A full description of the Three-ME model: Multi-sector Macroeconomic Model for the Evaluation of Environmental and Energy policy

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Abstract

Since 2008, the ADEME and the OFCE are involved in a research convention to develop the model Three-ME. This document provides a full description of new version of the model. Three-ME is a new model of the French economy especially designed to evaluate the medium and long term impact of environmental and energy policies at the macroeconomic and sector levels. To do so Three-ME combines two important features. Firstly, it has the main characteristics of neo-Keynesian models by assuming a slow adjustment of effective quantities and prices to their notional level, an endogenous money supply, a Taylor rule and a Philips curve. Compared to standard multi-sector CGEM, this has the advantage to allow for the existence of under-optimum equilibria such as the presence of involuntary unemployment. Secondly, Three-ME is a hybrid model in the sense that it combines the top-down approach of general equilibrium macroeconomic models with elements of bottom-up models of energy models developed by engineers. As in bottom-up models, the amount of energy consumed is related to their use, that is the number of buildings or cars, and the energy class to which they belong. This hypothesis is more realistic compared to the assumption made in the majority of top-down models where energy consumption is usually directly related to income through a nested structure of utility function.

Key word: neo-Keynesian model, hybrid modeling, macroeconomic modeling, energy and environmental policy modeling

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Contents

1 Introduction

Top-down Computable General Equilibrium Models (CGEM) are often used to evaluate a wide range of economic problems. They have the advantage to combine tractability with a high level of detail, being able to distinguish different countries, goods, type of consumer, etc[1] . Particularly important for the analysis of the economic impact of environmental and energy policy, they often account for an important number of sectors: e.g. GREEN has 11 sectors (Burniaux et al., 1992), GEMINI-E3 has 18 sectors of which 5 energy sectors (Bernard and Vielle, 2008), GEM-E3 has 14 sectors (Capros et al., 1997), IMACLIM-S has 10 sectors (Ghersi and C., 2009).

But CGEM have two important drawbacks. First, they rely on very restrictive assumptions relative to the functioning of the economy especially in the short and medium run. CGEM are supply models where the hypothesis of perfect price flexibility and money neutrality often insures the full and optimal use of production factors and thus rule out permanent or transitory under-optimum equilibrium such as the presence of involuntary unemployment. They neglect, the dynamic effects of the demand side, and especially the multiplier effect of the public investment, by assuming a total eviction between private and public spendings. This result is due to the hypothesis that the interest rate insures the equilibrium betwen investment and savings, in a framework where the money supply is exogenous. Neo-Keynesian macroeconomic models, also called Aggregate Demand-Aggregate Supply (AS-AD) models, try to give a more realistic representation of the actual functioning of the economy taking explicitly into account slow adjustments of prices and quantities, an endogenous money supply, thus allowing for permanent or transitory under-optimum equilibrium. This effort seems to have a cost in terms of the disaggregation level which is often limited. This is typically the case for currently running macroeconomic models for the French economy: e.g. MESANGE of the French ministry of Economy has three sectors (Allard-Prigent et al., 2002), E-Mod of the OFCE (Chauvin et al., 2002) and MASCOTTE of the French central bank (Baghli et al., 2004) have only one. However, earlier versions of these models in the 1980's and 1990's had a higher level of disaggregation, between 6 and 8 products (see Economie et Prévision, 1998). But still, neo-Keynesian macroeconomic models generally do not distinguish between the different types of energy or of transport which are particularly important for the assessment of environmental and energy policy[2]. They are thus likely to neglect the effect of activity transfers in terms of growth and employment from high to low intensive energy sectors.

A second limit particularly important for the analysis of the economic impact of environmental and energy policy is that CGEM provide an insufficient representation of endogenous energy efficiency phenomena[3]. For instance, households

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[1] For a survey on CGEM see Brécard et al. (2006); Böhringer and Löschel (2006)

[2] NEMESIS is an exception with 30 sectors covering 16 European countries (Brécard et al., 2006)

[3] This limit explains partially why existing models have trouble to represent and model realistically energy and environmental issues as recently acknowledge by a recent FP7 re-

consumption behaviors are generally represented through a nested structure of utility function which defines a simple correlation between the level of consumption of each goods and the revenue. The model does not include saturation point in the consumption of the households. The link between consumption and revenue is often log-linear, that is linear in relative terms: a 1% increase in the real revenue leads to a 1% increase in the consumption of each goods. To account for non-linearities, it is usual to introduce a Linear Expenditure System (LES) utility function . A LES specification assumes that a share of the base year consumption is incompressible and therefore the relation between income and consumption is not linear anymore. This specification allows for the distinction between consumption of necessity and luxurious goods.

Although the LES utility function improves the realism of the modeling of consumption behavior, it still rely on the theoretical representation that each good provides a direct utility to households. This is not a realistic assumption for certain goods such as energy. As formalized theoretically by Lancaster (Lancaster, 1966)(1971) and applied certain hybrid models (Laitner and Hanson, 2006), households do not consume energy for their direct utility but rather for the service they provide when combined with a capital goods such as a car or a house. There is no point buying gasoline if one does not have a car. A more realistic theoretical representation is therefore to assume that energy is an input used in combination with different types of capital in a households production function. This representation accounts for the fact that in reality certain services are not always externally purchased by households but rather directly produced by them. This is typically the case for transports. Households can directly purchase a transportation service produced by an activity such as public transport. Alternatively, they can invest in a capital by purchasing a car and buy the necessary amount of gasoline that fulfills their needs.

Compared to the assumption made in standard top-down representation, there is hardly any direct utility from energy alone. This is of course different for other goods, although many commodities have similar properties to energy. For instance, a big share of water consumption is related to the use of appliances. The number of appliances generally depends on the number of houses. The amount construction material used is closely related to the size of the house. Relating the consumption of these goods directly to the revenue, as assumed in the standard top-down representation, may therefore lead to unrealistic results, where the consumptions expressed in physical units exceed their saturation levels. Indeed, it is unlikely that an household will ever decide to buy 6 cars, 10 washing machines or to heat its house at 35°C even if it becomes richer in the future. Because of their monetary representation, one cannot exclude that standard top-down CGEMs produce such unrealistic development in the long

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search proposal (ENV.2012.6.1-2) on the "Development of advanced techno-economic modeling tools for assessing costs and impacts of mitigation policies" that states: "Currently available [techno-economic modeling] tools have relevant limitations such as the difficulty to represent pervasive technological developments, the difficulty to represent non-linearities, thresholds and irreversibility, and the insufficiently developed representation of economic sectors with a significant potential for mitigation and resource efficiency."

run due to the general increase of the standard of living. Only a physical (along with a monetary) representation allows for the inclusion of realistic floor and ceiling in the consumption of certain goods.

Three-ME (Multi-sector Macroeconomic Model for the Evaluation of Environmental and Energy policy) is a new model of the French economy developed by the ADEME, OFCE and TNO. Its main purpose is to evaluate the impact of environmental and energy policy measures on the economy at the macroeconomic and sectoral levels. Moreover it has the ambition to overcome the two limitation of standard top-down CGEM pointed above by introducing neo-Keynesian features such as inertia in price and quantities and bottom-up features in the modeling of consumption behavior.

Having the general structure of neo-Keynesian macroeconomic models, Three-ME seems more realistic than the standard CGEM for describing the actual dynamic of the economy at least in the short and medium run. In the long run, the model is neo-classical in the sense of Solow (Solow, 1956) since it converges toward a steady states where all variables grow at a constant rate. Disaggregated in 37 sectors with an explicit distinction between 13 types of energy sources and five types of transports, it allows for the neo-Keynesian short/medium term macroeconomic modeling approach to catch-up with the most advanced CGEM in terms of sectoral analysis. In addition, its hybrid structure regarding the specification of the behavior of households overcomes the restrictions imposed by nested utility approach assumed in standard CGEM.

Compare to the previous version of the model (Callonnec et al., 2011; Reynès et al. 2011), the following important improvements were made:

- The 4 energy sectors of the previous version (coal, petroleum, electricity and gas) have been subdivided into 13 sectors in order to better account for the impact of renewable energy.

- The model distinghishes now commodities from activities: the number of activities is not equal to the number of commodities and each activity can potentially produce any commodity. This is typically the case for energy sectors that may produce the same commodity. For instance, the commodity electricity is produced by several activities: nuclear, wind, solar, etc. Another exemple is the activity agriculture that produces several commodities: agricultural product, food, biofuel and biogas.

- Because of access restriction to National account investment data, investment decisions were modeled for the all the private sector. We now use detailed investment data disaggregated by sector and therefore identify a specific investment pattern for each activity.

- The modeling of households' behaviors has been improved. The number of households has increases from 1 to 5, classified by quintile of revenue. In the previous version, only 2 energy classes (efficient and other) for buildings and cars were distinguished. The new version includes 7 energy classes for buildings and cars that follows the standard A to G classification. The link between energy consumption and stock was rather *ad hoc* since we assumed a loglinear negative correlation between energy consumption and the share of the efficient stock. The level of energy consumption (and thus of CO2 emissions) is now directly related to the type of buildings or cars and is also expressed in physical units. Finally, the arbitrage between the different classes of investments is not directly a function of price of energy but of the relative user cost of each investment which includes the expected energy related costs.

Section 2 presents a non technical overview of the model by summarizing its main characteristics. Section 3 describes the demand and supply equilibrium and the way adjustment processes are specified. Section 4 describes the supply side. Section 5 and 6 presents respectively the household and the labor market equations. In each sectors, the wage equation is an augmented Phillips curve including possible hysteresis phenomena. Under the assumption of full hysteresis, this specification has the same properties as a Wage Setting (WS) curve in level. Section 7 presents the external trade equations. Section 8 describes the price structure and how firms in each sector determine their production price. The behavior of the European Central Bank (ECB) about the determination of the interest rate is also presented. Section 9 treats the public administrations equation block. Section 10 describes the way GreenHouse Gases (GHG) emissions are modeled. Appendix A describes the long term properties of the model. Appendix B derives the optimality program of the producer and the consumer assuming a generalized CES (GCES) production and utility function. Appendix C provides all the equations of the model and Appendix D provides a glossary of the terms used.

2 Overview of the model

The overall structure of the model is schematized in Figure 1. In the short term, Three-ME has the main characteristics of a standard neo-Keynesian macroeconomic AS-AD model in an open economy. An important one is that demand determines supply. The demand is composed of (intermediate and final) consumption, investment and export whereas the supply comes from imports and the domestic production. As a feed-back with eventually some lags, the supply affects the demand through several mechanisms. The level of production determines the quantity of inputs used by the firms and thus the quantity of their intermediate consumptions and investment which are two components of the demand. It determines the level of employment as well and consequently the households' final consumption. Another effect of employment on demand goes through the wage setting via the unemployment rate which is also determined by the active population. The active population is mainly determined by exogenous factors such as demography but also by endogenous factors: because of discouraged worker effects, the unemployment rate may affect the labor participation rate and thus the active population.

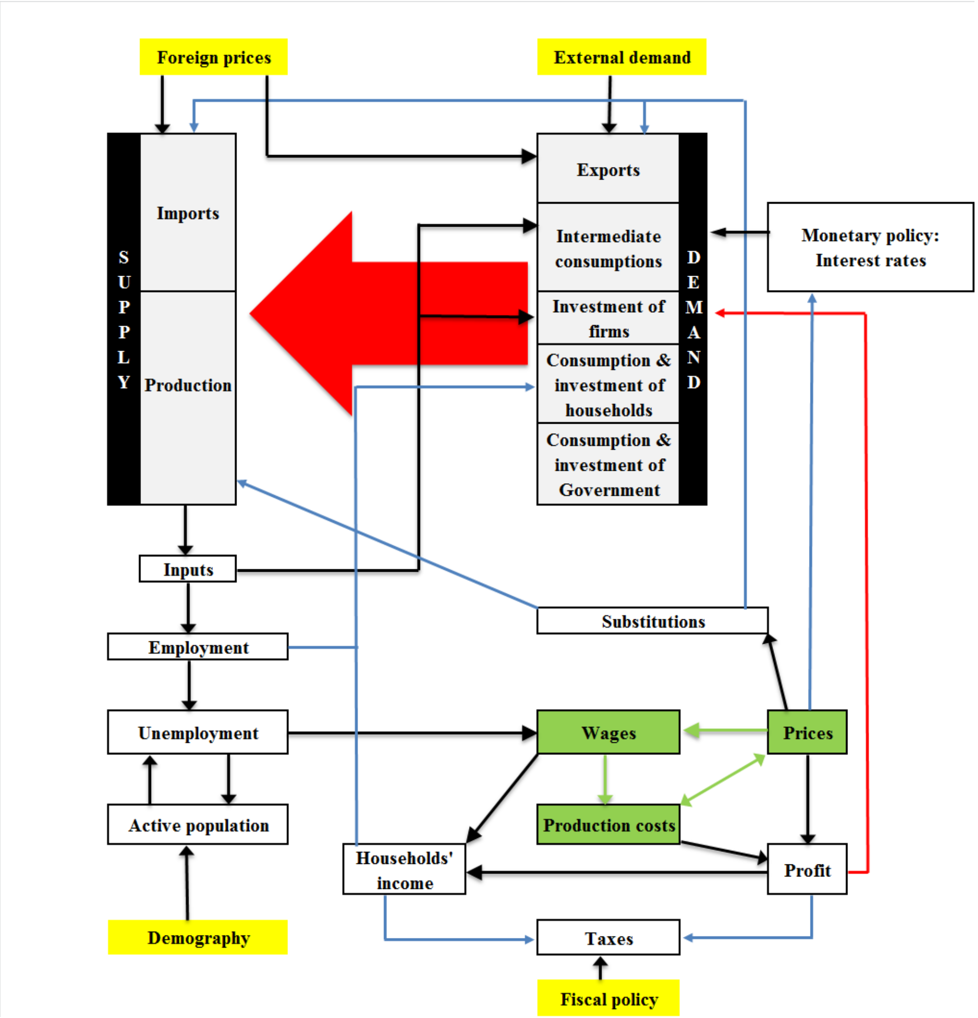


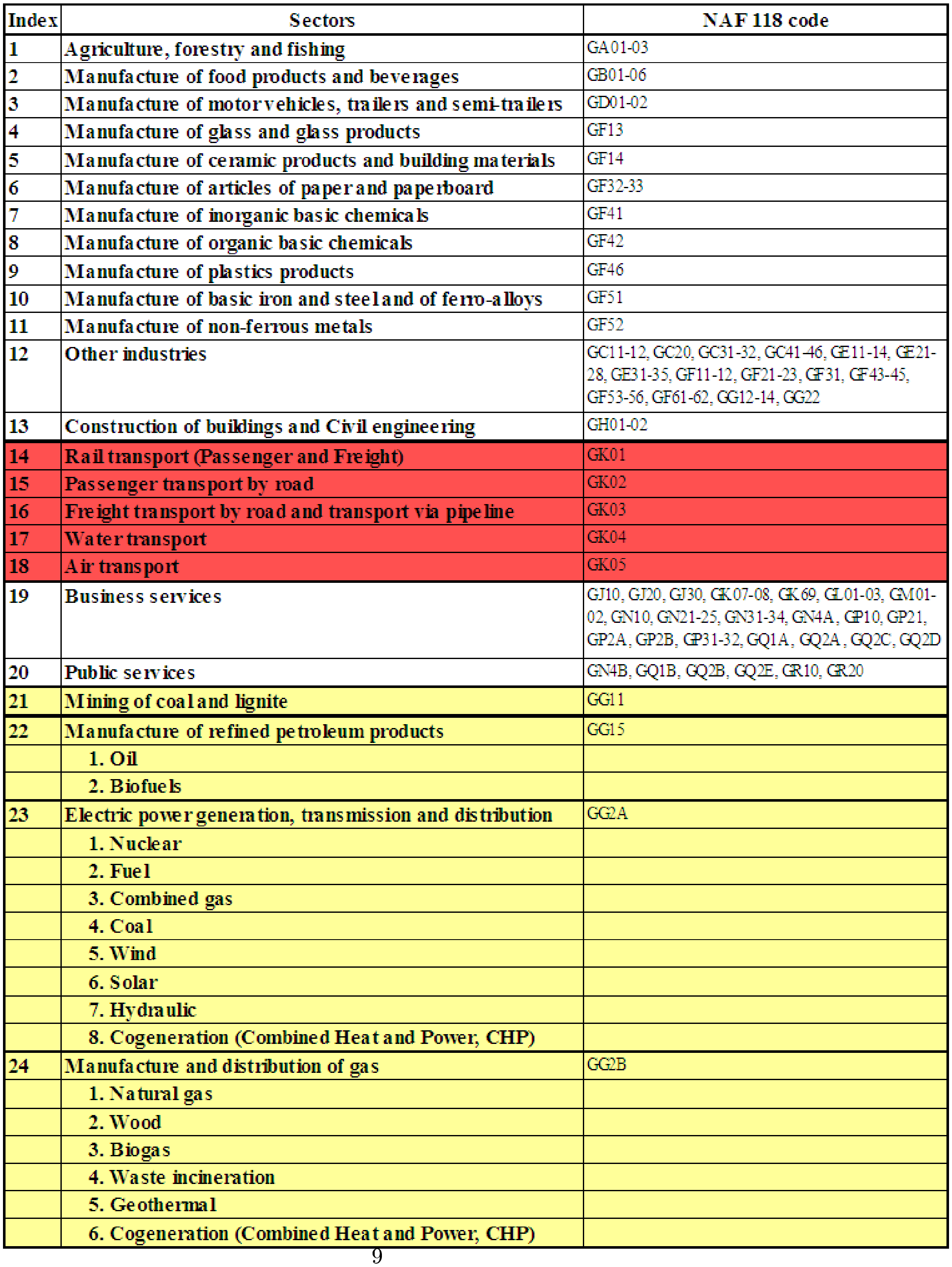
Figure 1: Overall structure of Three-ME

The unemployment rate is an important determinant of the wages dynamic which is defined by a Phillips curve. The inflationary property of the model is determined by the feedbacks between wages, production cost and prices. Prices are assumed to adjust slowly to their optimum level that corresponds to a mark-up over marginal costs. In the short term, the mark-up accounts for the tensions between production capacities (supply) and demand, which is a classical market property. Consequently, wages, which affect production costs, affect directly prices. Prices have in return an impact on wages because they are indexed on the consumer price inflation. Production costs are also directly affected by prices via the cost of intermediate consumptions and of investment.

This dynamic between wages, costs and prices affects the demand through several canals. Wages affect the household consumption because they are an important part of their income. Prices and costs affect profits and thus sectors' debts level. But they affect the households' consumption and investment too because they finance a part of the private debt of the economy.

Another canal is the monetary policy which is defined by a Taylor rule. The European central bank determines the interest rate level based on the European inflation and unemployment. This has an effect on the demand via the negative effect of the real interest rate on consumption and investment. Thus, in this model, interest rate is not directly determined by the equilibrium between investment and saving as generally assumed in the CGEM Models (see e.g. Shoven and Whalley, 1992). This is an important feature because the optimum cannot automatically be reached if this hypothesis does not prevail. In most CGE models, all incomes are spent, since all the savings finances instantaneously invesment. Combined with the hypothesis of a perfect prices flexibility, this assumption ensure that there is no demand constraint and that all the production factors are used (that is why there is no unvoluntary unemployment, unless there are some exogenous rigidities on the labor market). In neo-keynesian models, the equality between investment and savings is not ensured by the interest rate flexibility anymore. Therefore, the investment is not determined by the amount of savings. It depends on both the demand and the price of capital. The savings level adjusts to the level of investment, thanks to the fluctuations of the activity. In that case, investment is implicitly financed by monetary creation, that is by the credits offered by banks. Therefore the money is endogenous since investment depends on demand. As a consequence, in a context of prices rigidities and unvoluntary unemployment, a demand shock may have a positive effect on the production level in the short and medium terms since it influences the investment level, and therefore the amount of available capital factor. Money is not neutral anymore. The eviction effect of the public spendings is not total, since the global amount of investment is not fixed by a exogenous propensity to save. Nevertheless, there is generally an eviction effect because investment entails inflation and unemployment reduction, which lead to an increase in interest rate. In the long term, the public spending has a permanent and positive effect on the production level, if the direct and indirect incomes generated by the public investments are superior to the debt reimbursment and interest charges (that is the money destruction). In that case, one can concieve that climate change policy may provide some long term benefits, if the Net Present Value of the induced investments is positive and if the global variation of investment is positive or compensated by a reduction of the trade deficit or an increase in consumption due to a better labor intensity.

Table 1: Sectoral disaggregation in Three-ME



Index	Sectors	NAF 118 code
1	Agriculture, forestry and fishing	GA01-03
2	Manufacture of food products and beverages	GB01-06
3	Manufacture of motor vehicles, trailers and semi-trailers	GD01-02
4	Manufacture of glass and glass products	GF13
5	Manufacture of ceramic products and building materials	GF14
6	Manufacture of articles of paper and paperboard	GF32-33
7	Manufacture of inorganic basic chemicals	GF41
8	Manufacture of organic basic chemicals	GF42
9	Manufacture of plastics products	GF46
10	Manufacture of basic iron and steel and of ferro-alloys	GF51
11	Manufacture of non-ferrous metals	GF52
12	Other industries	GC11-12, GC20, GC31-32, GC41-46, GE11-14, GE21-28, GE31-35, GF11-12, GF21-23, GF31, GF43-45, GF53-56, GF61-62, GG12-14, GG22
13	Construction of buildings and Civil engineering	GH01-02
14	Rail transport (Passenger and Freight)	GK01
15	Passenger transport by road	GK02
16	Freight transport by road and transport via pipeline	GK03
17	Water transport	GK04
18	Air transport	GK05
19	Business services	GJ10, GJ20, GJ30, GK07-08, GK69, GL01-03, GM01-02, GN10, GN21-25, GN31-34, GN4A, GP10, GP21, GP2A, GP2B, GP31-32, GQ1A, GQ2A, GQ2C, GQ2D
20	Public services	GN4B, GQ1B, GQ2B, GQ2E, GR10, GR20
21	Mining of coal and lignite	GG11
22	Manufacture of refined petroleum products	GG15
	1. Oil	
	2. Biofuels	
23	Electric power generation, transmission and distribution	GG2A
	1. Nuclear	
	2. Fuel	
	3. Combined gas	
	4. Coal	
	5. Wind	
	6. Solar	
	7. Hydraulic	
	8. Cogeneration (Combined Heat and Power, CHP)	
24	Manufacture and distribution of gas	GG2B
	1. Natural gas	
	2. Wood	
	3. Biogas	
	4. Waste incineration	
	5. Geothermal	
	6. Cogeneration (Combined Heat and Power, CHP)	

The dynamic of prices is the driver of the substitution mechanisms of the model. The evolution of relative prices between imported and domestic goods defines the repartition between imported and domestic products to satisfy the internal (consumption and investments) and external (export) demand. The evolution of relative prices between types of goods and services defines the structure of consumption of the economy. Importantly for the analysis of environmental and energy policies, it defines the share of each energy and transport into (intermediate and final) consumptions.

Three-ME accounts for 5 types of households ranked according to their revenue decile. It also explicitly distinguishes between five types of transports and four types of energy (resp. red and yellow lines in Table 1). Energy intensity was the main criterion for the selection of the 24 sectors. In order to better account for the impact of renewable energy, the 4 energy sectors (coal, petroleum, electricity and gas) have been subdivided into 13 sub-sectors. This relatively high level of disaggregation is important to capture the complexity of the substitution mechanisms involved after a change in the relative price between energies. For instance, an increase in the oil price tends to lead to substitution from oil to the other energy in several ways. In addition to direct substitutions by producer and consumer, indirect effects occur via the increase of the production price of oil intensive sectors. This leads to intermediate and final consumptions structure less oil intensive. The decrease of the use of transport by road would be the most typical example.

Three-ME accounts also for endogenous energy efficiency and sobriety effects. In contrast with the substitution mechanisms, the reduction of a given energy consumption due to efficiency and sobriety effects does not imply the increase of the use of another energy. Sobriety consists in refraining from consuming energy by for instance staying home during the weekend instead of taking the car or by lowering the heating temperature in the house. In general, sobriety leads to a decrease in the welfare of the consumer. In contrast, in the case of efficiency, the same welfare is achieved with a lower quantity of energy. Energy efficiency implies an investment in a more efficient technology by for instance switching from a high to a low oil consumption car or by using more efficient insulation techniques for the house. In the model, endogenous efficiency phenomena are introduced through an explicit distinction between several types of housing and automobile investments, classified according to their energy consumption.

In Three-ME, efficiency and sobriety phenomenas decrease the consumer price since the share of energy into consumption decreases (see Section 5). This allows for directly capturing the so-called "rebound effect" in consumption behavior often observed at the micro level (Bentzen, 2004; Sorrell et al., 2009). There is a rebound effect when the effective energy saving from an investment in energy efficiency is less than the energy saving expected ex ante because the consumer uses a part of the reduction of her energy bill to increase her energy consumption. A typical example is the case of certain poor households who live in badly insulated houses and set a low heating temperature to reduce their energy bill. After an insulation investment, they will have the tendency to increase the heating temperature of their house keeping their energy bill more or less constant. This effect is explicitly taken into account in the model: an energy efficiency investment decreases the consumer price and thus increases the real income which leads to a higher level of (energy) consumption.

It is also important to mention that Three-ME is a hybrid model since it combines the top-down approach of general equilibrium macroeconomic models with elements of bottom-up models of energy "engineer" models. The top-down structure presented above gives the macroeconomic consistency and allows for taking into account the feedback between price and quantity that are generally absent in the bottom-up models where prices are exogenous. As in bottom-up models, the amount of energy consumed households is related to their use, that is to say to the number of buildings or cars, and the energy class to which they belong. There are limits on the number of vehicles or housing per households, energy consumption per vehicle or per housing which avoid to simulate unrealistic rebound or wealth effects. This framework differs from the majority of top-down models where energy consumption is usually directly related to income through a nested structure of utility function and not expressed in physical units which may lead to unrealistic representation of the future (e.g. heating temperature to 35 º C in the house, 5 cars per person). We therefore believe that this hybrid setting is particularly important for the analysis of environmental policy where the time horizon is long.

The short and medium run dynamic is largely driven by the demand side through multiplier and accelerator mechanisms. Because of the slow adjustment of price and quantity to their optimal value, the allocation of production factors is sub-optimal in the short and medium run. The long term is driven by the supply constrain. All adjustment processes are achieved: there is no error of anticipation and the effective quantities coincide with the optimal ones. The prices are fully adjusted and all markets are in equilibrium. The unemployment reaches its structural level. The economy thus converges toward a stable equilibrium growth path à la Solow (1956) where all real variables grow at the same rate defined as the sum of the growth rate of the technical progress and of the population. Therefore per capita real variables grow at the same rate as the technical progress. All prices grow at the rate of inflation which is defined by the exogenous rate of inflation in the rest of the world. The endogenous dynamic of the model is determined by capital accumulation of households and firms, the specification of the anticipation and of the adjustment dynamic.

Three-ME aims also to overcome the restriction imposed by nested Constant Elasticity of Substitution (CES) functions by assuming a more flexible form of the production function. This is a clear difference with most CGEM where the technology is generally represented by a series of nested CES production function (e.g. Bernard and Vielle 2008; Burniaux et al. 1992). Nested CES functions proposed by Sato 1967 have the advantage to allow for different elasticities of substitutions between production factors that are not in the same nested structure. But within the same CES, the elasticity of substitution is common to all factors. For instance, if several energy inputs are represented within the same CES, the elasticity of substitution is the same between all these energy inputs. This may be a very strong assumption in some cases. Three-ME does not impose this restriction by assuming a flexible function where the elasticity of substitution is not necessary common between all the inputs of the same nested structure. This allows for changing easily the hypotheses about the value of elasticity of substitutions without having to change the structure of the nest.

Three-ME model is programmed on the E-views 7 package software and simulated with the Broyden algorithm. Two version of the model can be simulated. The "analytic" version uses the standard Linear Expenditure System (LES) utility function to model the consumption of every commodities (including cars, transport and housing investments). This gives a benchmark particularly useful to test the consistency of the model and its basic properties. In particular, this version allows for testing the consistency of the data, of the calibration of parameters and variables and of the model specification with the standard assumption of stable equilibrium growth path à la Solow (1956). Indeed, with a LES hypothesis, and in the absence of error of parametrization and specification, the model can simulate a stable equilibrium growth path from the first period onward (see Appendix A). Implementing standard shock allows to see if the dynamic is consistent with a stable path. In the hybrid version, the modeling of the household include bottom-up elements for the consumption of car, transport, housing and energy. The other commodities are still modeled using the LES hypothesis.

3 Demand and supply equilibrium and adjustment processes

The demand and supply equilibrium simply models the national account equilibrium. The base year 2006 has been calibrated on the input-output tables and resources and uses tables of the French national accounts (available on www.insee.fr). In order to derive price indexes, each variables (GDP, consumption, investment, etc.) are defined in value and in volume (see Appendix C-1). For calibration convenience, each prices are calibrated to unity.

Compared to standard walrasian CGEMs, the equality between supply and demand is not achieved through the perfect flexibility of prices and quantities. In coherence with a neo-keynesian framework, prices and quantities are sticky and supply is determined by demand. Although we assume, for simplicity, that producer are always able to match the demand, supply shortages are taken into account by assuming that they increase the production price.

For quantity and prices, the adjustment process and expectations are specified according to the following equations:

$$ln(X_t) = \lambda_0^X.ln(X_t^n) + (1-\lambda_0^X)(ln(X_{t-1}) + \Delta ln(X_t^e)) \tag{1}$$

$$\Delta ln(X_t^e) = \lambda_1^X.\Delta ln(X_{t-1}^e) + \lambda_2^X.\Delta ln(X_{t-1}) + \lambda_3^X.\Delta ln(X_t^n) + \lambda_4^X.\Delta ln(X_{t+1}) \tag{2}$$

Where X_t is the effective value of a given variable (e.g. the production price, labor, capital, etc), X_t^n its notional (or desired) level, X_t^e its expected (anticipated) value at period t. The first equation assumes a geometric adjustment process. The taking into account of the anticipation warrants that in the long run the effective variables converge to their desired levels. The second equation assumes a general specification for expectation that combines backward-looking and forward-looking expectation. We assume further that in the long run expectations are accurate: $\sum_{i=1}^{4} \lambda_i^X = 1$. The above specification is simple and relatively general since it can be calibrated to match other usual specification such as Error Correction Model (ECM). We also assume that substitution effect adjust slowly:

$$SUBST\_X_t = \lambda_5^X.SUBST\_X_t^n + (1 - \lambda_5^X).SUBST\_X_{t-1} \tag{3}$$

The above three equations allows for a rich set of adjustment processes since they introduce different types of rigidity, i.e. on price and quantity, on expectations, and on substitution mechanisms. To illustrate, let us describe more specifically the case of labor (L) by introducing the notional labor demand and notional substitution effects (see Appendix C and D for more explanations on terms and notations):

$$\begin{aligned} \Delta l_{a,t}^n &= \Delta y_{a,t} - \Delta prog_{a,t}^L + \Delta SUBST\_L_{a,t} \qquad (4) \\ \Delta SUBST\_L_{a,t}^n &= -\eta_a^{KL} \varphi_{a,t-1}^K \Delta(c_{a,t}^L - c_{a,t}^K) - \eta_a^{LE} \varphi_{a,t-1}^E \Delta(c_{a,t}^L - p_{a,t}^E) \\ &\quad - \eta_j^{LM} \varphi_{a,t-1}^{Mat} \Delta(c_{a,t}^L - p_{a,t}^{Mat}) \end{aligned}$$

These notional value are those the producer would like to reach immediately if there were no adjustment constraint. Because of adjustment costs, we assume that this process takes time. We introduce inertia in substitution mechanisms to account for the fact that the impact of substitution is generally slower than the impact of production on the demand for inputs. Assuming that the adjustment process is defined according to Equations (1), (2) and (3), the full dynamic for labor is defined by the three following additional relations:

$$\begin{aligned} ln(L_{a,t}) &= \lambda_0^L.ln(L_{a,t}^n) + (1 - \lambda_0^L) ln(L_{a,t-1} + \Delta ln(L_{a,t}^e)) \\ \Delta ln(L_{a,t}^e) &= \lambda_1^L.\Delta ln(L_{a,t-1}^e) + \lambda_2^L.\Delta ln(L_{a,t-1}) + \lambda_3^L.\Delta ln(L_{a,t}^n) + \lambda_4^L.\Delta ln(L_{a,t+1}) \\ SUBST\_L_{a,t} &= \lambda_5^L.SUBST\_L_{a,t}^n + (1 - \lambda_5^L).SUBST\_L_{a,t-1} \end{aligned}$$

For the sake of concision, the representation of adjustment dynamic [Equations (1), (2) and (3)] is not reproduced for each variable. Only notional variables are presented in the rest of the document.

4 The producer

4.1 Domestic production

Three-ME assumes that each activities (or sectors) may produce more than one commodities. For instance the commodity electricity is produced by several sector (nuclear, wing, etc.). Therefore the production of commodity c by activity a is:

$$Y_{c,a} = \varphi_{c,a} YQ_c \tag{5}$$

YQ_c and $\varphi_{c,a}$ are respectively the aggregate (domestic) production of commodity c and the share of commodity c produced by activity a. Therefore the aggregate production of activity a is:

$$Y_a = \sum_c Y_{c,a} \tag{6}$$

4.2 Demand for production factors

As shown in Figure 2, the production structure is decomposed in three levels. The first level assumes a technology with four production factors (or inputs) sometimes referred as a KLEM (Capital, Labor, Energy, Material) technology, thus splitting intermediary consumptions into energy and material. Compared to most existing models, we do not necessarily assume a Constant Elasticity of Substitution (CES) between these factors. For instance the elasticity of substitution between capital and labor may differ from the one between capital and energy. To do so we use a generalized CES (GCES) function (see Appendix B). We added a fifth element in the first level: the transport and commercial margins. Stricto sensus, they cannot be considered as production factors since they intervene after the production process. Thus they are not substitutable with the production factors. But they are closely related to the level of production since once a good has been processed, it has to be transported and commercialized. At the second level, the investment, energy, material and margins aggregates are further decomposed. The investment level is determined by the capital stock assuming a constant depreciation ratio. At the third level, the demand for each factor or margin is either imported or produced domestically. The generalized CES function is also used to capture substitutions effect at the levels 2 and 3. Moreover, we assume at each level a degree 1 homogenous function, that is a constant return-to-scale technology.

Appendix B shows that the cost minimizing program of the firm in the case of a constant return-to-scale GCES technology leads to the following notional production factors (or) demand (Equation 125):

$$\Delta I_{j,a}^{Input\_n} = \Delta y_a - \Delta prog_{j,a}^{Input} + \Delta SUBST\_i_{j,a}^{Input} \tag{7}$$

$$\Delta SUBST\_I_{j,a,t}^{nput\_n} = - \sum_{\substack{j'=1 \\ j' \neq j}}^{J} \eta_{j,j'} \varphi_{j',a,t-1} \Delta (p_{j,a,t}^{Input} - p_{j',a,t}^{Input}) \tag{8}$$

$$with \quad \varphi_a^j = \frac{P_{j,a}^{Input} . I_{j,a}^{Input}}{\sum_j P_{a,j}^{Input} . I_{j,a}^{Input}} \quad and \qquad j = \{K, L, E, M\}$$

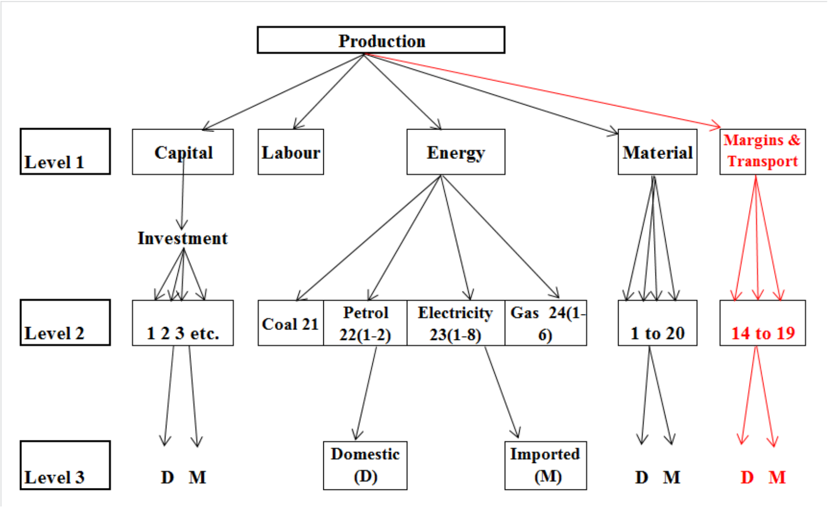


Figure 2: Production structure of Three-Me

where$I_{j,a}^{nput}$ and $I_{j,a}^{nput-n}$ is the effective and notional demand of input h (KLEM) in sector a, $\eta_{j,a}$ the elasticity of substitution between the production factors j and j' in sector a, $P_{j,a}^{rog}$ the technical progress of input j in sector a, $\varphi_{j',a}^{val}$ the value share of input j into the production of sector a. The superscript n refers to the adjective "notional" as opposed to "effective" as defined by neo-Keynesian disequilibrium theory (e.g. see Benassy 1975). The notional demand is the optimal demand of the firm derived from its maximization program. We may also use the adjective "desired" since it would be the demand the firm would like to achieve immediately if there were no constrains such as adjustment costs.

The above input demand is replicated for the four KLEM production factors: $I_{j,a}^{nput} = [K_a; L_a; E_a; M_a^{at}]$, referring respectively to capital, labor, energy and material. In the case of material, relation 7 can be interpreted as the equation of the Leontief technical coefficients which corresponds to the input to production ratio $I_{j,a}^{nput-n}/Y_a$. Unlike the Leontief model, they may here vary over time because of substitution mechanisms between inputs and because of the technical progress.

The investment in sector a ($IA_{a,t}$) is calculated by inverting the capital accumulation equation assuming a constant depreciation rate ($\delta_{a,t}$) of capital:

$$IA_{a,t}{=}K_{a,t} + (1-\delta_a).K_{a,t-1} \tag{9}$$

The depreciation rate is calibrated on national account data by inverting Equation [9], using the net fixed capital stock data for capital and the gross fixed capital formation data for investment.

Because of access restriction to National account investment data, investment decisions were initially modeled for the all the private sector. We now use detailed investment data disaggregated by sector and therefore identify a specific investment pattern for each activity. The commodity type c investment in activity a is specified by assuming that all type of investments are complementary (Leontief assumption):

$$\Delta ia_{c,a} = \Delta ia_a \tag{10}$$

Labor is assuming homogenous inside each sector, and is thus not disaggregated further.[4] On the contrary, the aggregate of energy and material inputs are disaggregated in a second level of production structure assuming a GCES function. The demand for energy c and material c (per activity) are respectively:

$$\begin{aligned} \Delta e_{c,a} &= \Delta e_a + \Delta SUBST\_E_{c,a} \qquad (11) \\ \Delta SUBST\_E^n_{c,a,t} &= -\sum_{i'=21}^{24} \eta_{c,c'} \varphi^{val}_{c'a,t-1} \Delta(p^E_{c,a,t} - p^E_{c',a,t}) \end{aligned}$$

$$\begin{aligned} \Delta mat_{c,a} &= \Delta mat_{c,a} + \Delta SUBST\_MAT_{c,a} \qquad (12) \\ \Delta SUBST\_MAT^n_{c,a,t} &= -\sum_{i'=1}^{20} \eta_{c,c'} \varphi_{c',a,t-1} \Delta(p^{Mat}_{c,a,t} - p^{Mat}_{c,'a,t}) \end{aligned}$$

In both cases, the demand for each type of energy and material is the function of the aggregates defined in the first level and of the relative prices between types of energy and material. Note that here there is no distinction between the effective and notional demand since we assume that the adjustment is instantaneous ($e_{c,a} = e^n_{c,a}$). However there is still an adjustment dynamic for substitution mechanisms.

Finally, in the third level, each type of investment products, energy and material can be domestically produced or imported. As in Armington (1969), a CES function is used to describe the possibilities of substitution between imported and domestic goods. For instance, in the case of the demand for imported and domestic energy c of the sector a, the specification is:

$$\begin{aligned} \Delta em_{c,a,t} &= \Delta e_{c,a,t} + \Delta SUBST\_EM_{c,a,t} \qquad (13) \\ \Delta SUBST\_EM^n_{c,a,t} &= -\eta_{cd,cm} \varphi^{EM}_{c,a,t-1} \Delta(p^{EM}_{c,t} - p^{ED}_{c,t}) \end{aligned}$$

$$\begin{aligned} \Delta ed_{c,a} &= \Delta e_{c,a} + \Delta SUBST\_ED_{c,a} \qquad (14) \\ \Delta SUBST\_ED^n_{c,a,t} &= -\eta_{cd,cm} \varphi^{ED}_{c,a,t-1} \Delta(p^{ED}_{c,t} - p^{EM}_{c,t}) \end{aligned}$$

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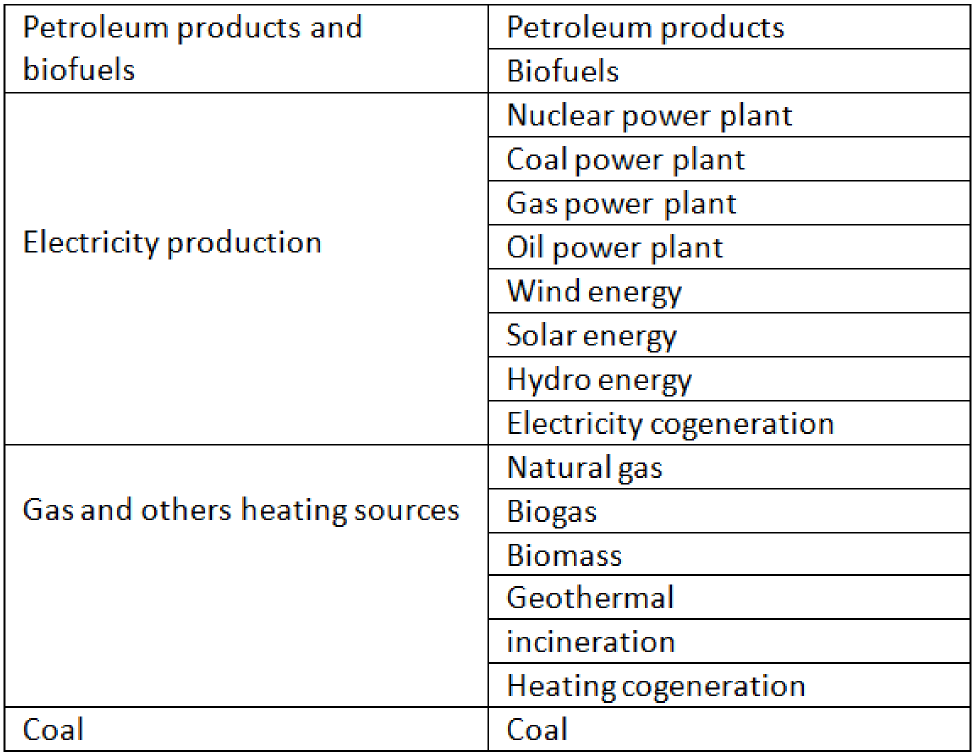
[4] On the contrary, the JULIEN model (Laffargue, 1996) applied to the French economy distinguishes two types of worker qualification. As suggested by econometric studies (e.g. Shadman-Mehta and Sneessens, 1995), this would allow to reproduce more accurately the recent evolution in the industry sector by accounting for different substitution pattern between each kind of labor and capital, and biased technical progress in favor of less qualified labor.



Petroleum products and biofuels	Petroleum products
	Biofuels
Electricity production	Nuclear power plant
	Coal power plant
	Gas power plant
	Oil power plant
	Wind energy
	Solar energy
	Hydro energy
	Electricity cogeneration
Gas and others heating sources	Natural gas
	Biogas
	Biomass
	Geothermal
	incineration
	Heating cogeneration
Coal	Coal

Figure 3: Energy production by source

4.3 The energy production

The production functions of the energetic subsectors (displayed in Figure 3) are the same as the others, but the market share of each energy source is exogenous. This assumption is realistic for the electricity production sector, since the government delivers the authorizations for installing power plants (nuclear or conventional ones).

We assume that the objectives given for the various renewable energy sources are reached thanks to a policy of guaranteed purchases tariffs, financed by a tax on energy consumption. This policy has already been enforced in France. Thus, the renewable energy producers are insured to receive a public subvention per KWh equal to the difference between the energy market price and the tariff, which is fixed above their unitary cost of production. The tariffs are designed to equalize the profit margins per unit of production in all sectors. Hence the investment choices in energy sectors do not obey to the same market rules than others. There are almost entirely determined by the public policy.

Like for other goods, the propensities to import the consumption or export the production depend on both, their previous amounts and the distortions between international and domestic prices.

Technical progress has been introduced in the production functions of energy sectors. It may be positive when the factors productivity is increasing thanks to innovations (that is still the case in most renewable energy production sectors), or negative, when the unitary costs of production increase due to stricter security rules, as for nuclear power plant (the production cost of the third generation is much more important than those of the second one). This effect is decreasing over time.

The energetic mix in the base year has been parameterized with the data from Department of Energy[5]. For each subsectors, the shares of labor, capital, intermediary consumption, and fuel consumption, intio the production costs have been parameterized with data from Department of Energy[6] and the French Environment and Energy Management Agency[7] (ADEME)

4.4 Debt in the private sector

The dynamic of the debt in the private sector (D_t^s) is determined by the accumulation Equation 15, which depends on the gap between the private investment spending and the Gross Operating Surplus (GOS_t) :

$$D_t^s = D_{t-1}^s(1+R_t^s) + P_t^{inv} I^s - GOS_{jt}^s + FP_t^{tax} \tag{15}$$

$$GOS_t^s = P_t^{va} VA_t + SY_t - IY_t - L_t W_t (1+T_t^{CSE}) \tag{16}$$

where SY_t and IY_t are respectively the subvention and tax on production. W_t is the gross wage and (R_t^s) the interest rate paid by the private sector. FP_t^{tax} is the tax on profit , T_t^{CSE} the rate of employer social security contribution.

5 LES modeling of households'behaviour

In a first version of the model, we assume a Linear Expenditure System (LES) utility function to model consumption decisions. This imply the demand for every expenditures (including energy) is directly related to the income:

$$\begin{aligned} EXP_{c,h}^n.PEXP_{c,h} &= PEXP_{c,h}.NEXP_{c,h} + \beta_{c,h}^{EXP}[DISPINC\_VAL_h.(1-MPS_h) \\ &\quad - \sum_c PNEXP_{c,h}.NEXP_{c,h}] \end{aligned} \tag{17}$$

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[5] Ministère de l'économie des finances et de l'industrie, « l'énergie en France, repères », col. Chiffres clés, ed. 2006, 40 p. Ministère de l'économie des finances et de l'industrie, « Bilan énergétique de l'année 2006 de la France », DGEMP, Observatoire de l'Energie, 2007, 25 p.

[6] Ministère de l'économie des finances et de l'industrie, « Coût de référence de la production électrique » décembre 2003, 163 p.

[7] In Numeri, « marchés, emplois et enjeu énergétique des activités liées à l'amélioration de l'efficacité énergétique et aux énergies renouvelables, situation 2008-2009 – perspectives 2010 », ADEME, SEP, octobre 2010, 379 p.

Where $EXP^{n}_{c,h}$ is the notional demand for expenditure c by household h, $PEXP_{c,h}$ the price of expenditure c by household h, $NEXP_{c,h}$ the necessary (minimum) expenditure c by household h, $DISPINC\_VAL_{h}$ household h's disposable income, MPS_{h} household h's marginal propension to save. The household h's marginal propension to spend in commodity c ($\beta^{EXP}_{c,h}$) is generaly constant in a LES setting assuming implicitly an elasticity of substitution of one between comodities. In a more general case where the elasticity of substitution (η^{LES_CES}) can vary from zero to infinity, it is possible to show that that the marginal propension to spend is not constant and depends on the price of expenditures:

$$\Delta\beta^{EXP}_{c,h} = (1-\eta^{LES\_CES}).\Delta\frac{PEXP_{c,h}}{PEXP^{CES}_{h}} \tag{18}$$

$$PEXP^{CES}_{h} = \left[\sum_{c}\beta^{EXP}_{c,h,0}.PEXP_{c,h}{}^{(1-\eta^{LES\_CES})}\right]^{\frac{1}{1-\eta^{LES\_CES}}} \tag{19}$$

6 An hybrid modeling of households'behaviour

The standard representation of the consumer maximization behavior used in most top-down CGEM assumes that energy sources provide utility on their one. Therefore their consumption is more or less proportional to their revenue because of the hypothesis of nested utility function. But in reality energy has no use in itself. Households buy energy to fulfill certain services such as housing (heating and the functioning of equipments) or transport. Therefore the quantity of energy consumed for heating purposes is more related to the size of the house than directly to the revenue of households. Of course, rich households generally have bigger houses and therefore their energy consumption will generally be higher. At the same time, one can expect that the energy consumption per square meters will be lower for poor households since they are generally more careful in trying to limit their energy bill. Indeed, micro data suggest that the poor households tend to lower heating temperature in their houses. The energy consumption per m^{2} tends to increase with the income decile but within limit since energy is more a necessity good than a luxury good. And hardly no one wants a heating temperature in their house of 35 °C even very rich people.

One way to model this is to assume that the production of certain services such housing and mobility is directly produced by households rather than purchase externally. Therefore we specify explicitly a household's production function for the services of housing and mobility (Figure 4). The block has two main components: housing, transport. We assume further that the expenses related to this production function is prioritary. For the other expenditures, we use a standard LES which allows to model in a simple way the distinction between necessity/luxury goods. In this section, we only present the key equations of our hybrid modeling of households block. The complete block is presented in

Appendix C whereas all the notations used are defined in the glossary of terms used (Appendix D).

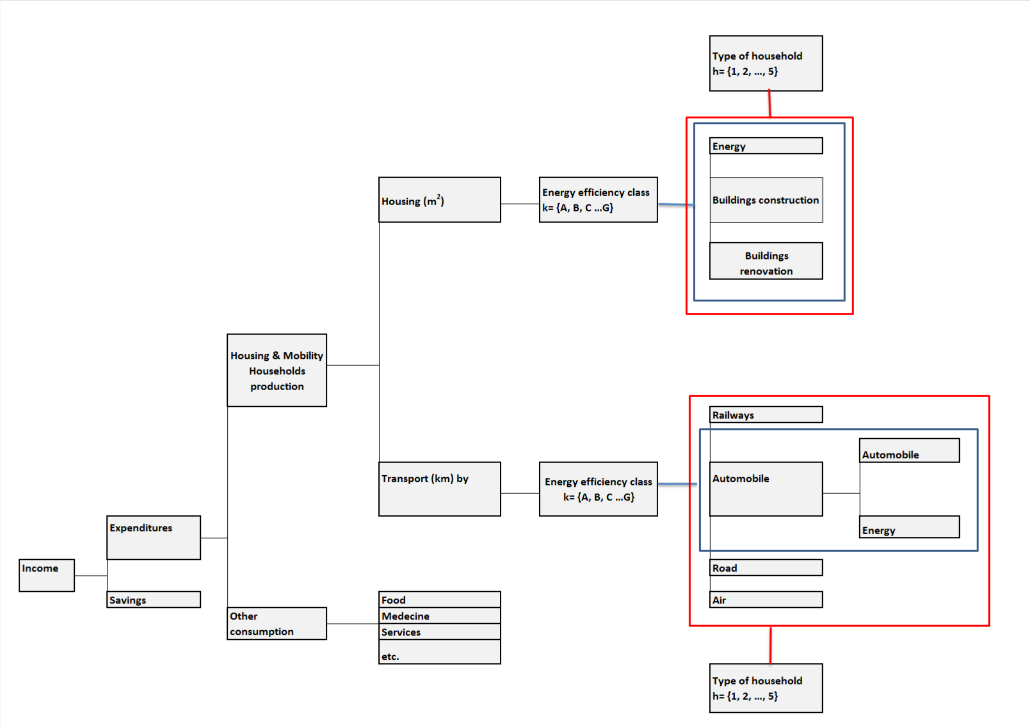

Figure 4: Household's structure of expenditures

6.1 Building stock dynamic

We differentiate buildings according to their energy efficiency class, k = {1,. . . ,K}. We assume that the building stock of class k expressed in m^2 is driven by the following dynamic:

$$\begin{aligned}\Delta BUIL_{h,k,t} \quad = \quad & \varphi_{h,k}^{NewBUIL}(\Delta BUIL_{h,t} + BUIL_{h,0,t}) \\ & + \sum_{k'=0}^{k-1} REHAB_{h,k',k} - \sum_{k'=k+1}^{K} REHAB_{h,k,k'} \\ & - \sum_{k'=0}^{k-1} \delta_{h,k,k'}^{BUIL} BUIL_{h,k,t-1} + \sum_{k'=k+1}^{K} \delta_{h,k',k}^{BUIL} BUIL_{h,k',t-1}\end{aligned} \tag{20}$$

$$BUIL_{h,0,t} = \sum_{k} \delta_{h,k,0}^{BUIL} BUIL_{h,k,t-1} \tag{21}$$

Where, for household h, $BUIL_{h,k,t}$ is the building stock of class k, $BUIL_{h,t}$ the total building stock, $BUIL_{h,0,t}$ the stock of buildings destroyed in the previous period and reconstructed in the current period. $\varphi_{h,k}^{NewBUIL}$ is the share of the new buildings constructed with a class k label ($\sum\limits_{k} \varphi_{h,k}^{NewBUIL} = 1$). $REHAB_{h,k,k'}$ is the number of m^2 rehabilitated from class k to class k' (with $k < k'$ and $REHAB_{h,k,k} = 0$), $\delta_{h,k',k}^{BUIL}$ the depreciation (or downgrading) rate from class k' to class k (with $k' > k$).

Equation (20) assumes that at each period t, the stock of buildings of class k:

- Increases by the share of the new buildings constructed according to class k standards: $\varphi_{h,k}^{NewBUIL}(\triangle BUIL_{h,t} + BUIL_{h,0,t})$.

- Increases by the amount of rehabilitated buildings from a lower class to class k: $\sum\limits_{k'=0}^{k-1} REHAB_{h,k',k}$.

- Increase by the downgraded buildings from a higher class to class k: $\sum\limits_{k'=k+1}^{K} \delta_{h,k',k}^{BUIL} BUIL_{h,k',t-1}$

- Decreases by the amount of rehabilitated buildings from class k to a higher class: $\sum\limits_{k'=k+1}^{K} REHAB_{h,k,k'}$.

- Decreases by the downgraded buildings from class k to lower class: $\sum\limits_{k'=0}^{k-1} \delta_{h,k,k'}^{BUIL} BUIL_{h,k,t-1}$, where "class 0" refers to destroyed building.

We assume for simplicity that the number of buildings are related to the size of the population:

$$\Delta buil_h = \Delta pop_h + \Delta m2percapita_h \tag{22}$$

Equations (20) and (21) are dynamically consistent since they imply that $\sum\limits_{k=1}^{K} BUIL_{h,k,t} = BUIL_{h,t}$ (provided this is verified and correctly calibrated for the initial period).

To provide a better intuition, the stock dynamic is charted in Figure 5. Blue arrows represent the depreciation mechanism. As time goes along, high energy classes loses efficiency and gets downgraded until they gets eventually destructed (pool $BUIL_{h,0,t}$). As in the model, this chart presents the general case where the downgrading is possible to any lower class. In reality, this process is generally gradual and buildings of high classes will go successively to lower classes instead of been directly destructed. Orange arrows represent the rehabilitation mechanism: by investing in renovation, households have the possibility to increase the energy efficiency of their house. Here too various transitions are possible, e.g. from class 1 to 2, then from 2 to 3 or directly from class 1 to 4. Naturally, the strongest the rehabilitation, the higher the cost. Finally, black arrows represent the (re)-construction process. There are new buildings because the total housing park increases ($\triangle BUIL$) and because destroyed buildings ($BUIL_{h,0,t}$) are reconstructed. Here as well, although new buildings are possible in any category, in practice, new construction follows high energy efficiency standards.

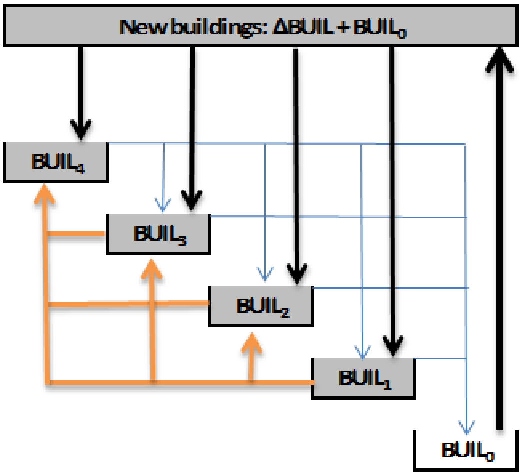

Figure 5: Overall structure of Three-ME

At each period, a proportion of the buildings of category k is rehabilitated: $\tau_{h,k}^{REHAB} = \sum_{k'} REHAB_{h,k,k'}/BUIL_{h,k}$. This proportion may not be constant over time. For instance, it may increase as the energy price increases because this gives an incentive toward more energy efficiency renovation. This can be modeled by assuming that $\tau_{h,k}^{REHAB}$ is endogenous and depends on the user cost of the building. Variation in that $\tau_{h,k}^{REHAB}$ may also be exogenous due to the imposition of stricter energy efficiency requirements embodied in $\tau_{h,k}^{REHAB*}$. Naturally, the proportion of the buildings that are renovated cannot exceed 1. But it appears logical to assume that it has also a lower bound ($\tau_{h,k}^{REHAB\_l}$) to account for irreversibility phenomena: even if the energy price starts to go down, it is possible that households will not lower their investment in energy efficiency. These considerations lead to the following specification:

$$\begin{aligned} \tau_{h,k}^{REHAB} &= \tau_{h,k}^{REHAB*} + \eta_{h,k} \frac{UC_{h,k}^{REHAB}}{UC_{h,k}} \\ 0 \leqslant \tau_{h,k}^{REHAB\_l} \leqslant \quad \tau_{h,k}^{REHAB} &\leqslant \tau_{h,k}^{REHAB\_h} \leqslant 1 \end{aligned} \tag{23}$$

Where $UC_{h,k}$ is the user cost of buildings of type k and $UC_{h,k}^{REHAB}$ is the user cost of the investment in the renovation of building k.

As explained previously, the rehabilitation of a building of a given class k can be done to different higher classes. It would be logical to assume that the choice between the higher classes is endogenous and depends on the relative cost of each option of renovation. However, because of the lack of data, it is difficult to model and calibrate this arbitrage. Moreover, this choice may be strongly determined by technical renovation standards with a small influence of relative prices. Therefore, we assume that this choice is exogenous, that is the share of class k buildings rehabilitated to class k' ($\varphi_{k,k'}^{REHAB}$) is exogenous:

$$REHAB_{h,k,k'} = \varphi_{h,k,k'}^{REHAB}.\tau_{h,k}^{REHAB} BUIL_{h,k,t-1} \tag{24}$$

$$\sum_{k'} \varphi_{h,k,k'}^{REHAB} = 1 \tag{25}$$

In Equation (23), we assume that the proportion of the buildings of category k to be rehabilitated depends on the user cost of buildings. We assume that the latter corresponds to the annual cost of the investment ($UC_{h,k}^{REHAB}$)which consists of two components: (1) the annual cost of the investment itself including eventual interests ($UC_{h,k}^{K\_REHAB}$), (2) annual energy cost ($UC_{h,k}^{E\_REHAB}$). This leads to the following relation:

$$UC_{h,k}^{REHAB} = UC_{h,k}^{K\_REHAB} + UC_{h,k}^{E\_REHAB} \tag{26}$$

$$UC_{h,k}^{E\_REHAB} = \sum_{k'=k+1}^{K} \varphi_{h,k,k'}^{REHAB}.UC_{h,k'}^{E} \tag{27}$$

Regarding the decision of rehabilitating the house to a higher class, the above user cost is compared to the user cost of a building remaining in class k:

$$UC_{h,k} = UC_{h,k}^{K} + UC_{h,k}^{E} \tag{28}$$

The annual investment and energy costs are defined by the following equations:

$$\begin{aligned} UC_{h,k}^{K\_REHAB} &= P_{h,k}^{REHAB\_\delta^{BUIL}} (R_{h,k}^{CASH\_REHAB} \\ &\quad + \frac{R_{h,k}^{LOAN\_REHAB} R_{h,k,t-1}^{I\_REHAB} LD_{h,k}^{REHAB}}{1-(1+R_{h,k,t-1}^{BUIL\_REHAB})^{-LD_{h,k}^{REHAB}}}) && (29) \\ R_{h,k}^{LOAN\_REHAB} &= 1 - R_{h,k}^{CASH\_REHAB} && (30) \\ LD_{h,k}^{REHAB} &\leqq 1/\delta_{h,k}^{REHAB} && (31) \end{aligned}$$

$$
\begin{aligned}
UC^{K}_{h,k} &= P^{REHAB}_{h,k,k}\delta^{BUIL}_{h,k}(R^{CASH}_{h,k} + \frac{R^{LOAN}_{h,k}R^{I\_BUIL}_{h,k,t-1}LD_{h,k}}{1-(1+R^{I\_BUIL}_{h,k,t-1})^{-LD_{h,k}}}) && (32)\\
R^{LOAN}_{h,k} &= 1 - R^{CASH}_{h,k} && (33)\\
LD_{h,k} &\leqq 1/\delta^{BUIL}_{h,k} && (34)
\end{aligned}
$$

$$
\delta^{REHAB}_{h,k} = \sum_{k'=k+1}^{K} \varphi^{REHAB}_{h,k,k'}\delta^{BUIL}_{h,k'} \tag{35}
$$

$$
\delta^{BUIL}_{h,k} = \sum_{k'=0}^{k-1} \delta^{BUIL}_{h,k,k'} \tag{36}
$$

$$
\begin{aligned}
UC^{E}_{h,k} &= \frac{\left(1+\dot{P}^{Ener\_m^2\_\mathrm{e}}_{h,k}\right)^{1/\delta^{BUIL}_{h,k}} - 1}{\dot{P}^{Ener\_m^2\_e}_{h,k}/\delta^{BUIL}_{h,k}}.P^{Ener\_m^2}_{h,k} && (37)\\
P^{Ener\_m^2}_{h,k}.BUIL_{h,k} &= PENER^{BUIL}.ENER_{h,k} && (38)\\
\dot{P}^{Ener\_m^2\_e}_{h,k,t} &= \lambda_0^{Ener\_BUIL}\dot{P}^{Ener\_m^2\_e}_{h,k,t-1} + (1-\lambda_0^{Ener\_BUIL})\dot{P}^{Ener\_m^2}_{h,k,t-1} && (39)
\end{aligned}
$$

Where $R^{CASH}_{h,k}$ is the share of investment that is paid cash, $R^{LOAN}_{h,k}$ the share of investment that is paid with a loan, $R^{I}_{h,k}$ the interest rate, $LD_{h,k}$ the duration of the loan, $P^{Ener\_BUIL}_{h,k}$ the average energy price paid in type k buildings, $P^{Ener\_BUIL\_e}_{h,k}$ its expected value and $ENER_{h,k}$ the energy consumption in buildings k. Note that $1/\delta^{BUIL}_{h,k}$ is the average duration of the investment. $P^{REHAB\_\delta^{BUIL}}_{h,k}$ is the average price of the investment in renovation calculated as follows:

$$
P^{REHAB\_\delta^{BUIL}}_{h,k} = \sum_{k'=k+1}^{K} \left(1-R^{SUB}_{h,k,k'}\right)\varphi^{REHAB}_{h,k,k'}P^{REHAB}_{h,k,k'}\delta^{BUIL}_{h,k'} \tag{40}
$$

Where $R^{SUB}_{h,k}$ is the (eventual) rate of subsidies on the investment in energy efficiency. The expenditures related to housing for buildings k at a given period therefore includes the expenses related to the debt (interest and reimbursement), the investment paid in cash and the cost of energy:

$$\begin{aligned} EXP\_HOUSING^{VAL}_{h,k} &= DEBT^{REHAB\_VAL}_{h,k,t-1}(R^{I\_REHAB}_{h,k,t-1} + R^{RMBS\_REHAB}_{h,k,t-1}) \\ &\quad + R^{CASH\_REHAB}_{h,k,t} P^{REHAB}_{h,k} REHAB_{h,k} \\ &\quad + DEBT^{NewBUIL\_VAL}_{h,k,t-1}(R^{I\_NewBUIL}_{h,k,t-1} + R^{RMBS\_NewBUIL}_{h,k,t-1}) \\ &\quad + R^{CASH\_NewBUIL}_{h,k,t}.P^{NewBUIL}_{h,k}.NewBUIL_{h,k} \\ &\quad + PENER_{h,k}.ENER_{h,k} \end{aligned} \tag{41}$$

$$\begin{aligned} DEBT^{REHAB\_VAL}_{h,k,t} &= DEBT^{REHAB\_VAL}_{h,k,t-1}(1 - R^{RMBS\_REHAB}_{h,k,t-1}) \\ &\quad + R^{LOAN\_REHAB}_{h,k,t}.P^{REHAB}_{h,k}.REHAB_{h,k} \end{aligned} \tag{42}$$

$$\begin{aligned} DEBT^{NewBUIL\_VAL}_{h,k,t} &= DEBT^{NewBUIL\_VAL}_{h,k,t-1}(1 - R^{RMBS\_NewBUIL}_{h,k,t-1}) \\ &\quad + R^{LOAN\_REHAB}_{h,k,t}.P^{NewBUIL}_{h,k}.NewBUIL_{h,k} \end{aligned} \tag{43}$$

$$\begin{aligned} R^{RMBS\_X}_{h,k} &= 1/LD^{X}_{h,k} \\ P^{REHAB}_{h,k}.REHAB_{h,k} &= \sum_{k'} P^{REHAB}_{h,k,k'}.REHAB_{h,k,k'} \end{aligned} \tag{44}$$

Where $R^{RMBS}_{h,k}$ is the rate of reimbursement of the debt. The evolution of the debt is standard: it increases from the investment paid with a loan and decreases with reimbursements). It is worth noticing that in the particular case where the building stock is integrally pay with a loan ($R^{CASH}_{h,k}=0$), the rate of reimbursement of the debt is equal to the depreciation ratio, (and the energy costs are omitted), then debt is always equal to the value of the building stock and the above equation collapse in the standard equation of the user cost of capital: $P^{REHAB}_{h,k}(R^{I}_{h,k,t-1} + \delta^{BUIL}_{k})$.

The investment price for rehabiliation and new buildings is indexed on the consumer price for commodities 13 (construction of building) :

$$\triangle lnP^{REHAB}_{h,k,k'} = \triangle lnPCH_{13} \tag{45}$$

$$\triangle lnP^{NewBUIL}_{h,k} = \triangle lnPCH_{13} \tag{46}$$

6.2 Automobile and Transport

We assume that transport needs are mainly driven by demography. Therefore the number of traveler-km increase proportionally with size of the population:

$$\Delta km_h^{traveler} = \Delta pop_h \tag{47}$$

$$\Delta km_{c,h}^{traveler} = \Delta km_h^{traveler} \tag{48}$$

$$\Delta exp_{c,h} = \Delta km_{c,h}^{traveler} \tag{49}$$

Where $KM_h^{traveler}$ is total number of traveler-kms traveled by household h, POP_h the population size of households h, $KM_{h,c}^{traveler}$ is number of traveler-km of type c transport (plane, train, etc.) traveled by household h, $EXP_{c,h}$ the volume expenditure in type c transport spend by household h.

The above relations may not be be fully proportional since it may be affected by trends: e.g. people may travel more in the future due to wealth increase; they may switch from one type of transport to another. These trends are therefore included in Equations 47 and 48.

The same logic applies for travels by cars. The number of traveler-kms traveled by cars ($KM_h^{traveler\_auto}$) is properational to the total number of traveler-kms (Equation 50). The number of automobile-kms (KM_h^{AUTO}) is proportional to the number of traveler-kms traveled by cars (Equation 51). The number of automobile ($AUTO_h$) is proportional to the number of automobile-kms (Equation 52). Here as well, the inclusion of trends is possible.

$$\Delta km_h^{traveler\_auto} = \Delta km_h^{traveler} \tag{50}$$

$$\Delta km_h^{AUTO} = \Delta km_h^{traveler\_auto} \tag{51}$$

$$\Delta auto_h = \Delta km_h^{AUTO} \tag{52}$$

The modeling of transportation by car is quite similar to the one of housing. In particular, we use the same equations to describe the dynamic of the automobile stock per energy class. However, it differs in two main respects: (1) there is no energy efficiency renovation; (2) an automobile is assumed to remain in its energy class during all its life duration. Hence the downgrading goes directly to destruction. Therefore, the equivalent of Equations 20 and 21 for the automobile stock is:

$$\begin{aligned} \Delta AUTO_{h,k,t} &= \varphi_{h,k}^{NewAuto}(\Delta AUTO_{h,t} + AUTO_{h,0,t}) \\ &\quad -\delta_{h,k}^{AUTO} AUTO_{h,k,t-1} \end{aligned} \tag{53}$$

$$AUTO_{h,0,t} = \sum_k \delta^{AUTO}_{h,k} AUTO_{h,k,t-1} \tag{54}$$

Where, for household h, $AUTO_{h,k,t}$ is the automobiles stock of class k, $AUTO_{h,t}$ the total automobiles stock, $AUTO_{h,0,t}$ the stock of automobiles destroyed in the previous period and reconstructed in the current period. $\varphi^{NewAuto}_{h,k}$ is the share of the new cars constructed with a class k label ($\sum\limits_k \varphi^{NewAuto}_{h,k} = 1$).

Equation (53) assumes that at each period t, the stock of cars of class k:

- increases by the share of the new cars constructed according to class k standards: $\varphi^{NewAuto}_{h,k}(\Delta AUTO_{h,t} + AUTO_{h,0,t})$.

- decreases by the number of cars destroyed: $\delta^{AUTO}_{h,k} AUTO_{h,k,t-1}$.

Otherwise the modeling of the the investment decisions is quite similar to one of housing. The share of the new cars constructed with a class k label ($\varphi^{NewAuto}_{h,k}$) depends on the user cost of the car (see Appendix C for details) that includes both the acquisition cost and the energy costs.

$$\begin{aligned} UC^{auto}_{h,k} &= P^{NewAuto}_{h,k}.NewAUTO_{h,k}(1 - R^{SUB}_{h,k})(R^{CASH\_AUTO}_{h,k} \\ &\quad + \frac{R^{LOAN}_{h,k} R^{I}_{h,k,t-1} LD_{h,k}}{1-(1+R^{I}_{h,k,t-1})^{-LD_{h,k}}}) \\ &\quad + \frac{\left(1+\dot{P}^{Ener\_auto\_e}_{k}\right)^{1/\delta^{auto}_k} - 1}{\dot{P}^{Ener\_auto\_e}_{k}/\delta^{auto}_k}.\dot{P}^{Ener\_auto}_{k} \end{aligned} \tag{55}$$

6.3 Energy consumption

In our hybrid setting, the energy consumption of households is not directly related to the revenue but to the servive it provides, that is to size of builidings and to tyhe number of cars. Therefore the energy consumption of households are directly related to the characteristics of stock of buildings and cars and of the households that possess them:

$$ENER\_HEAT_{h,k,e} = ENER\_HEAT^{PerM2}_{h,k,e} BUIL_{h,k} \tag{56}$$

$$ENER^{AUTO}_{h,k,e} = C^{PerKM}_{k,e}.KM_{h,k,e}.AUTO_{h,k,e} \tag{57}$$

Where $ENER\_HEAT_{h,k,e}$ is the energy e consumption in buildings k for heating, $C^{PerM2\_HEAT}_{h,k,e}$ is the energy e consumption per m^2 in buildings k. The index e refers to the different type of energy. For households h and automobile k, $ENER^{AUTO}_{h,k,e}$ is the energy e consumption, $C^{PerKM}_{k,e}$ the energy e consumption per km, $KM_{h,k,e}$ the number of km traveled per automobile, $AUTO_{h,k,e}$ is the number of automobile.

We assume that the energy consumption for others uses is also proportional to the number of m^2 of buildings (since it is proportional to the number of apparatus and equipments which are themselves proportional to the number of m^2 of buildings):

$$ENER\_OTH_{h,k,e} = ENER\_OTH_{h,e}^{PerM2} BUIL_{h,k} \tag{58}$$

Where $C_{h,e}^{PerM2\_OTH}$ is the energy e consumption per m^2 in buildings k for others uses than heating. In the above equations, the energy consumption is expressed in physical units. Applying relevant coefficient of conversation, this allows for expressing the aggregate energy consumption in tone petroleum equivalent (TPE).

The energy consumption per km, the number of km per year varies between households since it is generally higher for the highest decile. So it is logical to assume that these variables are related to the level of revenue. At the same time, given that housing is a necessity "commodity" for household, it is logical to assume a lower and a higher bound. To impose high and low boundaries, we assume the following logistic function specification:

$$X(\alpha) = [1 - \phi(\alpha)] X^l + \phi(\alpha) X^h \tag{59}$$

$$\phi(\alpha) = [1 + exp(\tau - \sigma.\alpha)]^{-1} \tag{60}$$

Where X^l and X^h are the two bounds (e.g. low and high) or regimes. σ the switching speed between the two regimes (when α increases), τ/σ the value of α when the regime switching arises. It is easy to verify that $\phi(-\infty) = 0; \phi(+\infty) = 1$.

Assuming that of α is the revenue, this function can be calibrated for each households to endogenize relevant parameters (such as energy consumption per m^2) and making them a function of the revenue. For a low and higher bound of respectively 0 and 1 and a change of regime at $\alpha = 2$, Figure 6 shows that the logistic function can be used to model small to high correlation between a relevant parameter (Y-axis) and the revenue (X-axis).

6.4 Other goods: LES function

Once the household has chosen the level of housing and mobility expenditures, it spends the rest of its desired level of expenditure in other goods. We assume a LES function in order to capture the "necessity" or "luxury" character of a given commodity:

$$\begin{aligned} PC_c.C_{i,c} &= PC_c.CMIN_{i,c} + \alpha_{i,c}^H. \\ & (PEXP_i.EXP_i - PEXPHM.EXPHM \\ & - \sum PC_c.CMIN_{i,c}) \end{aligned} \tag{61}$$

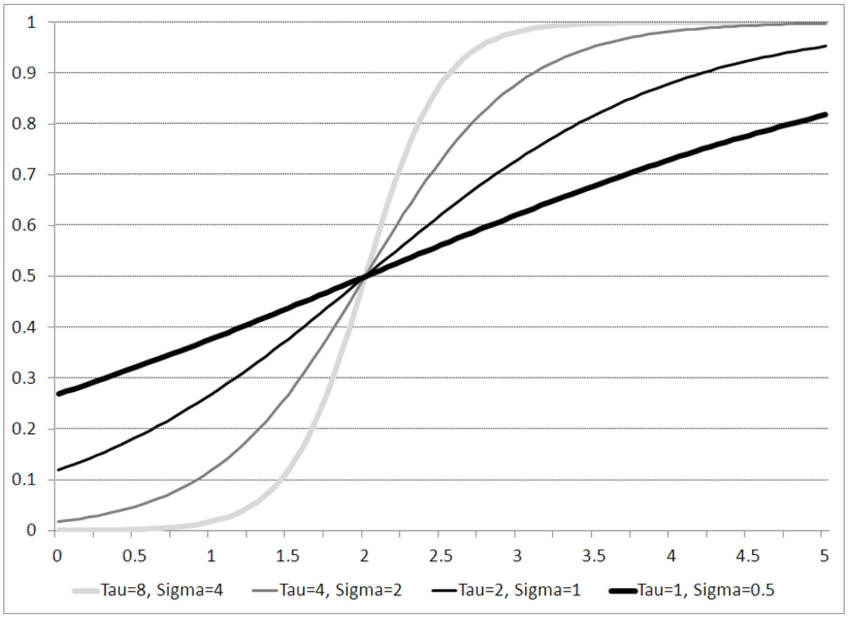

Figure 6: Logistic curve $X^l = 0$ and $X^h = 1$ and $Tau/Sigma = 2$

$$PEXPHM_c.EXPHM_c = PEXPH_c.EXPH_c + PEXPM_c.EXPM_c \tag{62}$$

$$\sum_c \alpha^H_{i,c} = 1 \tag{63}$$

$CMIN_{i,c}$ is the minimum consumption level. If it is equal to zero, the results are similar as those with a Cobb-Douglas function. The constraint $\sum\limits_c \alpha^H_{i,c} = 1$ ensures that households spend all their revenue (minus their desired level of savings). The marginal propension to consume $\alpha^H_{i,c}$ is modeled in the same way as Equation 18 and may therefore depend of relative prices between commodities.

7 The labor market

We assume that the average gross wage (that is including employee social security contributions) in activity a, W_a, is determined by a Phillips curve. Wages may be indexed on the consumer price inflation ($\rho_{2,a} > 0$) and on productivity gains of the sector j ($\rho_{3,a} > 0$) . Trade unions may accept lower wage increases in case of a degradation of the terms of trade, that is in case of competitiveness losses ($\rho_{4,a} > 0$) . In addition to the level of unemployment (U_t) , the variation of unemployment may influence the Phillips curve ($\rho_{6,a} > 0$) , because wages can be affected not only by the level but also by the evolution of employment (Phillips, 1958; Lipsey, 1960) or due to hysteresis phenomena[8] . Finally, it is possible that the wage dynamic differs across sectors because of differences in employment situation ($\rho_{7,a} > 0$) .

$$\Delta w^n_{a,t} = \rho_{1,a}+\rho_{2,a}\Delta p_t+\rho_3\Delta p^{rog}_{a,t}-\rho_{4,a}\Delta(p^m_{a,t}-p^y_{a,t})-\rho_5 U_t-\rho_6\Delta U_t+\rho_7\Delta(l_{a,t}-l_t) \tag{64}$$

The parameter ($\rho_{1,a} > 0$) reflects the labor market tensions and the bargaining power of trade unions.

L_t is the aggregated employment:

$$L_t = \sum_a L_a \tag{65}$$

It can be shown that the WS curve in level is a particular case of the Phillips curve 64: the case of full hysteresis (Reynès, 2010) that is the case where the level of unemployment does not have any effect on the wage setting($\rho_{5j} = 0$). Moreover, we assume a slow adjustment of wages: the effective wage growth adjusts to its notional level defined in 64 according to the adjustment process presented in Section 3.

In order to capture heterogeneity on the labor market, the population is segmented according to age and sex categories.The unemployment rate for each type of population based on their age and sex, is calculated according to its conventional definition:

$$U_{sex,age} = (LF_{sex,age} - EMPL_{sex,age})/LF_{sex,age} \tag{66}$$

$$LF_{sex,age} = PARTR_{sex,age}.POP_{sex,age} \tag{67}$$

where $LF_{sex,age}$ is the active population which is by definition the product between the labor participation ratio$PARTR_{sex,age}$ and the total population $POP_{sex,age}$ assumed to be exogenous.

Since the seminal works of Strand and Dernburg (1964) and Dernburg and Strand (1966), several studies have observed that the labor force participation depends on the labor market situation in particular because of a discouraged-worker effect. Thus, the labor participation ratio may be endogenous and depend negatively on the unemployment rate:

$$\Delta PARTR^n_{sex,age} = \Delta PARTR^{Trend}_{sex,age} + \beta_{sex,age}\Delta U \tag{68}$$

The calibration of the sensitivity of the labor participation ratio to the unemployment rate, $\beta_{sex,age}$, is based on work of Filatriau and Reynès (2011) wh estimate this parameter for 12 sex and age cohorts (See Figure 7). This studies

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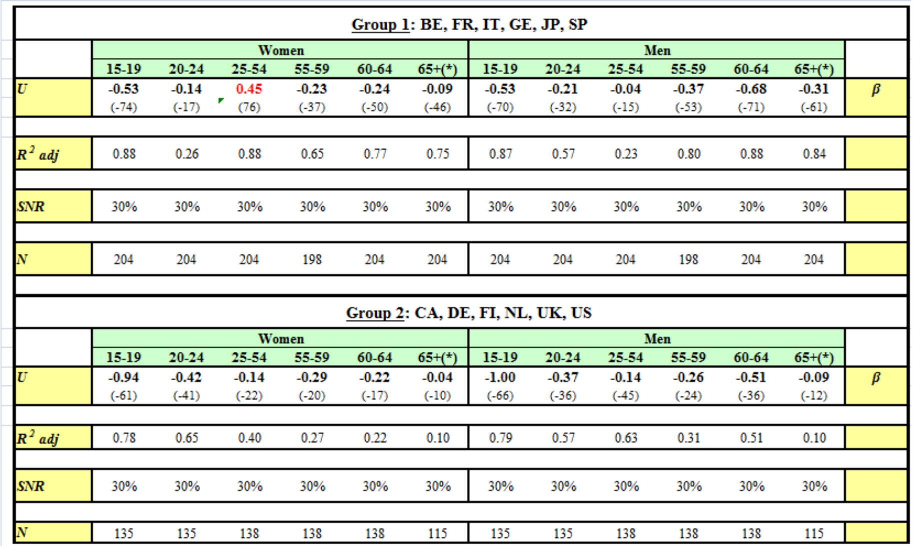
[8] Hysteresis occurs when the long-term unemployed workers exert no influence on wage-setting Blanchard and Summers, 1986; Lindbeck, 1993). However, some authors contest the use of the term hysteresis to describe this phenomenon (Cross, 1995).



Group 1: BE, FR, IT, GE, JP, SP

	Women						Men						
	15-19	20-24	25-54	55-59	60-64	65+(*)	15-19	20-24	25-54	55-59	60-64	65+(*)	
U	**-0.53** (-74)	**-0.14** (-17)	**0.45** (76)	**-0.23** (-37)	**-0.24** (-50)	**-0.09** (-46)	**-0.53** (-70)	**-0.21** (-32)	**-0.04** (-15)	**-0.37** (-53)	**-0.68** (-71)	**-0.31** (-61)	*β*
R^2 *adj*	0.88	0.26	0.88	0.65	0.77	0.75	0.87	0.57	0.23	0.80	0.88	0.84	
SNR	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	
N	204	204	204	198	204	204	204	204	204	198	204	204	

Group 2: CA, DE, FI, NL, UK, US

	Women						Men						
	15-19	20-24	25-54	55-59	60-64	65+(*)	15-19	20-24	25-54	55-59	60-64	65+(*)	
U	**-0.94** (-61)	**-0.42** (-41)	**-0.14** (-22)	**-0.29** (-20)	**-0.22** (-17)	**-0.04** (-10)	**-1.00** (-66)	**-0.37** (-36)	**-0.14** (-45)	**-0.26** (-24)	**-0.51** (-36)	**-0.09** (-12)	*β*
R^2 *adj*	0.78	0.65	0.40	0.27	0.22	0.10	0.79	0.57	0.63	0.31	0.51	0.10	
SNR	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	
N	135	135	138	138	138	115	135	135	138	138	138	115	

Figure 7: Estimation of the flexion effect by age and sex

find that the OECD labor participation of certain categories is particularly sensitive to the labor market situation. Typically for the youngest and the eldest, a discouraged-worker effect generally appears: their participation decreases when the situation on the labor market deteriorates. For France and other Continental European countries, an additional worker effect dominates for women between 25 and 55 years old. Accounting for an endogenous labor participation ratio seems empirically consistent and therefore allows for a more precise measure of the unemployment variation for each population and at the aggregate level.

8 External Trade

The external trade in Three-ME is treated with a relatively high level of detail. On the one hand, import behaviors are specific for each economic actor and each product. On the other hand, the model integrates explicit external demand functions of both the domestic production and the importations with a constant price elasticity.

8.1 Imports

Following the Armington's (1969) approach, the international trade is justified by the differentiation of products between regions of the world. This explanation assumes implicitly the imperfect substitutability between domestic and imported products. To determine the volume of imports by product, each economic actor minimizes the purchasing costs under the constraint of a predetermined absorption level and a CES substitution pattern. This can be formulated as:

$$\min\{PA_c.A_c = PAM_c.AM_c + PAD_c.ADQ_c\}$$

$$st \qquad Z_c.\left[\varphi_c^{vol}.(AM_c)^{(1-\eta_c)\eta_c} + (1-\varphi_c^{vol}).(AD_c)^{(1-\eta_c)\eta_c}\right]^{\eta_c(1-\eta_c)} \tag{69}$$

$$with \;\; A, a \quad = \quad MAT_c, E_c, IA_c, X_{c,}.CH_c, G_c; \qquad c \quad = \quad 1...24$$

where A_c represents the demand of each composite product by each Armington agent and PA_c its price, AM_c and AD_c are the import and domestic product quantities demanded by agent A, and PAM_c and PAD_c their respective prices. These prices are different between products but common between agents except for households who have to pay the value-added tax. Z_c and φ_c^{vol} are the scale and absorption parameters. η_c is the Armington elasticity of substitution between domestic and foreign goods and services. The import bloc is quite flexible since the elasticity of substitution can potentially be different for each type of use of a given product (such as intermediary consumption, investment, consumption, public spending, export, etc). The solution of the optimization program 69 gives the optimal demand for domestic and imported goods:

$$\begin{aligned} m_c^{a^n} &= a_c - \eta_c.(p_c^m - p_{c,t}) \\ q_c^{a^n} &= a_c - \eta_c.(p_c^q - p_{c,t}) \end{aligned} \tag{70}$$

8.2 Exports

In the same logic, exports are determined by the external demand for domestic products and the ratio between the export and world prices assuming a constant price elasticity. In other words, under the hypothesis of a "small open economy", the external demand and the export price are negatively related for a given world price[9] . The functional form for the export demand (x_c) for each product in Three-ME is a logarithm transformation of the one derived by Wilcoxen (1988):

$$\begin{aligned} \Delta x_{c,t} &= \Delta wd_{c,t} + \Delta SUBST\_X_{c,t} \\ \Delta SUBST\_X_{c,t}^n &= -\eta^x \Delta(p_{c,t}^X - tc.p_{c,t}^W) \end{aligned} \tag{71}$$

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[9] An alternative approach which is using frequently in CGEM, but less realistic, consists in assuming an infinite price elasticity between exports and the production of foreign competitors and that domestic producers do not have any difficulty to sell their products on the foreign market as long as the domestic price does not differ from the international price. In this case, the volume of exports is limited by supply Shoven and Whalley (1992).

where wd_c is the world demand and p^W_c its price expressed in national currency. p^X_c is the exports price that depends on the production cost and reflects the price competitiveness of domestic products. Finally, η^x is (the absolute value of) the price elasticity assumed constant. The unit elasticity between export and the world demand guarantees the long run stability the export market shares.

In Three-ME, part of the exports comes from imported products (re-exports). The repartition between domestic and imported products results from the minimization by foreign clients of the value of their imports from France (i.e. of French export)[10] :

$$\begin{aligned} \Delta xd_{c,t} &= \Delta x_{c,t} + \Delta SUBST\_XD_{c,t} \\ \Delta SUBST\_XD_{c,t} &= -\eta^{xd} \varphi^{XM}_{c,t-1} \Delta(p^{XD}_{c,t} - p^{XM}_{c,t}) \end{aligned} \tag{72}$$

$$\begin{aligned} \Delta xm_{c,t} &= \Delta x_{c,t} + \Delta SUBST\_XM_{c,t} \\ \Delta SUBST\_XM^n_{c,t} &= -\eta^{xd} \varphi^{XD}_{c,t-1} \Delta(p^{XM}_{c,t} - p^{XD}_{c,t}) \end{aligned} \tag{73}$$

where xd_c and xm_c are the optimal level of domestic and import products that are exported. η^{xd} is the elasticity of substitution between domestic and imported products. As the exchange rate is exogenous in the model, the external balance may differ from zero:

$$DC\_VAL_a = \sum_c PX_c.X_c - \sum_c PM_c.M_c \tag{74}$$

with PM_c as the import product price.

9 Prices Structure

The prices in THREE-ME follow a bottom-up structure. The production price is defined at the lowest level as a mark-up over the production cost (labor, capital, energy and other intermediary raw consumptions). The domestic price for commodities includes, in addition to the production price, commercial and transport margins, and taxes on products net from subsidies. Combined with the import price, we get a price for each commodity. Depending on the destination of the product, the price may vary since certain taxes or subsidies do not apply uniformaly to every clients. For instance, VAT affects primary consumers but not exports and subsidies affect only the domestic price. As a feedback, final demand prices affect the production price through several canals. The agregate consumer price defines inflation which is (at least partially) repercuted into wage and thus costs. Inflation also increases the real interest rate and therefore

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[10] The optimization program is $\begin{matrix} \min P^x_c.X^x_c = P^q_c.X^q_c + P^m_c.X^m_c \\ Subject \quad to \quad X_c = CES(X^d_c, X^m_c) \end{matrix}$:

the cost of capital because of the monetary policy of the central bank. Final demand prices affect also production costs via the price of intermediaty consumption and of investment. These interactions and feedbacks between prices, wages, and production costs are schematized in Figure 8.

9.1 Production prices

In order to describe as clearly as possible the construction of prices in ThreeME, we begin with the production prices fixed by firms. With the import prices, the system of production prices is the key element in the price structure since all other prices are derived from them by adding taxes or/and deducting subsidies according to the destination of each product. In the case of imperfect competition, firms choose the price that maximizes their profit as a mark-up $TMD_{a,t}$ over the unit cost of production:

$$PY^n_{a,t} = NCU_{a,t}.(1 + TMD_{a,t}) \tag{75}$$

where $PY^n_{a,t}$ is the optimal (or desired or notional) production price. $NCU_{a,t}$ is the net unit cost of production calculated by adding over the gross level all taxes on production and deducting operating subsidies. The mark-up rate is calibrated by inverting Equation [75] at the base state.

The effective price adjusts slowly to the desired level according to the geometric law of adjustment described in Section 3:

$$ln(PY_{a,t}) = \lambda^X_0.ln(PY^n_{a,t}) + (1 - \lambda^X_0)ln(PY_{a,t-1} + \Delta ln(PY^e_{a,t})) \tag{76}$$

$$\begin{aligned} \Delta ln(PY^e_{a,t}) \quad = \quad & \lambda^X_1.\Delta ln(PY^e_{a,t-1}) + \lambda^X_2.\Delta ln(PY_{a,t-1}) \\ & +\lambda^X_3.\Delta ln(PY^n_{a,t}) + \lambda^X_4.\Delta ln(PY_{a,t+1}) \end{aligned} \tag{77}$$

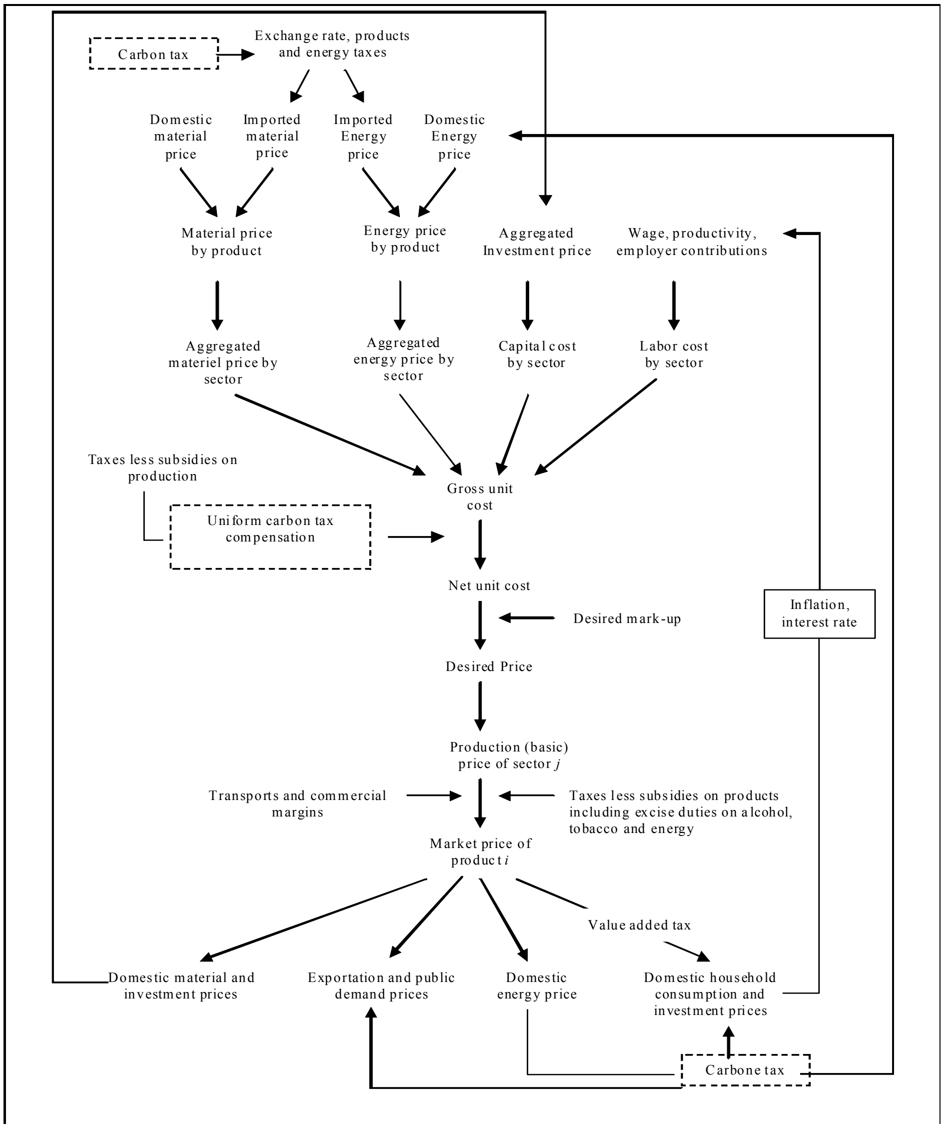

Figure 8: Prices structure

The steps that lead to the calculation of the gross unit cost $GUC_{a,t}$ are described in Figure 8. It follows a bottom-up approach starting from the most disaggregated price levels to reach the most aggregated one by determining the prices of composite factors in intermediate steps. At the bottom of the price structure, the composite prices for each energy and material in each sector depends on the product's geographic origin:

$$\begin{aligned} PMAT_{c,a}.MAT_{c,a} =& PMATD_c.MATD_{c,a} + PMATM_c.MATM_{c,a} \quad (78) \\ & for\, i = \{1, ..., 20\} \\ PE_{c,a}.E_{c,a} =& PED_c.ED_{c,a} + PEM_c.EM_{c,a} \quad (79) \\ & for\, i = \{21, ..., 24\} \end{aligned}$$

At the upper level, we calculate the prices for each composite factor in each sector:

The composite materiel price in sector a:
$PMAT_a.MAT_a = \sum_{a=1}^{20} PMAT_{c,a}.MAT_{c,a}$
The composite energy price in sector a: $PE_a.E_a = \sum_{a=21}^{2406} PE_{c,a}.E_{c,a}$
The user capital cost per unity produced in sector a:

$$CK_{a,t} = PI_{a,t}K_{a,t-1}(\delta_a + \varphi_a^{autof}\dot{K}_{a,t}) + PDEBT_{a,t-1}DEBT_{a,t-1}.r_{a,t-1}$$

where:
-$PI_{a,t}$: the investment price for all sector
-r_a: The long-run nominal interest rate.
The unit labor cost in sector a is:
$CL_a = W_a(1 + T_a^{CSE})/P_a^{rog}$
where:
-$W_{a,t}$: the average gross wage
-$T_{a,t}^{CSE}$: The employer social security contributions per activity
Finally, the unit cost of production before taxes net from subsidies in actvity a is equal to :

$$CU_a.Y_a = CK_aK_a + CL_aL_aPROG_a + PE_aE_a + PMAT_aMAT_a \quad (80)$$

9.2 Commodity price

Since the model distinghishes commodities from activities, the price a commodity is weighted average of the prices of activities that produce this commodities:

$$PYQ_c = \sum_c \varphi_{c,a}PY_a \quad (81)$$

Where PYQ_c is the price of commodity c (at basic price), PY_a the price of activity a. $\varphi_{c,a}$, the share of commodity c produced by activity a, may not be constant. They typically vary for energy sectors to account for the increasing impact of the production from renewable energy.

9.3 Domestic and import price

The selling price of each commodity ($PYQS_c$) domestically produced includes, in addition to the price of commodity at basic price (PYQ_c), tax and subsidies on product, and transportation and commercial margins:

$$\begin{aligned}
PYQS_c.YQS_c = {} & PYQ_c.YQ_c.(1 + T_c^{ENERTD} + T_c^{OTHD} + T_c^{SUB}) && (82)\\
& + PMTD_c.MTD_c + PMCD_c.MCD \\
& {}_c \quad if\, c \neq \{14, ..., 19\} \\
PYQS_c.YQS_c = {} & PYQ_c.YQ_c.(1 + T_c^{ENERTD} + T_c^{OTHD} + T_c^{SUB}) \\
& if\, c = \{14, ..., 19\} && (83)
\end{aligned}$$

$$\Delta yqs_c = \Delta yq_c \tag{84}$$

The same logic applies for the import price (see Appendix C). Notice that YQS_c is the volume of the production expressed at market price before VAT. It should not be seen as a composite of several "goods": production at base price and margins. Indeed, its does not increase when the volume of the commercial and transport margins increase. The price does instead. Its specification is $YQS_{c,t} = YQ_{c,t}\left(1 + T_{c,0}^{ENERT} + T_{c,0}^{OTHD} + T_{c,0}^{SUB} + \frac{MTD_{c,0}}{YQ_{c,0}} + \frac{MCD_{c,0}}{YQ_{c,0}}\right)$ which is equivalent to 84, that is to assuming that YQS_c is always proportional to YQ_c. Writing it following the specification composite of several goods, $YQS_{c,t} = YQ_{c,t}\left(1 + T_{c,0}^{ENERT} + T_{c,0}^{OTHD} + T_{c,0}^{SUB}\right) + MTD_{c,t} + MCD_{c,t}$, would lead to inaccurate results since a decrease in the quantity of transport used per unit of production would not lead to a decrease of the selling price.

9.4 Price for final demand

The price for final demand varies according to the destination depending if VAT applies or not. We provide below the specification for the final price of commodities produced domestically. The specification for imported goods is provided in Appendix C. We assume that no VAT applies on export or for the valorisation of inventories. Therefore, the final price for export and for the change in inventories is:

$$PXD_c = PDSD_c = PYQS_c \tag{85}$$

In theory, VAT applies only on households'final consumption and should therefore be paid only by consumers. In practice, desagregated data by commodity show that sectors pay also a small amount of VAT since (a) VAT is paid on product not consumed by households and (b) the apparent rate on several products would exceed the legal rate if the VAT was exclusively paid by households. Therefore, we distinguish 2 VAT rate: one paid on consumption and another one paid on intermediary consumption, sectoral investment and public spending:

$$PIAD_{c,t} = PMATD_{c,t} = PED_{c,t} = PGD_{c,t} = PYQS_{c,t} \frac{\left(1 + T_{c,t}^{VATD_{oth}}\right)}{\left(1 + T_{c,0}^{VATD_{oth}}\right)} \tag{86}$$

$$PCHD_{c,t.} = PYQS_{c,t} \frac{\left(1 + T_{c,t}^{VATD}\right)}{\left(1 + T_{c,0}^{VATD}\right)} \tag{87}$$

The above differentiation of the final price by destination is a substantial improvement compare to the assumption made in most CGE models that assume that VAT is proportional to production. In these models, an increase in export would lead to an increase in VAT receipts, which is not true in reality.

9.5 Consumer price index

The consumer price index is defined as a weighted average of prices of all total expenditure household components: Each commodity price is itself a weighted average between domestic and import commodities prices.

$$PCH.CH = \sum_{c} PCH_c.CH_c \tag{88}$$

9.6 Interest rate

In Three-ME, money supply is endogenous. The interest rate is determined at the euro area (EA) level according to a reaction function à la Taylor. We assume that the European Central Bank (ECB) sets the short-term interest rate taking into account inflation and the situation on the labor market in the euro area:

$$R^{Dir} = \theta_0 + \theta_1 \Delta.(\dot{P}_t^{ea} - \dot{P}_t^{ea*}) - \theta_2.\Delta(U_t^{ea} - U_t^{ea*}) \tag{89}$$

$$\dot{P}_t^{ea} = \sum_{e=1}^{E} \sigma_e \dot{P}_t^{e} \tag{90}$$

$$U_t^{ea} = \sum_{e=1}^{E} \sigma_e U_t^{e} \tag{91}$$

Where R_t^{Dir} is the nominal short run interest rate, $\dot{P}_t^{ea}$ the inflation rate within the EA, $\dot{P}_t^{ea*}$ the ECB inflation target, U_t^{ea} the unemployment rate in the EA and U_t^{ea*} the unemployment rate target. $\dot{P}_t^{e}$, U_t^{e} and σ_e are respectively the inflation rate, the unemployment rate and the GDP weight of country e in the EA. We assume further that the long-term interest rate adjusts slowly to the short-term interest rate as described in Section 3.

10 The government

According to the French national accounts, public administrations refer to the central and regional government services and social security administration. In Three-ME, we have aggregated these three components in order to focus on transfers between public administrations, household and sectors. These transfers are accounted for in the government's resources (REC_VAL) and expenditures (DEP_VAL) :

$$\begin{aligned} REC\_VAL \; = \; & PY_{20}.Y_{20,t} + PTAX.TAX + PIY.IY + PIS.IS_t + IR\_VAL \\ & + AIC\_VAL + PCSE^{TOT}.CSE^{TOT} + PCSS^{TOT}.CSS^{TOT} + DIV^{GOV\_VAL} \end{aligned} \tag{92}$$

with:

- The marketed part of public administrations production is evaluated at its net production cost:($Y_{20}.PY_{20}$) ;
- The composite $PTAX.TAX$ index which embodies the Value-added tax, the taxes on energies (TIPP, etc) and the others taxes on commodities
- The aggregate tax on activities: $PIY.IY = \sum\limits_{a,a\neq 20} TIYN_a.PY_a.Y_a$
- The aggregate subvention on activities: $PSY.SY = \sum\limits_{a,a\neq 20} TSYN_a.PY_a.Y_a$;
- The total firm profit tax $PIS.IS = \sum\limits_{a} T_t^{IS}.PEBE_{a,t-1}.EBE_{a,t-1}$;
- The taxes on household's financial wealth $AIC\_VAL = \sum\limits_{h}^{5} T^{AIC}.DISPINC\_VAL_h^{AI}$ which includes the sum of the social contribution from the activity sectors and from self-employed workers.
- The income tax $IR\_VAL = \sum\limits_{h=1}^{5} T^{IR}.DISPINC_h^{AI}$
- The total employer social contribution $PCSE^{TOT}.CSE^{TOT} = \sum\limits_{a} T_a^{CSE}.L\_S.W\_S_a + T_a^{CSE^{ROW}}.SB^{ROW}$ which includes the sum of the social contribution from the activity sectors and from the rest of the world.
- The total salary social contribution $PCSS^{TOT}.CSS^{TOT} = \sum\limits_{a} T^{CSS}.L_a.W\_S_a + T^{CSS\_SE}.L\_SE.W\_SE_{19} + PCSS.CSS^{ROW}$ which includes the sum of the social contribution from the activity sectors and from self-employed workers.
- The financial transfers from the others institutions

$$DIV\_VAL_b^{GO} \quad for \;\; b \quad = \quad HH, ROW, BK$$

Public subventions to sectors consist of subventions on production and products. Both types are applied on volume which means that changes in price caused by a shock do not affect the amount of subsidies:

$$\begin{aligned} DEP\_VAL = &(NCU\_20 \ast Y\_20) + PRESOC\_VAL + PG \ast G \\ &+ R\_G_{t-1} \ast DEBT\_G\_VAL_{t-1} - PSUB \ast SUB \end{aligned} \tag{93}$$

with:

- The net cost per unit of production in public activity:

$$\begin{aligned} NCU_{20}.Y_{20} = &CU_{20}.Y_{20} + PIY_{20}IY_{20} + PIS_{20}IS_{20} - PSY_{20}SY_{20} \\ &+ DIV\_HH\_VAL_{20} + DIV\_GOV\_VAL_{20} \\ &+ DIV\_ROW\_VAL_{20} + DIV\_BK\_VAL_{20} \end{aligned} \tag{94}$$

- The total public expenditures: $PG \ast G$
- The social benefits: $PRESOC\_VAL$

Domestic and imported government consumption are specified as follow:

$$\begin{aligned} \Delta gd_{c,t} &= \Delta expg_{c,t} + \Delta SUBST\_GD_{c,t} \\ \Delta SUBST\_GD^{n}_{c,t} &= \eta_{chd,chm}\varphi_{chm,c}\Delta(p^{GD} - p^{GM}) \end{aligned} \tag{95}$$

$$\begin{aligned} \Delta gm_{c,t} &= \Delta expg_{c,t} + \Delta SUBST\_GM_{c,t} \\ \Delta SUBST\_GM^{n}_{c,t} &= \eta_{chd,chm}\varphi_{chd,c}\Delta(p^{GM}_{c} - p^{GD}_{c}) \end{aligned} \tag{96}$$

Public subsidies consist of subsidies on production and products. We assume that subsidies on production are ad valorem and are therefore automatically indexed on the production price:

$$PSY_a.SY_a = TSYN_{a,t}.PY_a.Y_a \tag{97}$$

$$SY_a = TSYN_{a,0}.Y_a \tag{98}$$

On the contrary, we assume that subsidies on products are applied on volume which means that changes in price caused by a shock do not affect the amount of subsidies:

$$PSUB_{c,t}.SUB_{c,t} = T^{SUB}_{c,t}.YQ_{c,t} \tag{99}$$

$$SUB_{c,t} = T^{SUB}_{c,0}.YQ_{c,t} \tag{100}$$

The public deficit and debt accumulation equations are written as follows:

$$BF\_G\_VAL = DEP\_VAL - REC\_VAL \tag{101}$$

$$DEBT\_G\_VAL = DEBT\_G\_VAL_{t-1} + BF\_G\_VAL \tag{102}$$

11 Greenhouse gases emissions

In France, the anthropogenic CO2 emissions represent about 70% of the total gross greenhouse gases (GHG). They come from the burning of fossil fuels and decarbonation process. The modeling of the demand for fossil energy in Three-ME is detailed by economic agent, by kind of fossil energy and by emission process. This allows for a precise estimation of the variation in the national CO2 emissions. The calculation of emissions level consists in multiplying the fossil energy demand by the corresponding emission coefficients. These coefficients are specific for each economic actor, each sector and each energy sources depending on their carbon intensity.

The CO2 emissions due to the combustion of fossil energy by sectors and households are proportional to the quantity of fossil fuel energy consumed. They are therefore calculated according to the following equations:

$$\Delta ems_{e,a} = \Delta e_a \tag{103}$$

$$\Delta ems_{e,h} = \Delta (ch_{e,h}) \tag{104}$$

CO2 emissions from decarbonation during the production process for the non mineral metallic products, asthe glass or ceramic sectors; is assumed proportional to the quantity of intermediate raw material used in the production process:

$$\Delta ems\_dc_a = \Delta mat_a \tag{105}$$

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Appendix A Long term of the model

The long term steady state of the model is generally defined as a state where all variables grow at a constant rate. This state is coherent with the representation of a stable economy able to maintain a given configuration forever. This implies that rates such as the unemployment or labor participation ratios, tax rates are constant in the long run. This is coherent with the fact that these ratios lie by definition between 0 and 100% and thus cannot be affected by a trend forever.

Most shares should also be constant. For instance, the shares of investment or of consumption into GDP should be constant. Otherwise the effect of one of these two determinants of the GDP vanishes over time. The same argument holds for the share of one sector in the total in terms of labor or production: we expect an economy where all sectors remain in the long run, which implies that some economic mechanisms ensures stable share for each sectors.

Some exceptions are possible. As empirically observed, it seems realistic that the share of labor into the GDP decreases over time because of the technical progress. But the share of the efficient labor, that is including the technical progress, remains constant. Because of the globalization of the economy, the ratio between export and production may also increase permanently in the long run. But in the long run this effect is expected to be compensated by the increase in the ratio between import and production so that the share of the external balance into production still remains constant.

In the long run, all relative prices are expected to be constant. This implies that all prices grow at the same rate. This ensures that the economy is not affected by substitution mechanisms in the long run: firms do not want to change the share of each production factors into production and consumers are satisfied with share of each good into their aggregate consumption. It implies also that each agent is satisfied with their share of the global revenue: firms do not want to change the growth rate of their price whereas employees do not want to change the growth rate of their wage.

Assuming that ν, τ, μ, π andω are the growth rates of the population, of the technical progress, of the real economy (i.e. of the GDP), of prices (i.e. inflation), and of wages, the long run value of these rates cannot be chosen independently. First, the growth rate of the real economy should be equal to the sum of the growth rate of the population and of the technical progress:$\mu = \nu + \tau$. This condition is a direct consequence of the hypothesis of constant return to scales (homogeneous of degree 1) of the production function. In the long run, relative price are constant and the labor demand [12] implies that production grows at the sum of the growth rates of labor and technical progress$\Delta y_{jt} = \Delta l_{jt} + \Delta p^{rog}_{jt}$. In addition, the stability of unemployment rate implies that labor grows at the same rate as the population (Equation 66). In the long run, the price equation implies that the growth rate of wages should be equal to the sum of inflation and of the growth rate of the technical progress. This holds only if some economic mechanisms imply that the unemployment rate converge to the NAIRU. The latter depends on the parameter of the Phillips curve 64:

$$U_\infty = \left(\rho_1 - (1-\rho_2)\pi - (1-\rho_3)\tau/\rho_5\right) \tag{106}$$

In the model several stabilizing equation guaranty that the economy return to stationary path after a shock. Inflationary shocks degrade the external position of France by decreasing export and increasing imports. In addition, the Taylor rule combined with the negative impact of the real interest rate on the demand prevents inflationary shock to lead to an explosive inflation dynamic. The negative impact of the real interest rate on the activity has several possible canals:

- Consumption: in coherence with a life-cycle model and the possibility of an intertemporal allocation of their resource, households may increase their savings when the real interest rate increases and thus reduces their consumptions. They may also have Ricardian behavior in the long run by internalizing the government and firms' budget constraints. They may thus adjust their consumption in such way that the ratio between their savings and the national debt is constant in order to insure the sustainability of the debt.
- Investment: firms may choose their investment level that is coherent with the stability of their debt into the value-added.
- Tax and public spending: the government is expected to choose the tax rate and public spending levels that are coherent with a stable debt into the GDP.

The consistency of a dynamic model with a stationary equilibrium requires long term constraints which depend on the type of mathematical equation. We briefly detailed the main cases that are encountered in Three-ME and how the model can be calibrated in order to be at the stationary state from the first period of the simulation onward. It should be noted that this assumption is made in order to verify the coherence of the data calibration and of the dynamic properties of the model. Some of the constraints needed to guarantee a permanent stationary state are quite restrictive. For instance, it implies a strict relation between stock and flow values (e.g. capital and investment, see below) which may not be satisfied at the base year. Also, the growth rates of population, and of technical progress should always be constant, which is not verify empirically. For this reason, we simulate also realistic baseline scenarios where the model is fully calibrated on empirical data and on realistic projections for the exogenous variables.

A-1 Additive equations

In the model, many relations enter in an additive form:

$$Y_t = \sum_{i=1}^{I} X_{it} \tag{107}$$

These are in general definitions such as the GDP decomposition or income, etc. In case of an additive equation 107, the variable Y grows at the rate μ from the first period onward if all its components X grow also at that rate:

$$Y_t = \left(\sum_{i=1}^{I} X_{i,0} \right) (1+\mu)^t = Y_0 (1+\mu)^t \tag{108}$$

Moreover in that case all ratios between variables ($\frac{X_i}{Y}$ and $\frac{Xi}{Xj}$) are constant over time. In the case of the GDP equation this seems a realistic long run property. Otherwise the share of each component in the GDP is not stable over time and the long run growth rate of the GDP corresponds to the component's highest growth rate. Indeed, if the X-variables do not grow at the same rate, the growth rate of $Y(\mu)$ converges to the highest X-variable growth rate. And the share of the X-variable with a lower growth rate tends toward zero. This mathematical property may imply unrealistic constraint on the model if one wants to be at the steady states at the first period of the simulation. This is particularly true if one wishes to calibrate the model on real data. We can give 2 examples: For instance, it is unrealistic to assume that a negative inventory change will decrease indefinitely because the level of inventories becomes at some point negative. One possibility is to amend the calibration in order to impose a zero-inventory change at the base years.

In the real world, most countries' imports and exports do not grow at the rate of the GDP but at a higher rate because of the trade globalization. In fact Equation 107 allows that several X-variables grow at a different rate than Y in the long run as long their sum grows at the same rate as Y. Consequently, imports and exports may grow faster that the GDP forever as long as their effect cancel out, that is as long as the foreign trade balance grows at the rate of the GDP. If the long run foreign trade balance is zero, imports and exports grow at the same rate. If not, they grow at the same rate asymptotically, the smallest (in absolute value) growing faster. This implies mechanism that imposes import and export to grow consistently.

The most common way is to assume that the exchange rate adjusts in order to reach the external balance objective.

A-2 Unit elasticity logarithm equations

Many relations in the model impose a unit-elasticity specified in logarithm form:

$$\ln(Y_t) = \ln(X_t) + \alpha \tag{109}$$

This specification is used for all production factor demand since we systematically assume a constant return-to-scale technology. If the coefficient α is calibrated in the initial period as a simple inversion of equation 109 and constant over time, this specification implies that Y always grows at the same rate as X.

In the production factors demand, α depends on the relative prices and thus may vary over time in case of shock or if they are not in equilibrium in the initial period. In that case, the growth rate of Y and X differs over time but they tend to converge toward each other provided that mechanisms in the price equation guaranty the long run stability of relative prices.

A-3 Accumulation equations

The model contains several accumulation equations: capital stock dynamic, public and private debt, household savings. All can be represented with the following equation:

$$Y_t = Y_{t-1}(1+\beta) + X_t \tag{110}$$

In the case of capital accumulation, β is the depreciation rate and is negative. In the case of debt or saving equation,β is the interest rate and is thus positive. Dividing both sides by Y_{t-1} give the growth rate of the stock variable:

$$\dot{Y}_t = \beta + X_t/Y_{t-1} \tag{111}$$

At the steady states, X should grow at the same rate as Y which is defined by Equation 111. Consequently, being at the stationary states from the first period onward implies that X cannot be calibrated on real data. At the stationary state, X is calibrated as an inversion of Equation110:

$$Y = \left(\frac{\mu - \beta}{1+\mu}\right)^{-1} X \tag{112}$$

Appendix B Generalized CES function and factors demand

This appendix derives the optimality program of the producer and the consumer assuming a generalized CES (GCES) production and utility function. We show that the GCES function can be approximated in the neighborhood of the optimal stationary state by a Cobb-Douglas function for which the technical coefficients vary with the relative prices. This result greatly facilitates the deduction of linear demands functions for input and goods.

B-1 GCES production function and factors demand

Let us define a GCES production function as a H inputs-production function with different elasticities of substitution between each pair of input. We still assume a constant elasticity of substitution between 2 inputs along the isoquant. Let us assume that technology may be represented by a continuous and twice differentiable function, linearly homogeneous, strictly increasing ($Q'(x_{ht}) > 0$) and concave ($Q''(x_{ht}) < 0$) reflecting the law of diminishing marginal returns:

$$Q_t = Q(X_{ht}) \tag{113}$$

Where X_{ht} is the quantity of input (or production factor) $h = [1; H]$ used to produce the quantity of production (or output) Q_t . For algebraic simplicity, we assume a technology with constant returns to scale (i.e. the production function 113 is homogeneous of degree 1) and the absence of technical progress. We shall relax these constraints latter. Driven by maximizing profit behaviour, the producer chooses her demand for each input by minimizing her production cost 114 subject to the technical constraint113:

$$C_t = \sum_{h=1}^{H} P^X_{ht} X_{ht} \tag{114}$$

Where P^X_{ht} is the price of input h. The Lagrangian to this problem is:

$$L_t = C_t - \lambda(Q_t - Q_t(X_{ht}) \tag{115}$$

The necessary first order conditions are $L'(X_{ht}) = 0$ for all h and $L'(\lambda) = 0$. The second order conditions ensure that the optimum is a minimum is always verified because of the convexity of the cost function 114 and strict convexity of the isoquants formed by the production function 114[11] . The well-known first order condition says that at the optimum, the ratio between marginal productivities of two inputs equals the one between their prices:

$$\frac{Q'(X_{ht})}{Q'(X_{h't})} = \frac{P^X_{ht}}{P^X_{h't}} \tag{116}$$

The production function 113 can be linearized with the following first-order Taylor expansion:

$$\dot{Q}_t = \sum_{h=1}^{H} \frac{Q'(X_{ht}).X_{ht}}{Q(X_{ht})} \dot{X}_{ht} \tag{117}$$

Euler's Theorem states that a function which is homogeneous of degree 1 can be express as the sum of its arguments weighted by their first partial derivatives:

$$Q(X_{ht}) = \sum_{h=1}^{H} Q'(X_{ht}).X_{ht} \tag{118}$$

The fact that in equilibrium, the remuneration of the production factors must be equal to the value of the production provides another useful relation:

$$\sum_{h=1}^{H} P^X_{ht} X_{ht} = P^Q_t Q_t \tag{119}$$

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[11] According to the technological constraint 113, the strict convexity of the isoquant ($X''_{ht}(X_{ht}) > 0$) implies that $Q''(X_{ht}) < 2Q'(X_{ht})$. This condition is always verified since by assumption the left-hand side is negative ($Q''(x_{ht}) < 0$) while the right-hand side is positive ($Q'(x_{ht}) > 0$).

The combination of equations 116 to 118 gives at the neighbourhood of the stationary state a linear specification of the production function:

$$\dot{Q}_t = \sum_{h=1}^{H} \varphi_{ht} \dot{X}_{ht} \Leftrightarrow q_t = \sum_{h=1}^{H} \varphi_{ht} x_{ht} \tag{120}$$

Where φ_{ht} is the share (in value) of input h in the production sometimes called Leontief technical coefficient

$$\varphi_{ht} = \frac{P^X_{ht} X_{ht}}{P^Q_{ht} Q_{ht}} \tag{121}$$

We have just shown that at the neighbourhood of the optimum any linearly homogeneous, twice differentiable, strictly increasing and concave production function can be approximated by a Cobb-Douglas with technical coefficients that varies over time. Moreover these technical coefficients correspond to the input share into production. They are stable in the long run because the specification of Three-ME guaranties the stability of ratios between prices and of input to production ratios. Suppose further that the direct elasticity of substitution – in the sense of Hicks (1932) and Robinson (1933) – between inputs h andh' $\eta_{hh'}$ is not necessarily the same between each couple of production factors. This elasticity measures the change in the ratio between two factors of production due to a change in their relative marginal productivity, i.e. in the marginal rate of substitution (in the slope of the iso-production curve):

$$-\eta_{hh'} = \frac{\partial \ln(X_{ht}/X_{h't})}{\partial \ln(Q'(X_{ht})/Q'(X_{h't})} \Leftrightarrow \partial \ln(X_{ht}/X_{h't}) = \eta_{hh'} \partial \ln(Q'(X_{ht})/Q'(X_{h't})) \tag{122}$$

Integrating 122 with respect to time and then combining it with the optimality condition 116 gives:

h

$$\frac{X_{ht}}{X_{h't}} = \xi_{hh'} \left(\frac{P^X_{ht}}{P^X_{h't}} \right)^{-\eta_{hh'}} \tag{123}$$

Where $\xi_{hh'}$ is the constant of integration which we calibrate to one for algebraic simplicity. Rewriting 123 in terms of input share, $\varphi_{ht}/\varphi_{h't} = \xi_{hh'}(P^X_{ht}/P^X_{h't})^{1-\eta_{hh'}}$, gives the well-known result that the inputs share is constant over time only in case of unit elasticity of substitution between all factors of production (Cobb-Douglas technology).

The first order conditions 123 and the production function 120 constitute a system of H linearly independent equations and H unknowns. Its resolution give the demand for each factor as a positive function of output and negative function of relative prices between production factors:

$$x_{ht} = \sum_{h=1,h\neq h'}^{H} \eta_{hh'}\varphi_{h't}(p^X_{ht} - p^X_{h't}) \tag{124}$$

The introduction of technical progress and non constant return-to-scale is straightforward and does not alter the results. In the first case one can simply define $X_{ht} = I^{nput}_{ht} P^{rog}_{ht}$ as the efficient input, which includes the technical progress P^{rog}_{ht} ,I^{nput}_{ht} being the effective input. In the second case, one can simply define production as an homogenous function of Q of degree $\theta : Y_t = Q^\theta_t$ In case of a technology with increasing (resp. decreasing) return-to-scale, $\theta > 1$ (resp. < 1). Integrating technical progress and non constant return-to-scale leads to the following input demand:

$$I^{nput}_{ht} = \theta^{-1} y_t - p^{rog}_{ht} - \sum_{h=1,h\neq h'}^{H} \eta_{hh'}\varphi_{h't}(p^X_{ht} - p^X_{h't}) \tag{125}$$

Assuming constant return to scale, this log-linear specification has been recently estimated for the Euro area by Lemoine et al. (2010) using the Kalman filter to extract the trend of technical progress.

B-2 GCES consumer utility function and demand for goods

In Three-ME, the demand for goods is treated in a similar way as the demand for input. Let us assume that at a first stage the consumer divides (eventually via an intertemporal maximization program) her revenue between expenditures and savings. For a given level desired volume of expenditure Q , the consumer is then assumed to minimize the cost of this expenditure. The substitutability between the different consumption goods (or expenditures), X_h , is measured through a J goods-utility function having the same property as the production function defined in 113. Formally the optimization program is the same as the one of the producer. It consists in minimizing the cost of expenditure 114 subject to the utility function constraint 113. The demand for goods is thus 124.

Notice that minimizing the cost of expenditure subject to a utility function constraint give the same result as the standard approach which consists in maximizing the utility 113 subject to a budget constraint 126:

$$\sum_{h=1}^{H} P^X_{ht} X_{ht} = P^Q_t Q_t \tag{126}$$

The Lagrangian to this problem is:

$$L_t = Q_t - \lambda\left(\sum_{h=1}^{H} P_{ht}^X X_{ht} - P_t^Q Q_t\right) \tag{127}$$

The necessary first order conditions ($L'(X_{ht}) = 0$ for all h and $L'(\lambda) = 0$) are the well-know conditions that the ratio between marginal utilities of two goods equals the one between their prices (Equation 116 and thus123) and the budget constraint (Equation 126). Using a first-order Taylor expansion on Equation 126 (divided byP^Q) in the neighbourhood of the stationary equilibrium characterized by the stability of price ratios ($P_h^X/P_{h'}^X$,P_h^X/P^Q), allows for rewriting the budget constraint as 120. As we have now the same system to solve as in the producer case (Equations123 and 120), the demand for good is thus 124. Notice that the particular case of a CES function $(\eta_{hh'}/P_{h'}^X)$, 124 simplifies. To see this, let us first use a first-order Taylor expansion on Equation 126 (divided by Q_t) in the neighbourhood of the stationary equilibrium characterized by the stability of ratios between volumes $(X_h/X_{h'}X_h/Q)$,. This conveniently allows expressing the consumer price as a weighted average of the prices of goods, the weight being the share into consumption (Equation 121):

$$\dot{P}_t^Q = \sum_{h=1}^{H} \varphi_{ht} \dot{P}_{ht}^X \Leftrightarrow p_t^Q = \sum_{h=1}^{H} \varphi_{ht} p_{ht}^X \tag{128}$$

Assuming a constant elasticity of substitution $(\eta_{hh'} = \eta)$ between goods and combining the price equation 128 to 124, the demand for goods simplifies and depends only on the relative price between the price of goods and the consumer price:

$$x_t = q_t - \eta(p_{ht}^X - p_t^Q) \tag{129}$$

Not surprisingly this relation is the same as the one deduced from a direct maximization of CES utility function subject to a budget constraint (see Blanchard and Fischer, 1989; Blanchard and Kiyotaki, 1987; Dixit and Stiglitz, 1977). The only difference is that the consumer price index(P_t^Q) is a linear approximation of the Dixit-Stiglitz index which is a CES function of the price of goods. As demonstrated by Arrow et al. (1961), Leontief and Cobb-Douglas functions are particular cases of a CES function where η tends to 0 and 1 respectively.

Appendix C Equations of the model

Appendix D Glossary of terms used