank the department for its hospitality and stimulating research environment. Furthermore, I have benefitted from discussions with Holger Gerhardt and Marco Runkel on the topic of this paper. Of course, all errors are my own.
† Correspondence TU Darmstadt, Department of Economics, FB 1/ VWL 1, Schloss, D-64283 Darmstadt, Germany. phone: +49-6151-162219; fax: +49-6151-165553; e-mail: rehme@hrzpub.tu-darmstadt.de
1 Introduction

As a consequence of the current global financial and economic crisis many countries have mounted fiscal stimulus packages in order to fight the negative effects resulting from the economic downturn. In a large number of countries one such measure has been to increase capital depreciation allowances. For instance, under the economic stimulus package of the United States the allowance has been increased to fifty percent that can be deducted from taxable income within the first year. In Germany, which is usually considered to be very conservative about granting capital depreciation allowances, it is now possible to write off 45 percent of a business investment within the first year. Furthermore, France introduced more generous depreciation allowances whereby investment outlays can be written off at a significantly higher rate (up to 66 percent) and more quickly than before. Similarly, Brazil reduced the time period from 24 to 12 months for which investment outlays for machinery and equipment can be recovered, as long as they are intended for production of goods and services.\(^1\) The motivation for these measures in those countries and many others is seen in the investment promoting effect of capital depreciation allowances. This is well known.

However, the long-run distributional implication of depreciation allowances is less clear. For instance, if redistributive transfers have to be financed with taxes, then higher capital depreciation allowances reduce tax revenues and therefore appear to be a bad instrument for redistributive policies. This is the problem that is being analyzed in this paper.

In particular, in the present analysis capital income taxes are coupled with depreciation allowances in a neoclassical growth setup. Therefore, I relate to the literature on optimal capital income taxation. According to the “celebrated” result by Judd (1985) and Chamley (1986) capital income taxes are no good instrument for pure redistribution in a neoclassical growth framework.\(^2\)

The intuition for the result is astounding. Even workers who may not own cap-

\(^1\)For the U.S. see the Economic Stimulus Act of 2008 (Treasury Department, Tres. Reg. Sec. 1.168(k)-1), for Germany, see “Konjunkturpaket 2008” (Bundesministerium der Finanzen, November 2008), for France see “La Loi de Finance 2009”, December 2008, and for Brazil see “Provisional Measure 428”, May 2008, as part of Brazil’s new Productive Development Policy (PDP).

\(^2\)Sargent and Ljungqvist (2004) call this a “celebrated result”. Similar results have been obtained by many authors as, for example, Lucas (1990). Guo and Lansing (1999), fn. 1, point out that a similar result had already been discussed by Arrow and Kurz (1970), p. 191-203, in the context of a neoclassical growth model with inelastic labour supply and productive public expenditures.
ital and may, therefore, not accumulate resources might benefit more from higher steady state wages resulting from nondistorted accumulation with zero taxes than having redistributive transfers now at the expense of a lower steady state capital stock and so wages in the long run.

The authors then contemplated other capital income policy packages, including consumption taxes, and basically found the same result as in, for instance, Judd (1999). However, that capital income taxers are not good instruments for redistribution need not always hold, as was, for instance, shown by Lansing (1999). He found a counterexample for a world where agents have logarithmic utility.

Also, in an endogenous growth framework with productive government expenditure financed by a capital income tax, Rehme (1995) shows that zero capital income tax rates are not optimal when the latter are coupled with investment subsidies. In that paper there is full expensing of investment outlays by assumption so that the tax scheme amounts to a consumption tax on capital owners. That the optimal capital income tax rate may be nonzero in growth contexts has, for instance, been shown by Uhlig and Yanagawa (1996) and others.

In this paper I relate to these findings in a simple two-agent, neoclassical growth framework. It is shown that coupling capital income taxes with accelerated depreciation allowances to finance pure redistributive transfers to the non-accumulated factor or production ("workers") may also imply a nondistortionary policy package, similar to a consumption tax on "capitalists". Most governments redistribute resources but also grant depreciation allowances to be deducted from collectable tax revenues. This appears to be a pervasive phenomenon in most countries. Hence, these realistic features may justify the policy package under consideration.

In this context, I follow Sinn (1987), ch. 3, who introduced a simple way to analyze the possibility of granting accelerated depreciation allowances. Notice that accelerated depreciation allowance are usually granted above the true depreciation rate of a capital good. We will distinguish between them in this paper, but do not really address the question how to determine the true depreciation. This is common in the literature. The details are presented in the main text.\footnote{The importance of studying depreciation allowances in more detail can, for instance, be motivated further by historical events. For example, Sinn notes that in the United Kingdom up to the mid 1980s there was a 75 percent depreciation of industrial buildings in the first year and straight line depreciation at that time. Plants could be written off 100 percent in the first year. In the United States at the beginning of the Reagan administration the so-called Accelerated Cost Recovery System (ARCS), introduced in 1981, had provisions according to which the majority of plants could be fully depre-}
As is known, depreciation allowances are similar in nature to granting investment subsidies. See, for example, Atkinson and Stiglitz (1980), ch. 5.3. Thus, the present paper also relates to the finding of e.g. Jones, Manuelli, and Rossi (1997) who show for a representative agent framework that an investment subsidy can offset the growth distortion associated with a capital income tax and that a consumption tax is the optimal second best policy. For a related argument see also Guo and Lansing (1999). A similar point was made by Kaldor (1955) and Fisher (1937) as well who basically proposed that taxable “income” should be “income after savings are taken out”. See Fisher (1937), p. 54. In the present paper, however, we contemplate a (simple) two-agent, closed economy framework with a capital-income-cum-depreciation-allowance (CICDA) tax scheme.

In this paper the following results are then derived under that tax scheme: For arbitrary, i.e. non-optimizing behaviour of the agents (capital owners and workers) and the government higher depreciation allowances are generally good for economic growth, but bad for redistribution. In turn, higher capital income taxes are bad for growth, but good for redistribution. Furthermore, it is shown that when there is a sudden drop in the real return to capital, as happens most often during economic crises, the government should increase the accelerated capital depreciation allowance or cut the capital income tax rate, if it wishes to stabilize the real return to investment. All these results would correspond to what one usually expects.

However, when the agents and a benevolent government that represents the weighted interests of the workers and the capitalists act optimally, it turns out that the government would mostly find it optimal to grant a maximal depreciation allowance. The reason is that that would remove the distorting effect policy has on capital accumulation. Thus, the paper shows that full expensing of investment outlays is optimal for the long run. This result does not depend on the social weights attached to the interests of different factor owners. Even an entirely pro-labour government would choose maximal depreciation allowances, even though this could

---

3 In that sense the assumption of full expensing of investment outlays made in Rehme (1995) is endogenized in the present paper and found to be optimal in the present neoclassical growth framework.

4 In that sense the assumption of full expensing of investment outlays made in Rehme (1995) is

ciated over just five years. That was clearly less than the economic life of plants. The ARCS would have entailed massive losses in tax revenues and so the government “took fright” and abolished the arrangement, as, among other things, the aim of the Reagan administration was to cut taxes. Shortly after, Reagan was re-elected in 1984 and a more conservative rule was put into place in the U.S. tax reform of 1986.
mean less tax revenues and so less redistributive transfers. Thus, in the model accelerated capital depreciation allowances serve as an important indirect redistribution device, because transfers ultimately depend on the capital income tax rate chosen.5

As regards the latter, it turns out that capital income tax rates may optimally be non-zero under quite plausible conditions. As one might expect from actual taxation by governments the optimal choice of capital income taxes and so redistribution in the long run depends on the social weight of those who receive redistributive transfers, the physical wear and tear of capital, the distribution of pre-tax income among individuals, and the intertemporal elasticity of substitution.

The analytic results are checked against a numerical simulation based on standard calibrated parameters for the U.S. from the business cycle literature. That exercise sheds light on the important link between political preferences and positive capital income tax rates.

In summary, the main message of the paper is that capital depreciation allowances may well serve as a redistributive device, especially in the long run and when the private sector and the government would act optimally. The results suggest that the current fiscal measures may serve a quite beneficial role for long-run economic growth and redistribution if they were kept in place for longer periods of time.

The paper is organized as follows: Section 2 presents the model. Section 3 analyzes the optimality for depreciation allowances and tax rates in long-run equilibrium. Section 4 provides concluding remarks.

## 2 The Model

The economy consists of a government, identical competitive firms and two types of infinitely-lived, equally patient and price taking individuals called workers and capitalists. All agents derive utility from the consumption of a homogenous, maleable good. The population is normalized so that the number of each type equals one. The model abstracts from uncertainty, technological progress, and population growth. The workers supply one unit of unskilled labour inelastically and do not

---

5The result would a fortiori hold for an open economy. As shown by Sinn (1984) and Sinn (1985) higher depreciation allowances might entail large capital movements to a country where, given everything else is equal, the allowances are higher. The capital inflow would then clearly benefit the workers in terms of their wages and any transfers granted to them.
Thus, all the wealth is concentrated in the hands of the capitalists who do not work.

### 2.1 Capitalists

In each period the *capital owners* choose how much of their income to consume or invest, and they take prices and policy as given. By assumption capital is broadly defined and includes human capital. See Mankiw, Romer, and Weil (1992). Thus, the model also captures distributional issues between owners of physical and human capital on the one hand and unskilled workers on the other.

Their instantaneous budget constraint is given by

$$c_t + i_t = r_t k_t - T_t \quad \text{and} \quad i_t = \dot{k}_t + \delta k_t,$$

where $c_t$ denotes consumption of the capitalists, $i_t$ their (gross) investment, and $T_t$ taxes to be paid to the government.\(^7\) Thus, the capitalists derive income from renting their (broad) capital, $k_t$, to competitive firms at the rate $r_t$. Their investment must cover the change in net assets, $\dot{k}_t$ and the depreciation of the capital stock, $\delta k_t$. The latter is assumed to happen in a linear way and is determined by technological wear and tear of capital that is not under the control of agents. For simplicity we assume that it is constant over time. In this paper $\delta k_t$ is taken to capture the true (technological) depreciation of capital. Finally, the capital owners have to pay taxes $T_t$ to the government which is taken into account when they make their decisions about consumption and investment.

The capital owners derive the following intertemporal utility stream

$$\int_0^\infty u[c_t] e^{-\rho t} \, dt$$

where $\rho$ is the constant rate of time preference, common to all agents, that is, capitalists and workers. The instantaneous utility function $u[c_t]$ satisfies the usual properties $u' \geq 0, u'' \leq 0$ and $\lim_{c_t \to -\infty} u' = 0$ and $\lim_{c_t \to 0} u' = 0$ where $u' = \frac{du[c_t]}{dc_t}$ and $u'' = \frac{d^2u[c_t]}{dc_t^2}$.

---

\(^6\)The assumption may be rationalized by imposing transaction costs on the workers when borrowing small amounts. Thus, the model uses the commonly used framework of Kaldor (1956) and Pasinetti (1962), which is also employed by Judd (1985) and Lansing (1999).

\(^7\)In the paper and for convenience we denote variables that are continuous functions of time by subscript $t$. Thus, for example, $c_t \equiv c(t)$. 

---

5
\[ u'' = \frac{d^2 u(t)}{dt^2}. \] Below we will analyze the capitalists’ problem as one where they maximize their intertemporal utility (2) subject to their budget constraint (1) and given initial capital.

### 2.2 Workers

The (unskilled) workers do not invest and are not taxed by assumption. They supply one unit of labour inelastically at each date and derive utility from consuming their entire wage and transfer income. Their total income, \( x_t \), depends on wage income, \( w_t \), and lump-sum transfers, \( TR_t \), granted by the government,

\[ x_t = w_t + TR_t. \tag{3} \]

Their intertemporal utility is given by \( \int_0^\infty v[x_t] e^{-\rho t} dt \) where \( v[x_t] \) need not be the same as that of the capitalists, but it is also assumed to satisfy \( v' \geq 0, v'' \leq 0 \) and the conditions \( \lim_{x_t \to \infty} v' = 0 \) and \( \lim_{x_t \to 0} v' = \infty \) where \( v' = \frac{dv[x_t]}{dx_t} \) and \( v'' = \frac{d^2 v[x_t]}{dx_t^2} \).

### 2.3 Firms

The firms operate in a perfectly competitive environment and maximize profits. The capital owners rent capital to and demand shares of the firms, which are collateralized one-to-one by capital. The markets for assets, capital and labour clear at each point in time so that the firms face a path of uniform, market clearing rental rates for capital and labour, \( r_t \) and \( w_t \). Given perfect competition the firms rent capital and hire labour in spot markets in each period. Output serves as numéraire and its price is set equal to 1 at each date, implying that the price of capital, \( k_t \), in terms of overall consumption stays at unity.

Aggregate production is constant returns to scale in capital and labour inputs. Since the labour input equals one, \( k_t \) can also be interpreted as the capital labour-ratio. The production function \( f(k_t) \) for the representative firm is assumed to be increasing and strictly concave in \( k_t \) with \( \lim_{t \to \infty} f'(k_t) = 0 \) and \( \lim_{t \to 0} f'(k_t) = \infty \). Profit

---

8 The working population is normalized so that there is one worker and one capitalist.
maximization implies
\[ r_t = f'(k_t) \quad (4) \]
\[ w_t = f(k_t) - f'(k_t)k_t \quad (5) \]

and perfect competition and the free entry and exit of firms means that profits, \( f(k_t) - r_t k_t - w_t \), are zero.

### 2.4 Government

As in Judd (1985) and Lansing (1999), I rule out a market for government bonds and assume that the government can commit itself to following a tax policy announced at \( t = 0 \). We assume that the government taxes capital income. But taxable income is such that the government allows for a depreciation allowance as is the case in most countries, that is, it allows for a deduction of taxable income related to the depreciation of capital. As shown above, we will consider *true* economic capital depreciation, \( \delta k_t \), and the possibility of *accelerated* tax depreciation. For modelling the latter I follow Sinn (1987), ch. 3, and relate it to capital investment. In particular, like Sinn we assume that a certain proportion \( p_t \), where \( 0 \leq p_t \leq 1 \), of an investment expenditure is depreciated immediately and the remainder \( 1 - p_t \) gradually over time by keeping the tax depreciation at a level of \( 1 - p_t \) times the true economic depreciation.\(^9\) At each point in time gross investment of the capital owner is \( i_t = \dot{k}_t + \delta k_t \). As the true economic depreciation is \( \delta k_t \), the flow of immediate depreciation on new investment is \( p_t i_t \) and the flow of depreciation on existing assets is \( (1 - p_t)\delta k_t \). Thus, the current flow of tax depreciation is\(^{10}\)
\[ D \equiv p_t i_t + (1 - p_t)\delta k_t = p_t\dot{k}_t + \delta k_t. \quad (6) \]

The government taxes capital income net of the depreciation allowance and uses the tax revenues for transfers to the workers. The latter are not taxed by assumption. The total tax revenues are denoted by \( T_t \) and the transfers to the workers by \( TR_t \). The government runs a balanced budget by assumption and, thus, we have

\(^9\)Notice that \( p_t \) is allowed to vary over time. Below we will assume that it is a choice variable for the government.

for the government that

\[ T_t = \theta_t \left[ r_t k_t - p_t \dot{k}_t - \delta k_t \right] = TR_t \]  

(7)

where \( \theta_t \) denotes the tax rate on (net) capital income.

### 2.5 The Behaviour of the Private Sector

From equations (1) and (2) we can state the **capital owners’** problem as follows

\[
\max_{c_t} \int_0^\infty u[c_t] e^{-\rho t} \, dt \\
\text{s.t.} \quad \dot{k}_t = \frac{\left(1 - \theta_t\right)(r_t - \delta)k_t - c_t}{1 - \theta_t p_t} \quad \text{and} \quad k(0) = \text{given},
\]

(8)

where the budget constraint in (8) has been obtained by inserting the tax \( T_t \) to be paid by the capital owners in (7) into their budget constraint in (1) and rearrangement of the resulting expression.

The current value Hamiltonian for this problem is

\[ H = u[c_t] + \lambda_t \left(\frac{(1 - \theta_t)(r_t - \delta)k_t - c_t}{1 - \theta_t p_t}\right) \]

and the necessary first order conditions for its maximization are

\[
H_c : \quad u' - \frac{\lambda_t}{1 - \theta_t p_t} = 0 \]  

(9a)

\[
H_k : \quad -\lambda_t \left(\frac{(1 - \theta_t)(r_t - \delta)}{1 - \theta_t p_t}\right) + \rho \lambda_t = \dot{\lambda}_t \]  

(9b)

plus the transversality condition \( \lim_{t \to \infty} k_t \lambda_t e^{-\rho t} = 0 \) and the requirement that equation (8) holds.\(^{11}\) The (non-negative) co-state variable \( \lambda_t \) represents the capital owners’ shadow price of an additional unit of capital in terms of their utility.

When the factor input and goods markets are in equilibrium the **workers’** income is given by

\[ x_t = w_t + TR_t = f(k_t) - r_t k_t + \theta_t \left[ r_t k_t - p_t \dot{k}_t - \delta k_t \right] \]

(10)

\(^{11}\)As \( H \) is concave in \( c_t \) and \( k_t \), the necessary conditions are also sufficient.
where we have used equations (4) and (5). In equilibrium the overall resource constraint is such that the agents satisfy their budget constraints. By substitution of (8) into (10) one then obtains

\[ x_t = f(k_t) - r_t(k_t - \theta_t(r_t - \delta)k_t - \theta_t p_t \left(\frac{(1 - \theta_t)(r_t - \delta)k_t - c_t}{1 - \theta_t p_t}\right), \]

which can be simplified to

\[ x_t = f(k_t) - \left(\frac{1 - \theta_t}{1 - \theta_t p_t}\right) r_t k_t - \theta_t (1 - p_t) \delta k_t + \frac{\theta_t p_t c_t}{1 - \theta_t p_t}, \tag{11} \]

Thus, in equilibrium the total income of the workers is increasing in the consumption of the capital owners, for given \( \theta_t \) and \( p_t \).

### 2.5.1 Arbitrary Behaviour and Policy

Next we analyze the effects of policy changes when the agents and the government satisfy their budget constraints but otherwise act in arbitrary (unspecified) ways at a particular point in time. By arbitrary behaviour it is meant that the capital owners and the government have not solved for their optimal decisions yet. However, it is not assumed to be absolutely unspecified since we look at situations where the agents at least obey their budget constraints.

If we look at changes in the accelerated depreciation allowance, \( p_t \), when the agents and the government act in unspecified ways we then obtain

\[
\frac{dx_t}{dp_t | \theta_t, c_t} = -\frac{\theta_t (1 - \theta_t) r_t k_t}{(1 - \theta_t p_t)^2} - \left[ \frac{-\theta_t \delta k_t (1 - \theta_t p_t) + \theta_t^2 (1 - p_t) \delta k_t}{(1 - \theta_t p_t)^2} \right]
\]

\[ + \frac{\theta_t c_t (1 - \theta_t p_t) + \theta_t^2 p_t c_t}{(1 - \theta_t p_t)^2} \]

\[ = \frac{\theta_t}{(1 - \theta_t p_t)^2} \left( c_t - (1 - \theta_t)(r_t - \delta)k_t \right). \tag{12} \]

Concentrating on non-negative growth of the capital stock, we must have \( c_t \leq (1 - \theta_t)(r_t - \delta)k_t \) so that it seems that \( \frac{\partial x_t}{\partial p_t | \theta_t, c_t} \leq 0 \) in general. Thus, an increase in accelerated depreciation allowances does not appear to be a good redistribution device as it does not seem to raise after-tax wages. This should be expected as higher \( p_t \) means that ceteris paribus less taxes for redistribution are collected.
In turn, an increase in taxes produces

\[
\frac{dx_t}{d\theta_t|p_t,c_t} = -\left(\frac{-(1-\theta_t)p_t + p_t(1-\theta_t)}{(1-\theta_t)p_t^2}\right) r_t k_t - \left(\frac{(1-p_t)(1-\theta_t)p_t + p_t\theta_t(1-p_t)}{(1-\theta_t)p_t^2}\right) \delta k_t + \left(\frac{p_t - \theta_t p_t^2 + \theta_t p_t^2}{(1-\theta_t)p_t^2}\right) c_t
\]

which is positive for given $0 \leq p_t \leq 1$, $c_t \geq 0$ and $r_t > \delta$. Thus, capital income taxes net of depreciation seem to be a positive redistribution device because they seem to raise the after-tax wages.

Next, consider the growth effects of both policy instruments, given arbitrary behaviour and given everything else. To this end consider equation (8). Here we find

\[
\frac{d\dot{k}_t}{d\theta_t|p_t,c_t} = \frac{(1-p_t)(r_t-\delta)k_t + p_t c_t}{(1-\theta_t)p_t^2} \leq 0 \quad \text{and} \quad \frac{d\dot{k}_t}{dp_t|\theta_t,c_t} = \frac{\theta_t((1-\theta_t)(r_t-\delta)k_t - c_t)}{(1-\theta_t)p_t^2} \geq 0.
\]

Thus, for policies in the interior of the unit interval, i.e. $\theta_t, p_t \in (0,1)$, and given everything else (like consumption etc.) higher capital income taxes are bad for growth and more accelerated depreciation is growth enhancing when there is positive taxation of capital income and net income of the capital owners is greater than their consumption.\(^\text{12}\)

Given the reactions of net investment and the after-tax wages due to changes in arbitrary $\theta_t$ and $p_t$ we may summarize the above results as follows:

**Proposition 1** When the agents and the policy maker obey their budget constraints and otherwise act in arbitrary (possibly non-optimal) ways at a particular point in time and a capital-income-cum-depreciation-allowance tax scheme (CICAD) is used, an increase in the capital income tax rate $\theta_t$ does not appear to raise net investment, $\dot{k}_t$, but generally increases after-tax wages. In turn, an increase in the accelerated depreciation allowance $p_t$ generally raises net investment, if $\theta_t > 0$, and generally lowers after-tax wages. Thus, when behaviour and policy are unspecified, for instance, when agents and the government do not optimize, depreciation allowances do not seem to be a good redistribution device in the short run.

\(^{12}\)That may be one reason why many countries have increased the capital depreciation allowances in the current financial and economic crisis.
These results hold for the short-run. Below we introduce optimizing behaviour and will find that the above results need qualification, once we introduce optimizing behaviour and look at the long run.

2.5.2 Arbitrary Private Sector Behaviour and Investment Return Stabilization

In the model agents face given price paths. Although the setup is not really capturing business cycle phenomena we can still get a flavour what accelerated depreciation allowances might entail in an economic downturn. The latter most often entails a significant drop in the real return to capital \( r_t \). A standard mechanism to model such a drop is to argue that there is a shock to technology so that the marginal product of capital falls where the latter equals the real return on capital in equilibrium. Here we leave that (i.e. such equilibrium responses) outside the model and simply argue that for some reason (e.g. the financial markets, animal spirits or whatever) the real return of capital falls due to some process that is outside the model. We will look at the model’s implications for one policy response due to such a drop in \( r_t \). More precisely, we consider a government that wishes to ‘stabilize’ the return on investment by means of its two policy instruments considered in this paper.

By equation (8) one easily verifies that the real return on investment is given by

\[
R_t \equiv \frac{(1 - \theta_t)(r_t - \delta)(1 - \theta_t p_t)}{(1 - \theta_t p_t)}. \quad (14)
\]

Suppose there is a significant drop in \( r_t \) and the government reacts to that by changing \( p_t \) or \( \theta_t \) in order to keep \( R_t \) constant. Heroically, we assume that the government can react immediately, once it perceives the drop in \( r_t \). For such a government we would then have

\[
dR_t = 0 = R_r \, dr_t + R_p \, dp_t + R_\theta \, d\theta_t. \quad (15)
\]

If the government counteracts the drop in \( r_t \) only by changing capital depreciation allowances, keeping the (path of the) tax rates \( \theta_t \) and \( R_t \) constant, one finds that

\[
\frac{dp_t}{dr_t} = -\frac{(1 - \theta_t p_t)}{\theta_t (r_t - \delta)}. \quad (16)
\]

Thus, we assume that the price path \( r_t \) features a particular point in time where there is a sharp or noticeable drop in the real return on capital.
which is negative for $\theta_t, p_t \in (0, 1)$ and $r_t > \delta$ which I assume to hold. Thus, if the real return on capital $r_t$ falls, i.e. when $(dr_t < 0)$, and the government wants to ‘stabilize’ the real return on investment, it must increase the accelerated depreciation allowance.

In turn, if the government fixes (the path of) $p_t$ and adjusts the tax rates one gets

$$\frac{d\theta_t}{dr_t} = \frac{(1 - \theta_t)(1 - \theta_t p_t)}{(r_t - \delta)(1 - \delta)}$$

which is positive under the assumptions made. Thus, a drop in $r_t$ requires a tax cut, i.e. lower $\theta_t$.

**Proposition 2** When there is a drop in the real return to capital $r_t$, then the government should increase the accelerated capital depreciation allowance $p_t$ or cut tax the capital income tax rate $\theta_t$ by compensating amounts, respectively, if it wishes to ‘stabilize’ the real return to investment.

Hence, a government that wishes to ‘stabilize’ the real return to investment should increase the capital depreciation allowance or cut capital income taxes in response to a drop in the real return to capital during an economic downturn. Therefore, the model suggests that many countries seem to have indeed pursued something like the objective to ‘stabilize’ the real return to investment in the current economic crisis, especially by adjusting $p_t$. This is because in most situations it is a lot easier to adjust depreciation allowances than changing tax rates, in particular statutory tax rates that often require relatively longer processes of political debates and legislation.

Of course, one should be aware of the fact that the policy objective of ‘stabilizing’ the investment return would also imply a reverse reaction of $p_t$ and $\theta_t$ when there is a sudden boost in $r_t$ that may accompany an economy upswing. Furthermore, the result applies only to the investment return. Other ‘stabilization’ objectives such as e.g. transfer or tax revenue ‘stabilization’ may imply other responses as regards $p_t$ and $\theta_t$.

2.5.3 Non-Distortion of Accumulation

One important consequence of the Judd (1985) and Chamley (1986) result that capital income taxes be optimally zero in the long run is that the capital accumulation
process will not be disturbed by political interference in that case. The impact of accumulation distortion can be inferred from the Euler equation in (9b),

\[-\lambda_t \left( \frac{(1-\theta_t)(r_t - \delta)}{1-\theta_t p_t} \right) + \rho \lambda_t = \dot{\lambda}_t.\]

This equation shows how agents evaluate the evolution of the state variable \(k_t\) in terms of their welfare, which then leads them to pursue a particular accumulation programme. Policy would in general distort this evaluation which is captured by the term \(\frac{1-\theta_t}{1-\theta_t p_t}\).

The government does not distort this evaluation in a long-run equilibrium with \(\dot{\lambda} = 0\) in (9b) when \(\theta_t = 0\) or \(p_t = 1\), \(\forall\ t\). In this paper both solutions are in principle possible and we will analyze that in more detail below.

3 The Long-Run Optimal Capital Depreciation Allowance and Income Tax

We assume that there is a government that is benevolent by respecting the private sector optimality conditions and representing the agents’ interests by attaching weights to their welfare. The government keeps the agents on their respective supply and demand curves, and chooses a policy that can be realized as a competitive equilibrium.\(^{14}\) From now on time subscripts are dropped for convenience whenever it is clear that a particular variable depends on time. Furthermore, I now denote partial derivatives by a subscript, that is, for a function \(x(y)\) we let \(x_y \equiv \frac{\partial x}{\partial y}\). Given this, the government solves the following problem

\[
\begin{align*}
\max_{k,c,\theta,p,\lambda} & \int_0^\infty \{\gamma v(x) + u(c)\} e^{-\rho t} dt \\
\text{s.t.} & \quad x = f(k_t) - \left( \frac{1-\theta}{1-\theta p} \right) rk - \frac{\theta(1-p)\delta k}{1-\theta p} + \frac{\theta p c}{1-\theta p} \\
& \quad u'(c) - \frac{\lambda}{1-\theta p} = 0 \\
& \quad -\left( \frac{(1-\theta)(r-\delta)}{1-\theta p} \right) \lambda + \rho \lambda = \dot{\lambda} \\
& \quad \frac{(1-\theta)(r-\delta)k-c}{1-\theta p} = \dot{k} \\
& \quad \theta, p \geq 0 \text{ and } \lim_{t \to \infty} \lambda t e^{-\rho t} = 0
\end{align*}
\]

\(^{14}\)Similar setups are used by Judd (1985), Judd (1999), and Lansing (1999).
where \( \gamma \in (0, \infty) \) represents the social weight attached to the welfare of the workers.\(^\text{15}\) If \( \gamma \to 0 \), the government is only concerned about the capitalists, whereas it only cares about the workers when \( \gamma \to \infty \). The current value Hamiltonian for this problem is given by

\[
\mathcal{H} = \gamma v[\cdot] + u[c] + \mu_1 (u' - \frac{\lambda}{1 - \theta p}) + q_1 \lambda \left( -\left( \frac{1 - \theta}{1 - \theta p} \right) (r - \delta) + \rho \right) + q_2 \left( \frac{(1 - \theta)(r - \delta)k - c}{1 - \theta p} \right)
\]

where \( q_1 \) is the social marginal value of the private marginal value \( \lambda \) which measures how valuable more capital is in terms of utility of the capital owners. Furthermore, \( q_2 \) is the social marginal value of more capital \( k \). The shadow price \( \mu_1 \) measures how to keep the capital owners on their demand curve. Furthermore, we can directly substitute (18a) in the objective function so that the Hamiltonian really depends on

\[
v[x] = v \left[ f(k) - \left( \frac{1 - \theta}{1 - \theta p} \right) rk - \frac{\theta(1 - p)\delta k}{1 - \theta p} + \frac{\theta pc}{1 - \theta p} \right]
\]

which simplifies the analysis. I also assume that the government inherits a capital income tax rate \( \theta(0) \) that is less than one and takes this as given at time zero.\(^\text{16}\)

The necessary first order conditions are

\[
\begin{align*}
\mathcal{H}_k : & \quad \gamma v'[\cdot] x_k + q_2 \left( \frac{(1 - \theta)(r - \delta)}{1 - \theta p} \right) = \rho q_2 - q_2 \quad (19a) \\
\mathcal{H}_c : & \quad \gamma v'[\cdot] x_c + u'[\cdot] + \mu_1 u''[\cdot] - q_2 \frac{1}{1 - \theta p} = 0 \quad (19b) \\
\mathcal{H}_\theta : & \quad \theta \left\{ \gamma v'[\cdot] x_\theta - \mu_1 \frac{\lambda p}{(1 - \theta p)^2} + q_1 \lambda \left( \frac{(1 - p)(r - \delta)}{(1 - \theta p)^2} \right) - q_2 \left( \frac{(1 - \theta)(r - \delta)k + pc}{(1 - \theta p)^2} \right) \right\} = 0 \quad (19c) \\
\mathcal{H}_p : & \quad p \left\{ \gamma v'[\cdot] x_p - \mu_1 \frac{\lambda p}{(1 - \theta p)^2} - q_1 \lambda \left( \frac{\theta(1 - \theta)(r - \delta)}{(1 - \theta p)^2} \right) + q_2 \left( \frac{\theta(1 - \theta)(r - \delta)k - c}{(1 - \theta p)^2} \right) \right\} = 0 \quad (19d) \\
\mathcal{H}_\lambda : & \quad -\frac{\mu_1}{1 - \theta p} + q_1 \left( \rho - \left( \frac{1 - \theta}{1 - \theta p} \right) (r - \delta) \right) = \rho q_1 - \dot{q}_1 \quad (19e)
\end{align*}
\]

where (19c) and (19d) have to hold with complementary slackness due to the re-

\(^{15}\)Notice that the model’s normalization implies that we consider a representative capital owner and worker each. The weight \( \gamma \) really captures that in the political process the number of agents plays a role in the welfare function of a benevolent government. Thus, \( \gamma \) should be viewed as larger than one in most circumstances. As a stronger microfoundation of the political process is beyond the scope of the paper, I follow the common procedure to attach fixed (exogenous) weights on the representative agents’ welfare and argue that a benevolent government cannot usually ignore the welfare of the workers and takes their presence as given.

\(^{16}\)This assumption is used to make the tax problem nontrivial. It rules out taxing the initial capital stock via a so-called capital levy that would constitute a lump sum tax, since initial capital is in fixed supply. See Judd (1985), and Chamley (1986) or, for example, Sargent and Ljungqvist (2004), ch. 15.3.
quirement that $\theta$ and $p$ cannot be negative.\(^\text{17}\) Furthermore, the equations (18b), (18c) and (18d) and the transversality conditions $\lim_{t \to \infty} q_1 \lambda e^{-\rho t} = 0$ and $\lim_{t \to \infty} q_2 ke^{-\rho t} = 0$ have to hold. Lastly, note that $x_p$ and $x_q$ are given by equations (12) and (13) and equation (11) implies that

$$x_c = \frac{\theta p}{1 - \theta p} \quad \text{and} \quad x_k = f' - \frac{(1 - \theta) r - \theta (1 - p) \delta}{1 - \theta p}. \quad (20)$$

At time zero, the initial value of the consumer's marginal utility is unconstrained, i.e. initial $\lambda$ is unconstrained.\(^\text{18}\) Thus, the associated costate variable $q_1$ at time 0 is zero, i.e. $q_1(0) = 0$. But in the model it turns out that $q_1(t) = 0$ for all $t$. This can be shown by the following arguments.

Let us focus on interior solutions and notice that the coefficients on $q_2$ in (19c) and (19d) correspond to $x_\theta$ and $x_p$, respectively. The latter are given by (13) and (12). Given this, equations (19c) and (19d) can rearranged as follows

$$\mathcal{H}_\theta : (\gamma v' - q_2) x_\theta = \mu_1 \lambda \frac{p}{(1 - \theta p)^2} - q_1 \lambda \frac{(1 - p)(r - \delta)}{(1 - \theta p)^2}, \quad (21)$$

$$\mathcal{H}_p : (\gamma v' - q_2) x_p = \mu_1 \lambda \frac{\theta}{(1 - \theta p)^2} + q_1 \lambda \frac{\theta(1 - \theta)(r - \delta)}{(1 - \theta p)^2}. \quad (22)$$

Without loss of generality, see equation (12), let us assume that $x_p$ is strictly negative $x_p < 0$.\(^\text{19}\) When we substitute for $(\gamma v' - q_2)$ from (21) in (22) we obtain

$$\mathcal{H}_p : \lambda \mu_1 \frac{p}{(1 - \theta p)^2} x_\theta - q_1 \lambda \frac{(1 - p)(r - \delta)}{(1 - \theta p)^2} x_\theta = \mu_1 \lambda \frac{\theta}{(1 - \theta p)^2} x_p + q_1 \lambda \frac{\theta(1 - \theta)(r - \delta)}{(1 - \theta p)^2} x_p$$

$$\mu_1 = \Omega \cdot q_1 \quad (23)$$

where $\Omega \equiv \left[\frac{(1 - p)(r - \delta)}{x_\theta} + \frac{\theta(1 - \theta)(r - \delta)}{x_p}\right] \left[\frac{p}{x_\theta} - \frac{\theta}{x_p}\right]^{-1}$.\(^\text{17}\)

\(^\text{17}\)For example, one might argue that negative $\theta$ is a form of wage tax and should not be ruled out a priori. However, as can be verified from (19c) negative $\theta$ is only possible in the model when $\gamma = 0$ and the government would not really be that benevolent anymore. For the purposes of this analysis and following Judd (1985) I rule out negative $\theta$ and $p$.

\(^\text{18}\)This is a standard result. It can be inferred from the fact that, as is usual, for the capital owners’ problem initial $\lambda$ and initial $c$ are not restricted by an initial condition. See Chamley (1986), p. 616, or Turnovsky (1995), p. 403, for the same point.

\(^\text{19}\)The argument would also work if $x_p > 0$, which we have ruled out. But that would not matter for the subsequent reasoning. If $x_p = 0$, the argument below becomes even simpler. If $x_p = 0$, then we immediately get that $\mu_1$ can be expressed as a function of $q_1$. Substituting in (19e), we again get a simple linear homogeneous differential equation in $q_1$, and the same arguments as below apply.
Next, we substitute this expression in (19e) and get

\[-q_1 \frac{\Omega}{(1-\theta p)} + q_1 \left( \rho - \left( \frac{1-\theta}{1-\theta p} \right) (r-\delta) \right) = \rho q_1 - \dot{q}_1.\]

This is a homogeneous, linear differential equation. Integrating from time 0 up to some time \( t \) yields

\[q_1(t) = q_1(0)e^{-\int_0^t \Delta_s \, ds} \quad \text{where} \quad \Delta_s \equiv \left[ -\frac{\Omega}{(1-\theta p)} - \left( \frac{1-\theta}{1-\theta p} \right) (r-\delta) \right].\]

As \( q_1(0) = 0 \), we have indeed found that \( q_1(t) \) is 0 for any \( t \). Clearly, this also holds for \( q_1 \) in steady state. Thus,\(^{20}\)

**Lemma 1** \( q_1(t) = 0 \) for all \( t \in [0, \infty) \).

We will now restrict the analysis to the long-run when the economy is at a steady state, balanced growth position with \( \dot{k} = \dot{\lambda} = \dot{\epsilon} = q_1 = q_2 = 0 \). Lemma 1 then implies that \( \mu_1 = 0 \) for all \( t \) and so in the steady state, too. Then (19c) and (19d) imply that \( \gamma v' [\cdot] = q_2 \).

Next suppose the government attaches some positive weight on the workers’ welfare, \( \gamma > 0 \), and their marginal utility is positive, \( v' [\cdot] > 0 \). Furthermore, notice that \( x_k = f - \frac{1-\theta}{1-\theta p} rk - \frac{\theta(1-p)}{1-\theta p} \delta k \). Then with \( q_2 = \gamma v' \) the first order condition for the capital stock in (19a) becomes

\[f' - \frac{1-\theta}{1-\theta p} r - \frac{\theta(1-p)}{1-\theta p} \delta + \frac{(1-\theta)(r-\delta)}{1-\theta p} = \rho.\]

Rearranging and simplifying this yields

\[(1-\theta p) f' - (1-\theta)r - \theta(1-p)\delta + (1-\theta)(r-\delta) = \rho(1-\theta p)\]

\[(1-\theta p) f' - (1-\theta p)\delta = \rho(1-\theta p)\]

\[f' = \rho + \delta.\]

\(^{20}\)An intuitive explanation for this property of the model is the following. Initially the constraints on \( \lambda \) and the marginal utility of consumption are nonbinding. Given the impact of the depreciation allowance on consumption and accumulation of the capital owners, any policy package \( \theta, p \) will lead to the nonbinding of these constraints on \( \lambda \). In a sense inappropriate choices of \( \theta \) and \( p \) may lead to too less or too much consumption from a social point of view. Thus, the social planner chooses a policy package of \( \theta \) and \( p \) that balances these effects and attaches a zero value, \( q_1 = 0 \), on \( \lambda \) and the marginal utility of consumption for all \( t \) in the optimum.
Thus, the marginal product equals the time preference rate and the rate of true capital depreciation in the steady state. This also pins down the steady state capital ratio. Furthermore, as the firms set \( f' = r \), we know that the rate of return on capital must also equal \( \rho + \delta \) in steady state, i.e.

\[
    r = f' = \rho + \delta.
\]  

(24)

From the private sector accumulation relationship, \( \dot{k} = 0 \), we then have that

\[
    c = (1 - \theta) \rho k
\]

as the long-run equilibrium consumption of the capital owners. Notice that the capital income tax rate is now also directly relevant for the capitalists’ consumption. In fact, it is tantamount to a tax on their consumption in the long-run equilibrium, as we will see below.

The capitalists’ consumption, therefore, depends negatively on the capital income tax rate. From equation (18c) with \( \dot{\lambda} = 0 \) in steady state we have \( \lambda \left( \rho - \frac{(1-\theta)(r-\delta)}{1-\theta p} \right) = 0 \) where \( \lambda \geq 0 \). But then it follows that, with \( r - \delta = \rho \), the following has to hold

\[
    \rho = \frac{(1-\theta)\rho}{1-\theta p}
\]

\[
    \theta_\rho = \theta.
\]  

(25)

This relationship is only satisfied if either \( \theta = 0 \) or \( p = 1 \) or both. It has the important implication that, no matter what government is in power, it is not optimal to distort capital accumulation.

**Proposition 3** No matter whether the government is relatively more pro-labour or pro-capital, the optimal policy scheme would be not to distort capital accumulation by setting \( \theta = 0 \) or \( p = 1 \) or both.

That provides an example that a government can use different distortionary instruments to offset any distortions. In the present case it is the coupling of accelerated depreciation allowances, which are potentially growth enhancing, with capital income taxes, which distort growth. The result implies that in the optimum the instrument mix removes the distortion.

But this leaves the question of which of these optimal policies will eventually be
compatible with the economy under study. Suppose we follow the zero tax policy in the (interior) optimum, then with $\mu_1 = 0$ we still must have that $\gamma v' = u'$ in the optimum. Given $\theta = 0$ we must then have

$$\gamma v'[f - r k] = u'\rho k$$

from the FOC on consumption $\mathcal{H}_c$, that is, equation (19b). As we have not restricted the exact form of the utility function and as we have not specified any value for $\gamma$, it seems unlikely that this condition will generally be satisfied.\(^{21}\) Notice that $f, k$ are determined by $\delta$ and $\rho$ and do not adjust in a steady state. Thus, it appears that a solution with $\theta = 0$ does not easily satisfy all the conditions for an interior solution for the economy being analyzed.\(^{22}\) Hence, the interior solution for an equilibrium with $\theta = 0$ and any $p \in (0, 1)$ is only optimal under restrictive circumstances.

Thus, for the analysis below I concentrate on the policy where $p = 1$ and there is maximal accelerated depreciation of the capital stock. Notice that (19a) holds if $f' = r = \rho + \delta$. This pins down the capital stock to $\tilde{k}$ in steady state. Thus, as $t \to \infty$ the capital stock $k$ approaches some time invariant constant $\tilde{k}$. From now on the tilde will denote variables in long-run, steady state equilibrium.

The nondistortionary optimal policy $p = 1$ has the following implication: By equation (11) we have that

$$x = f(k) - \left( \frac{1 - \theta}{1 - \theta p} \right) r k - \frac{\theta(1 - p)\delta k}{1 - \theta p} + \frac{\theta pc}{1 - \theta p}.$$

When $p = 1$ in the long-run equilibrium this is

$$\bar{x} = f(\tilde{k}) - \tilde{r} \cdot \tilde{k} + \theta \rho \tilde{k}. \quad (26)$$

Thus, the income of the workers is implicitly increasing in the consumption of the capital owners \textit{and} in $\theta$, because that raises tax revenues that can be transferred to the workers raising their total income when $p = 1$.

**Proposition 4** \textit{High capital income depreciation allowances are good for the workers}

\(^{21}\)This argument also applies to the case of utility functions that feature a constant elasticity of intertemporal substitution.

\(^{22}\)Of course, corner solutions may be possible when the parameters would imply them. But here we focus on interior solutions first.
in an indirect way, because with a maximal accelerated depreciation allowance, the redistributive effect of capital income taxes becomes strongest in a long-run equilibrium. In that sense depreciation allowances are an indirect redistributive device in the long run.

Clearly, the result is in contrast to what the model predicts for the short run and for arbitrary behaviour.\textsuperscript{23} Thus, the effects of the policy instrument \( p \) depends on behaviour and the time horizon considered.

Notice that when in steady state, the transfers to the workers are given by

\[
\tilde{TR} = \theta \rho \tilde{k},
\]

which is increasing in the capital income tax rate.

With \( p = 1 \) we again have that \( q_2 = \gamma v' \left( \cdot \right) \) by (19c) for an interior equilibrium, and substitution of this into (19b) establishes that \( \gamma v' = u' \) must hold. As the capital stock is fixed at \( \tilde{k} \), which depends on \( \rho \), and as \( c = (1 - \theta) \rho \tilde{k} \), the latter condition boils down to finding \( \theta \) such that

\[
\gamma v' \left( f(\tilde{k}) - (\rho + \delta)\tilde{k} + \theta \rho \tilde{k} \right) = u' \left( (1 - \theta) \rho \tilde{k} \right)
\]

(27)

where \( \tilde{r} = \rho + \delta \). Clearly as \( \gamma \to \infty \) and the government is entirely pro-labour, the LHS becomes infinite and as a consequence \( \theta = 1 \) would be optimal, since \( \lim_{c_t \to 0} u' \left( \cdot \right) = \infty \).\textsuperscript{24}

On the other hand, if \( \gamma \to 0 \), then \( \mu_1 = q_1 = 0 \) still applies. In this case equation (19c) boils down to

\[
\theta \left\{ -q_2 \frac{(1 - p)(r - \delta)k + pc}{(1 - \theta p)^2} \right\} = 0
\]

(28)

where \( q_2 > 0 \) from (19b). Thus, \( \theta = 0 \) would be optimal by the Kuhn-Tucker conditions, as the expression in braces would then be negative.

**Lemma 2** If the workers and the capitalists have different utility functions under the accelerated depreciation and capital income tax scheme with maximal depreciation,

\textsuperscript{23}Recall that proposition 1 showed that \( p \) may be a bad instrument for redistribution.

\textsuperscript{24}Rehme (1995) and Rehme (2002) obtain a similar result in an endogenous growth framework where redistribution occurs via productive government input financed by a capital income tax cum investment subsidy scheme.
p = 1, and

1. the government represents the capitalists only (\(\gamma \to 0\)), then the optimal capital income tax is zero in the long run and redistribution from capital to labour is zero.

2. the government represents the workers only (\(\gamma \to \infty\)), then the optimal capital income tax is nonzero in the long run and redistribution from capital to labour is maximal.

Notice that the result does not depend on production externalities or any other things, the capital income taxes may be used for, except for using an accelerated capital depreciation allowance scheme.

Of course, governments do not always place so much weight on the workers. In fact, by implicit differentiation one verifies below that the optimal tax rate, if it exists, is increasing in \(\gamma\). Thus, as the workers get more social weight they would choose higher capital income taxes in this tax scheme with capital depreciation allowances.\(^{25}\)

It is not entirely clear why workers should evaluate a consumption good any differently than a capital owner. For that reason we now assume that \(v[x] = u[c]\) for any \(x = c\) so that the two groups have the same utility function. As I am only interested in conditions under which the capital income tax is zero or not in the long-run, let us assume that the utility functions are of the constant elasticity of intertemporal substitution (CIES) type: \(u[c] = \frac{c^{1-\beta} - 1}{1-\beta}\) and \(v[x] = \frac{x^{1-\beta} - 1}{1-\beta}\). Then (27) would be given by

\[
\gamma \left( f(\tilde{k}) - (\rho + \delta)\tilde{k} + \theta \rho \tilde{k}\right)^{-\beta} = (1-\theta)\rho \tilde{k}^{-\beta}
\]

(29)

where \(\rho + \delta = \tilde{r}\). This equation is not easily solvable. Notice, however, that the LHS is decreasing and the RHS is increasing in \(\theta\). Thus, in general an algebraic solution of the equation should exist. Below I will simulate solutions for \(\theta\) based on calibrations for widely used parameter values in the business cycle literature. Before doing this

\(^{25}\)Notice that \(\tilde{k}\) would be the same under any other capital income tax scheme for which it is shown that the long-run capital income tax should be zero. This is an important point, because overall welfare (sum of utilities) may be higher under our tax scheme in comparison to those other capital income tax schemes.
we look at the reactions an optimal $\theta$, let us call it $\tilde{\theta}^*$, will satisfy. It is clear that the optimal solution will be a function of the form $\tilde{\theta}^* = \theta(\gamma, \tilde{k}, \beta, \rho, \delta)$.

But $\tilde{k}$ will be a function of parameters too. To this end assume that the production function is of the standard type $f = Ak^\alpha$ where $0 < \alpha < 1$. Then $f' = \alpha Ak^{\alpha - 1}$ and $f'' = \tilde{r}$ and so $\tilde{k} = \left(\frac{\alpha A}{\tilde{r}}\right)^{1-\alpha}$. Thus, in steady state the output-capital ratio is given by $\frac{\tilde{f}}{\tilde{k}} = A\tilde{k}^{\alpha - 1} = A\frac{\tilde{r}}{\alpha A} = \frac{\tilde{r}}{\alpha}$. With this we can rearrange equation (29) and divide by $\tilde{k}$ to get

$$\gamma^{\frac{1}{\beta}} = \frac{1}{\rho} \left(\frac{\tilde{f}}{\tilde{k}} - \tilde{r}\right) + \theta \left[1 + \gamma^{\frac{1}{\beta}}\right].$$

Now notice that $\frac{\tilde{f}}{\tilde{k}} = \tilde{r}/\alpha$. Thus, for an interior solution the optimal $\theta$, called $\tilde{\theta}^*$, satisfies

$$\theta = \tilde{\theta}^* = \left[\gamma^{\frac{1}{\beta}} + \frac{\tilde{r}}{\rho} \left(1 - \frac{1}{\alpha}\right)\right]^{-1} \left[1 + \gamma^{\frac{1}{\beta}}\right]^{-1}$$

where $\tilde{r} = \rho + \delta$. As $\frac{1}{\alpha} > 1$ we can conclude that for positive optimal tax rates $\tilde{\theta}^*$, the social weight going to the workers must be sufficiently large, i.e. $\tilde{\theta}^* > 0$ if

$$\gamma > \left[\frac{1}{\rho} \left(\frac{1}{\alpha} - 1\right)\right]^{\beta}.$$  

Otherwise, the optimal tax rate must be zero as we have ruled out negative capital income taxes.

In order to get a feeling for the nature of the solutions let us use a numerical simulation based on calibrations from the business cycle literature. In particular, we rely on Walsh (2003), ch. 2, who bases his parameters on Cooley and Prescott (1995), p. 22, and Cooley and Hansen (1995), p. 201. For quarterly data for the United States he uses the following calibrated values for a standard money-in-the-utility function, real business cycle model.\footnote{The numbers reported for the U.S. command wide support in that literature. As the numbers are related to discrete time models, I have converted the discrete time discount rate of 0.989 to our continuous time set-up. The corresponding value of $\rho$ was 0.011 as reported in the table.}
Table 1: Baseline Parameter Values

<table>
<thead>
<tr>
<th>α</th>
<th>δ</th>
<th>ρ</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.019</td>
<td>0.011</td>
<td>2</td>
</tr>
</tbody>
</table>

Based on Walsh (2003), p. 75.

With these values it turns out that $\gamma > 23.5$ satisfies the condition for positive optimal capital income tax rates when there is maximal depreciation with $p = 1$. Table 2 reports the results of a numerical simulation for the calibrated economy varying $\gamma$. I only concentrate on solutions where the optimal tax rate is positive.

Table 2: Optimal Capital Income Tax Rates $\tilde{\theta}^*$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>200</th>
<th>500</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\theta}^*$</td>
<td>0.03</td>
<td>0.10</td>
<td>0.20</td>
<td>0.27</td>
<td>0.33</td>
<td>0.41</td>
<td>0.46</td>
<td>0.61</td>
<td>0.75</td>
<td>0.82</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The results are calculated for the optimal maximal capital depreciation allowance $p = 1$ and as a function of the social weight factor $\gamma$.

The numbers suggest that the social weight $\gamma$ is an important determinant of the optimal tax rate. That corresponds to common intuition. Governments that give more weight to the interests of the workers seem to choose higher capital income tax rates.27

Next, we look at the effects of parameter changes on the optimal tax rate. In order to calculate the effects of $\gamma$ and $\beta$ we notice that $\tilde{\theta}^*$ is of the form $\tilde{\theta}^* = F(\Gamma)$ where $\Gamma \equiv \gamma^{\frac{1}{\beta}}$. Thus,

$$\frac{\partial \tilde{\theta}^*}{\partial \gamma} = \frac{\partial F}{\partial \Gamma} \cdot \frac{\partial \Gamma}{\partial \gamma} \quad \text{and} \quad \frac{\partial \tilde{\theta}^*}{\partial \beta} = \frac{\partial F}{\partial \Gamma} \cdot \frac{\partial \Gamma}{\partial \beta}.$$  

27The value of $\gamma$ is outside the model. It may seem that one has to require a very large value of $\gamma$ to obtain value of the capital income tax rate that seems realistic. But one way to think about $\gamma$ is to associate it with the number of the agents in this economy. If each capitalist employed 60 workers in his/her firm, then the predicted values would not seem so unrealistic. For instance, the current (highest marginal) capital income tax rate in the United States is currently around 35 percent.
It is then quite straightforward to verify that

$$\frac{\partial \tilde{\theta}^*}{\partial \gamma} = \frac{(\alpha \rho + (1 - \alpha) \tilde{r})}{\rho \alpha \left[1 + \gamma^\frac{1}{\beta}\right]^2} \left(\frac{1}{\beta} \gamma^{\frac{1}{\beta} - 1}\right) > 0$$

(32)

because the products on the RHS of that expression, i.e. $\frac{\partial F}{\partial \Gamma}$ and $\frac{\partial \Gamma}{\partial \gamma}$ are positive. Hence, the optimal tax rate is strictly increasing in the social weight going to the workers. This holds for all the parameters in the model.

Furthermore, for the effect of the preference parameter $\beta$ I find

$$\frac{\partial \tilde{\theta}^*}{\partial \beta} = \frac{(\alpha \rho + (1 - \alpha) \tilde{r})}{\rho \alpha \left[1 + \gamma^\frac{1}{\beta}\right]^2} \left(-\frac{1}{\beta^2} \gamma^{\frac{1}{\beta} - 1} \ln \gamma \right)$$

(33)

where the first expression on the RHS is positive, i.e. $\frac{\partial F}{\partial \Gamma} > 0$. As $\frac{1}{\beta}$ reflects the elasticity of intertemporal substitution a higher $\beta$ means that people are less willing to accept deviations from a uniform pattern of consumption over time. Higher $\beta$ is, therefore, associated with the wish to smooth consumption more. Thus, the effect of $\beta$ on $\tilde{\theta}^*$ is in general not clear. However, if the social weight of the workers is sufficiently strong, i.e. $\gamma > 1$, then more consumption smoothing (higher $\beta$) would entail a lower capital income tax rate, $\tilde{\theta}^*$, in the optimum.

Noticing that $\tilde{r} = \rho + \delta$ one can easily infer from (30) that

$$\frac{\partial \tilde{\theta}^*}{\partial \rho} > 0, \quad \frac{\partial \tilde{\theta}^*}{\partial \delta} < 0, \quad \text{and} \quad \frac{\partial \tilde{\theta}^*}{\partial \alpha} > 0.$$

(34)

When concentrating on interior solutions where $\gamma$ satisfies equation (31), an explanation for these properties of $\tilde{\theta}^*$ is the following: More impatience (higher $\rho$) is associated with higher steady state consumption of the capital owners. As the contemplated tax scheme is tantamount to a tax on the capitalists’ consumption, a higher $\rho$ allows for more resources to be transferred to the workers. Thus, the optimal tax rate would be higher in that case.

From equation (29) one can verify that a higher true depreciation rate $\delta$ lowers the marginal utility of the workers. Thus, a unit of depreciated capital would be valued less by them. To compensate for that lower taxes are required to keep the workers and the capital owners on their demand curves so that $\gamma u' = u'$. The pre-tax factor income distribution plays role in the model. Higher $\alpha$
corresponds to a higher share of pre-tax factor income going to the capital owners. A benevolent government would see the redistributive goal and would correspondingly correct for a more unequal factor income distribution by levying higher capital income taxes.

Let us summarize our findings in the following proposition.

**Proposition 5** Let the agents possess the same constant relative risk aversion utility functions. Under a capital-income-cum-depreciation-allowance tax scheme (CICDA) it is optimal to grant the maximal depreciation allowance, \( p = 1 \), in a long-run equilibrium. If, furthermore, the long-run capital income tax is non-zero, there will be positive transfers. The optimal capital income tax rate \( \tilde{\theta}^* \) is non-zero if the social planner attaches sufficient weight on the welfare of the workers, \( \gamma > \left[ \frac{\tilde{r}}{\rho} \left( \frac{1}{\alpha} - 1 \right) \right]^{\beta} \). In contrast, if \( \gamma < \left[ \frac{\tilde{r}}{\rho} \left( \frac{1}{\alpha} - 1 \right) \right]^{\beta} \), then \( \tilde{\theta}^* = 0 \) is optimal. Hence, under CICDA the income distribution (\( \alpha \)), the physical wear and tear of capital (\( \delta \)), preferences (\( \rho, \beta \)) and the social weight of the workers (\( \gamma \)) determine whether the optimal capital income taxes are zero or not in the long run. Depreciation allowances serve as an indirect redistributive mechanism in a long-run equilibrium.

Thus, under the (optimally nondistorting) capital income tax scheme under consideration (CICDA) distributional and preference parameters matter and that may provide a counterexample to Judd (1985) and Chamley (1986). Importantly, the proposition establishes that there may be instances when capital income taxes are optimally non-zero in the long run. Thus, even though the social planner only concentrates on the first order conditions of the private sector and does not explicitly know the agents’ final decision rules, and even though he/she has freedom to choose consumption and capital independently, capital income taxes may optimally be non-zero in the long-run when coupled with accelerated capital depreciation allowances. The latter are maximal in the long-run optimum and can be taken to bring positive capital income taxes and so redistribution about. In the model the short-run (see proposition 1) and long-run effects of capital depreciation allowances turn out to have quite different implication for redistribution.
4 Conclusion

In this paper it is analyzed whether politically determined capital depreciation allowances are bad instruments for redistribution. When coupling the latter with capital income taxes it turns out that an increase in accelerated capital depreciation allowances is a bad tool for redistribution, but good for economic growth, when the private sector and the government act non-optimally in the short run. In turn, capital income taxes are bad for economic growth and good for redistribution under these conditions. On the other hand, it is shown that higher depreciation allowances and lower taxes may stabilize the real return to investment in an economic downturn.

However, for the long run and with optimizing behaviour things are quite different. The coupling of capital income taxes with accelerated capital depreciation allowances for financing pure redistribution imply maximal depreciation allowances and often nonzero capital income taxes. The policy package under consideration in this paper is nondistortionary for accumulation in the optimum. It is found that whether or not capital income taxes are optimally zero in the long run depends on realistic conditions for taxation policy. The most important conditions identified in this paper are: (a) the social weight of those who receive redistributive transfers, (b) the distribution and so inequality in pre-tax factor incomes, (c) the physical wear and tear of capital and (d) the intertemporal elasticity of substitution. The results imply that pure redistribution may optimally be financed by capital income taxes when using accelerated capital depreciation allowances as a complementing instrument.

The results suggest that it might be a good thing to keep in place, for a longer period of time, the more generous depreciation allowance schemes that have been introduced in response to the current economic crisis.
References


