OBSERVABLE AND UNOBSERVABLE VARIABLES IN THE THEORY OF THE EQUILIBRIUM RATE OF UNEMPLOYMENT, A COMPARISON BETWEEN FRANCE AND THE UNITED STATES

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Abstract

This paper confronts, theoretically and empirically, two estimation methods for the Equilibrium Rate of Unemployment (ERU). By introducing observable variables into the TV-NAIRU approach and unobservable variables into the structural approach, we show how these two methods can converge even though their diagnoses differ appreciably in the French case. We considerably improve the econometric and explanatory properties of the French TV-NAIRU model by identifying some of its determinants (interest rates, labour productivity). Moreover, by distinguishing between the concepts of long-term and medium-term ERU, we separate the medium- from the long-term and the observable from the unobservable components of the ERU.

Keywords: equilibrium unemployment, TV-NAIRU, Phillips curve, structural model, Kalman filter

JEL-Code: C13, C32, E24, E31

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Despite abundant research since its discovery by Phelps (1967, 1968) and Friedman (1968), the concept of the Equilibrium Rate of Unemployment (ERU) is still clouded by many uncertainties. Is the ERU stable or variable, unique or multiple, observable or unobservable, a variable having a precise definition or an abstract construct varying according to each theoretical model? Is it related to inflation stability in the short, medium or long run? Does it determine the inflationary process, or on the contrary, does the latter determine it? Lastly, can it diverge from the effective unemployment rate? Can it depend on the latter?

Taking up the concept of *Non Accelerating Inflation Rate of Unemployment* (NAIRU) formalised by Phelps (1967, 1968), the ERU is the unemployment rate that stabilises inflation. It is a sustainable target for the unemployment rate. This paper confronts, theoretically and empirically, the two main estimation methods used in the literature\(^1\), which are based on two opposing conceptions of the ERU. According to the structural wage-price setting approach, the ERU is a pure theoretical construct. It is not a direct determinant of inflation since it is determined by the inflationary process. Its calculation requires the estimation of a structural model and several concepts of ERU can be defined, depending on the time horizon. On the contrary, according to the Time Varying (TV) NAIRU reduced approach inspired by the Gordon (1997) triangle model, the ERU is a variable that directly determines inflation. Regarded as unobservable, it follows a stochastic process and is estimated from a reduced Phillips curve\(^2\) using the Kalman filter.

Both methods are applied for France and the United States using quarterly national accounts data over the 1970-2003 period. The empirical results highlight some weaknesses of the standard TV-NAIRU approach. At the econometric level, the TV-NAIRU estimations are not very robust because they are particularly sensitive to certain *ad hoc* statistical assumptions. From a theoretical point of view, the importance of temporary shocks is underestimated and the representation of ERU

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\(^1\) Hodrick-Prescott (HP) filter and other smoothing techniques of the unemployment rate are not tackled here. As already pointed out in several studies, their theoretical and empirical interest is fairly limited (Le Bihan et al., 1997; Passet et al., 1997; or Richardson et al., 2000).

\(^2\) As in Phillips (1958), the Phillips curve corresponds to a negative relation between the growth rate of wages and the unemployment rate. Specifications relating directly inflation to the unemployment rate are described as reduced Phillips curve.
as an unobservable variable is disputable since the ERU depends also on observable variables. Lastly, as the determinants of the TV-NAIRU are unidentified, this method is of limited interest for economic policy.

The structural approach, which is compatible with the reduced approach, overcomes some of these weaknesses. It separates the impacts of permanent and temporary shocks by distinguishing the concepts of the long-term and the medium-term ERU. It reduces the unobservable component of the ERU by identifying some of its determinants.

The first section of this article is devoted to the theory of the ERU. It presents the structural model and shows that it can be reformulated as a reduced Phillips curve. We point out the interpretation problems of the short-term ERU that can be overcome by defining the more general concept of medium-term ERU. The distinction between the medium- and long-term ERUs based on a clear theoretical definition allows us to separate out the medium-term and long-term and the observable and unobservable components of the ERU. Then, after having clarified the meaning of observable and unobservable in the theory of the ERU, it appears preferable to regard the ERU as a theoretical construct than as an unobservable variable. The second section presents the estimation of ERUs using both methods. By introducing unobservable variables into the structural approach and observable variables into the TV-NAIRU, we show how these two methods can theoretically converge even though empirically their diagnoses are quite different in the French case.

1. Theory of the equilibrium rate of unemployment

Several specifications of the structural model are possible. In particular, a controversy opposes proponents of the Phillips curve to those of the WS curve (Layard et al., 1991; Blanchard and Katz, 1999; Sterdyniak et al., 1997; Chagny et al., 2002). We chose a Phillips curve, for three reasons. From a theoretical point of view, it reflects the asymmetry between the wage and price setting, where only the employers have a target for the share of value added going to labour (Debonneuil and Sterdyniak, 1984; Chagny et al., 2002). Mathematically, it is a more general model than the WS curve (Reynès, 2003). Lastly, compared to the WS curve, the Phillips curve is compatible with its reduced form without any particular assumption
Observable and unobservable variables in the theory of the equilibrium rate of unemployment

Concerning the anticipations or the adjustment dynamics (Heyer et al., 2004). Compared to the famous WS curve à la Layard et al. (1991), a general specification of the Phillips curve does not postulate a priori a unit indexation of wages on prices and productivity:

\[
\dot{w}_t = Z + a\dot{p}_{C_t} - bU_t - b_2\dot{U}_t + d\dot{\pi}_t + \epsilon_{wt}
\]  

(1)

where \(W\) is the wage, \(P_C\) the consumer price, \(U\) the unemployment rate, \(\Pi\) the labour productivity and \(Z\) a coefficient representative of wage-push factors\(^3\).

The variation of the unemployment rate may intervene in the Phillips curve because of hysteresis phenomena\(^4\), or because wages can be influenced not only by the level but also by the evolution of employment (Phillips, 1958; Lipsey, 1960),

The consumer price is a function of the import price \(P_M\) and the value-added price \(P_V\):

\[
\dot{p}_{C_t} = n\dot{p}_{M_t} + (1-n)\dot{p}_{W_t} + \epsilon_{p_{C_t}}
\]  

(2)

The value-added price setting results from profit maximisation in an imperfect competitive market. The firm’s desired price level \(P_V^d\) corresponds to a desired mark-up \(M^d\) over unit labour costs \((C_u)\) (Debonneuil and Sterdyniak, 1984)\(^5\):

\[
p_{V_t}^d = c_{V_t} + m_t^d, \text{ with } c_{V_t} = w_t + T_C - \pi_t
\]  

(3)

where \(T_C\) is the employer’s social contribution rate.

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\(^3\) The lower-case variables are in logarithm. \(T\) as an exponent refers to trend values. \(t\) and \(L\) are respectively the time and the lag operator. Variables in first and second difference are respectively referred to as \(\dot{x}_t(= x_t - x_{t-1})\) and \(\ddot{x}_t(= \dot{x}_t - \dot{x}_{t-1})\). All coefficients are positive and long-run. Lag variables are considered as follow: \(\ddot{x}_t = \sum_{i=0}^{n} \varphi_i L x_t\) where \(\sum_{i=0}^{n} \varphi_i = 1\). The weighting coefficients \(\varphi_i\) are estimated. The average over \(h\) periods of the variable \(x\) is denoted by \(\overline{x}_t = \overline{x}_{t+h} = \frac{1}{h} \sum_{i=1}^{h} x_t\). Lastly, \(\epsilon_{\infty} \sim N(0, \sigma^2)\) is the residual of equation \(i\) assumed to have a normal distribution.

\(^4\) Hysteresis occurs when the long-term unemployed workers exert no influence on wage setting (Blanchard and Summers, 1986; Lindbeck, 1993). However, some authors contest the term of hysteresis to describe this phenomenon (Cross, 1995).

\(^5\) In another possible specification, firms do not consider only their short-term costs but set their price on the basis of their medium-term costs, thus including the capital cost \((C_K)\): \(p_{V_t}^d = \alpha c_{V_t} + (1-\alpha)\epsilon_{t\alpha} + m_t^d\).
The adjustment process of prices follows an error correction model: the variation of the value added price is a negative function of the gap between the desired price and the effective price, i.e. between the actual mark-up ($M$) and its desired level. Moreover, firms may incorporate inflation in their price setting (if $\nu \neq 0$):

$$p_{t-1} = \lambda u_{t-1} + \lambda' m_{t-1} + \mu (m_{t-1} - m_{t-1}^d) + \nu p_{t-1} + \epsilon_{p,t}, \text{ with } m_{t} = p_{t} - c_{t}$$

The desired mark-up depends on the tightness of the market, i.e. on the production capacity utilisation ratio ($T_{CU}$):

$$m_{t}^d = \psi_{t} + \psi_{t} T_{CU}$$

The structural model composed of (1), (2), (4) and (5) can be reformulated as a reduced Phillips curve:

$$\pi_t = (Z_{LT}^T + Z_{MT}^T + \text{Adjust}_t + \epsilon_{p,t} - bU_t - b_z \dot{U}_t) / (a + (1 - \lambda - \nu) / \mu)$$

Inflation has long-term ($Z_{LT}$) and medium-term ($Z_{MT}$) determinants:

$$Z_{LT} = Z - (1 - a) \dot{p}_C - (1 - d) \pi_t$$

$$Z_{MT} = n(p_m - \dot{p}_{t-1}) + \dot{T}_C + \dot{m}_t^d + \lambda' / \mu m_{t-1}$$

In the long run, prices and costs grow at the same rate, permanent shocks have reached their trend level and indexation mechanisms are implemented. In the medium run, adjustment processes have an impact on inflation:

$$\text{Adjust}_t = (\nu (p_{t-1} - \dot{p}_C) + \lambda (c_{t-1} - \dot{p}_C) - (p_{t-1} - \dot{p}_C)) / \mu$$

Lastly, for exact prediction of inflation, one has to take into account the econometric residuals:

$$\epsilon_{p,t} = \epsilon_{st} + \epsilon_{p,t} + \hat{\epsilon}_{p,t+1} / \mu$$

### 1.1. The medium- and long-term ERUs

Calculated from equation (6), the long-term ERU (ERU$_{LT}$, $U_{LT}$) is the unemployment rate that stabilises inflation in the long run:

$$U_{LT} = (Z - (1 - a) \dot{p}_C - (1 - d) \pi_t) / b$$

In order to measure the impact of transitory shocks, some studies calculate the short-term ERU “which stabilises inflation over two consecutive periods” (Richardson et al., 2000). Being generally extremely erratic, this concept has little relevance for economic policy since the authorities cannot immediately achieve such a variation in
the effective unemployment rate in order to stabilise an inflationary shock. The more
general concept of medium-term ERU (ERU\textsubscript{MT}) can overcome this problem. Let us
reformulate (6) in averages:

\[ \bar{\rho}_C = (\bar{Z}_{LT} + \bar{Z}_{MT} + \bar{\text{Adjust}}_t \varepsilon_{pt} - b \bar{U}_t - b_2 \bar{U}_t + \bar{\rho}_{\text{Ct}} - \bar{\rho}_{\text{Ct}}) / (a + (1 - \lambda - \nu) / \mu) \] (12)

The ERU\textsubscript{MT} corresponds to the unemployment rate trajectories that stabilise inflation
between two periods: \( t \) and \( t + h \), where \( h \) represents the medium-term horizon.
Hence, it guarantees that \( \bar{\rho}_C = (\bar{\rho}_{\text{Ct} + h} - \bar{\rho}_{\text{Ct}}) / h = 0 \). Combining (11) reformulated in
averages with (12) gives the ERU\textsubscript{MT} (\( U_{\text{MT}} \)):

\[ \bar{U}_{\text{MT}} = \bar{U}_{LT} + (\bar{Z}_{MT} + \bar{\text{Adjust}}_t + \varepsilon_{pt} + b_2 (\bar{U}_{t-1} - \bar{U}_{LT})) / (b + b_2) \] (13)

If \( h = 1 \), the ERU\textsubscript{MT} is the short-term ERU. Introducing (13) into (12) leads to:

\[ \bar{\rho}_C = -(b + b_2) (\bar{U}_t - \bar{U}_{\text{MT}}) / (a + (1 - \lambda - \nu) / \mu) \] (14)

Inverting this equation gives the sacrifice ratio (SR) which measures the cost in
terms of unemployment of a disinflationary policy (Gordon and King, 1982; or Ball,
1994):

\[ RS = \bar{U}_t - \bar{U}_{\text{MT}} = -\bar{\rho}_C (a + (1 - \lambda - \nu) / \mu) / (b + b_2) \] (15)

\subsection{1.2. Unobservable variable or theoretical construct?}

Equation (6) can be written as the Gordon (1997) triangle model which says that
inflation depends on the past inflation, the unemployment gap and temporary shocks:

\[ \dot{p}_C = \dot{p}_{C-1} - B(U_t - U_{LT}) + Z'_{\text{MT}} \] (16)

With \( B = b / (a + (1 - \lambda - \nu) / \mu) \) and

\[ Z'_{\text{MT}} = (Z_{\text{MT}} + \text{Adjust}_t - b_2 \bar{U}_t + \varepsilon_{pt} + a + (1 - \lambda - \nu) / \mu) \]

This equation has two interpretations. According to the TV-NAIRU approach, the
ERU is an exogenous unobservable variable of (16). Estimated simultaneously with
(16) using the Kalman filter, the ERU is specified as a stochastic process. Following
Gordon (1997), it is often a random walk\textsuperscript{6}:

\[ U_{LT} = U_{LT-1} + \varepsilon_{U_{t-1}} \] (17)

\textsuperscript{6} The literature adopts other estimation methods and other specifications of (16) and (17) (see Heyer et
al., 2004).
Conversely, according to the structural approach, the ERU is a pure theoretical construct. The ERU is not a direct determinant of (16) since it is a mathematical reformulation of (16) for certain trajectories of the endogenous and exogenous variables: inflation stability and permanent shocks to their long-term path. Thus, the ERU cannot be determined exogenously to (16) and its calculation requires the estimation of the structural model.

The TV-NAIRU interpretation is unsatisfactory for several reasons. Firstly, the concept of ERUMT is not defined. Secondly, assumed unobservable by most studies, the TV-NAIRU does not depend on exogenous variables. While a wide range of stochastic processes are possible, the TV-NAIRU specification is generally imposed without theoretical justification or econometric validation. Finally, a purely stochastic process cannot explain the fluctuations of the TV-NAIRU. That is why recent studies try to find exogenous determinants of the TV-NAIRU (McMorrow and Roeger, 2000; Heyer and Timbeau, 2002; Logeay and Tober, 2003). But in that case it is problematic to consider the TV-NAIRU as a perfectly unobservable variable.

These drawbacks mainly stem from a questionable definition of the unobservable disconnected from its econometric meaning. Every econometric model is made up of observable and unobservable components. The observable component corresponds to the variables of the model and is used to estimate the unobservable one, the relation between these variables, i.e. the coefficients. If the model is linear, the estimated equation is:

\[ Y_t = QX_t + \varepsilon_y \]  \hspace{1cm} (18)

where \( Y \) is the vector of the observable variable to be explained, \( X \) the matrix made up of the vectors of the observable explanatory variables, \( Q \) the vector of the unobservable coefficients to be estimated and \( \varepsilon_y \) the vector of the unobservable measurement errors or noise.

The Ordinary Least Squares (OLS) method estimates the coefficients \( (Q) \) and \( \varepsilon_y \), the two unobservable components of the model. These coefficients, which reflect economic behaviour, are assumed to be stable over time. The Kalman filter method allows \( Q \) to vary over time:

\[ Q_t = AQ_{t-1} + GZ_t + \varepsilon_Q \]  \hspace{1cm} (19)

where \( Z \) is the matrix made up of the vectors of the exogenous variables, \( A \) and \( G \)
two determinist matrixes and \( \epsilon \) the vector of the innovations or signal. Equations (18) and (19) are respectively the measurement and the transition equations. They constitute a space-state model\(^7\) and correspond respectively to equations (16) and (17) used to estimate the TV-NAIRU. The TV-coefficients (\( Q \)), called state variables, move by a greater amount, the higher the Signal-to-Noise Ratio (SNR), i.e. the ratio between the signal variance and the noise variance (\( \sigma^2_{\epsilon} / \sigma^2_{\epsilon} \)).

In the light of these considerations, the TV-NAIRU appears to be more a mathematical construct than an unobservable variable. Indeed, it is a ratio between two coefficients of (16): the “constant” (which varies) divided by \( B \). Moreover, it could also be a function of any variables having a long-term effect on inflation (equation (6)). Hence, in both approaches, the ERU is in fact a theoretical construct. Guaranteeing inflation stability, it is a function of the observable and unobservable components of the inflation model and can thus depend on observable and unobservable variables. It does not determine the inflationary process, but on the contrary it is determined by it. Moreover, the ERU is not a modelling of the effective unemployment rate since none of the determinants of the latter are modelled\(^8\). The ERU is a hypothetical trajectory of the unemployment rate: the trajectory that would stabilise inflation. It is not unique, since it is possible to distinguish several concepts depending on the time-horizon considered for inflation stability.

2. Estimations of structural ERUs and TV-NAIRUs

Estimations of the French and American ERU\(_{LT}\) by the reduced and structural approaches give generally convergent results: for the recent period, the ERU\(_{LT}\) would be around 5% in the United States and around 10% in France\(^9\). Our empirical work

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\(^7\) For example Durbin and Koopman (2001), for econometric details on random coefficient models.

\(^8\) Our definition hence contrasts with some studies using also the terminology of ERU: general equilibrium models such as in Cahuc and Zylberberg (1996) or Caballero and Hammour (1998) model the employment and the labour force, whereas partial equilibrium models model job creations and job destructions (Caballero \textit{et al.}, 1997).

\(^9\) For example, see the structural estimations of Heyer \textit{et al.} (2000), Chagny \textit{et al.} (2002) or L’Horty and Rault (2003) and the TV-NAIRU estimations of Staiger \textit{et al.} (1996), Gordon (1997), Richardson \textit{et al.} (2000), Irac (2000) or Laubach (2001). Some studies have shown that these estimations are
confirms this result for the United States whereas the diagnosis is more mixed in the French case.

We used four approaches. The first is the standard structural approach in which all coefficients are constant. In the case of the wage-price setting, this hypothesis could be rejected because of the changes in the last 30 years on the goods and labour markets. Hence, in the second approach, called structural/Kalman, we test stochastic time variations of the coefficients. We could also have tested the impact of observable variables reflecting institutional characteristics of the goods and labour markets (trade union membership, replacement ratio, etc.) but this generally does not give conclusive results (Chagny et al., 2002). The standard structural approach is a constrained version of the structural/Kalman one in which the variance of all the TV-coefficients is zero.

The third approach is the unobservable TV-NAIRU ($U_{TVN}$) model proposed by Gordon (1997) (equations (16) with some lags and (17); $\phi_2 = \phi_5 = 0$):

$$\begin{align*}
\hat{p}_{Ct} &= \hat{p}_{Ct-1} - \phi_1(U_t - U_{LN}) - \phi_2 \hat{U} + \phi_3(\hat{p}_{Mt} - \hat{p}_{Ct}) + \epsilon_{pTVNt} \\
U_{LN} &= U_{LN-1} - \phi_4(\hat{\pi}_t^a - \hat{\pi}_{t-4}^a) + \phi_5(r_t - r_{t-4}) + \epsilon_{U_{lt}}
\end{align*}$$

Finally, the fourth approach, called observable TV-NAIRU, tests the effect of observable variables on the TV-NAIRU. For France, the annual labour productivity growth ($\hat{\pi}_t^a = \pi_t - \pi_{t-4}$) and the real interest rate ($r_t$) (i.e. the long-term rate minus the annual growth of consumer prices) have a significant impact.

In the structural and reduced Phillips curve estimations (tables 2 and 3), the Wald test accepts the hypotheses of unitary indexation on inflation only for the United States. In order to have results that are homogeneous with the literature, this hypothesis is imposed in the French TV-NAIRU model but not in the structural model because a non-unitary indexation has implications in terms of the ERU (equation (11)).

often imprecise (Staiger et al., 1996; Laubach, 2001). For the sake of conciseness, we did not treat the rather technical problem of statistical imprecision. Instead, we concentrated on some crucial theoretical uncertainties of the concept of ERU that make the statistical uncertainty somewhat secondary. However, in Heyer et al. (2004), we show how the main estimation methods of the statistical uncertainty tend to exaggerate the imprecision. According to our own estimations, the precision seems good enough to make the concept of ERU interpretable: in both approaches the ERU standard error is around 0.5%.
2.1. Medium- and long-term ERUs using the structural approach

After estimating the structural model (1), (2), (4) et (5) (tables 3 to 5 of the appendix), the medium and long-term ERUs can be calculated (equations (11) and (13)). The inflation trend and the productivity growth trend are calculated using a HP filter. Our estimations are in accordance with the literature: the American ERUL_{LT} fluctuates around the standard level of 5% whereas the French one is above 10% after an increase of more than 6 points since the beginning of the 1970’s (graph 1).

**Graph 1 - French and American ERUL_{LT}**

Three phenomena help to explain these two different ERU stories. Because of the non-unitary indexation of wages on prices in France, disinflationary policies have a cost in terms of ERUL_{LT} whereas they are neutral for the United States. The indexation of wages on productivity being higher in the United States, the elasticity between the ERUL_{LT} and labour productivity is higher in France: a fall of 1% in the rate of annual productivity growth leads to an increase of 0.9% point in the ERUL_{LT} for the United States and of 1.3% for France. Lastly, the productivity slowdown came to an end in the United States in the 1980’s but not in France. Graph 2 summarises these differences. Curve 1 shows the evolution of the French ERUL_{LT}. Curve 2 shows what would this evolution have been in the case of unit indexation of
wages on prices. Curve 3 assumes in addition that the French indexation of wages on productivity is the same as the American one. And finally, if the trend of the French productivity growth had been identical to the American one, the French and American ERUL_T would have had the same evolution (curve 4).

**Graph 2 - Evolutions of the French and American ERUL_T**

In the United States, a 1% point decrease in the ERUL_T can be achieved by a 1.07% increase in the rate of annual productivity growth. In France, there are several possibilities: either a 0.77% rise in the rate of annual productivity growth, or a 2.96% rise in inflation, or an appropriate combination of rises in productivity growth and inflation.

The structural/Kalman approach provides evidence concerning the French disindexation of wages on prices after 1982 following the introduction of an austerity policy by the socialist government. Variations in \( a \) are tested simultaneously with those in the constant (\( Z \)) so that possible evolutions of the latter are not fallaciously interpreted as being those of \( a \). The random walk specification is not very satisfactory because the amplitude of the disindexation depends strongly on the chosen SNR. We preferred a logistic function specification because it can estimate an initial (\( a^i \)) and a final (\( a^f \)) indexation regime, the switching speed between the
two regimes ($\sigma$) and the date of regime switching ($\tau / \sigma$):

$$a_t = (1 - \phi_t)a_t + \phi_t a'$$

(21)

where $\phi_t = (1 + e^{-\sigma t})^{-1} \to 0$ (resp. 1) as $t \to -\infty$ (resp. $+\infty$).

According to our estimation (graph 3; table 3, equation c), the transition between the two indexation regimes would be very short and 1982 would be the date of regime switching. For the 1970’s, the hypothesis of unit indexation is accepted by the Wald test, whereas since the 1980’s, less than 60% of price increases would be passed on to wages\(^{10}\).

**Graph 3 - France: indexation on prices ($a_t$)**

Compared to the standard structural estimation (table 3, equation a), $a$ is higher at the beginning of the sample and lower at the end whereas the contrary is true for $Z$. This explains why the structural/Kalman ERULT (graph 1, curve 2) is quite similar to the standard structural ERULT (curve 1). In terms of ERULT, the evolutions of $a$ are compensated by those of $Z$, so that the structural/Kalman approach brings little

\(^{10}\) Whereas this disindexation has often been found econometrically (for example, Ralle and Toujas-Bernatte, 1990), some authors argue that the indexation is still unitary but the inflation target has being modified (Blanchard and Sevestre, 1989).
additional information compared to the standard one. That is why, for the calculation of the ERU_{MT}, we suppose all coefficients constant.

At the end of the 1990’s and at the beginning of the 2000’s, the unemployment rate of both countries is lower than the ERU_{LT} (table 1, columns 3 and 5) without any increase in inflation (column 1). These low unemployment rates relatively to their generally estimated non-inflationary level (5% for the United States and 10% for France) led to a controversy about the permanent or transitory character of this evolution. On the “permanent” thesis side, a new-economy effect (for a discussion see Gordon, 1998) and hysteresis phenomena (Heyer and Timbeau, 2002) is thought by some to have brought about a fall of ERU_{LT}. On the “transitory” thesis side, some studies highlight the role of favourable transitory shocks (Gordon, 1998; Chagny et al., 2002). The structural model supports this latter thesis. Indeed, columns 4 and 5 of table 1 reveal an ERU_{MT} lower than its long-term level at the end of the period. This temporary fall in the ERU stems partly from observable phenomena. Because of the presence of the variation of the unemployment rate in the wage equation, the unemployment rate can be maintained temporarily below the ERU_{LT} without inflationary pressures (column 7). Adjustment mechanisms also played a positive role during this period (column 8). The fall of the ERU_{MT} is also explained by a succession of favourable temporary shocks (columns 9 to 12): an improvement in the terms of trade, a decrease in the employer’s social contribution rate and a fall in the productive capacity utilisation ratio. Some evolutions in the ERU_{MT} result from unobservable “shocks” measured by the residuals of the econometric equations (columns 13 to 16). The importance of the unobservable shocks varies considerably from one period to another. Lastly, in our model, the gap between the unemployment rates and the ERU_{MT} is a measure of the sacrifice ratio. According to this indicator, in both countries, disinflationary policies are estimated to have cost between 0.1 and 0.3 point of the unemployment rate per quarter (column 2).
Table 1 - Gap between the medium- and long-term ERUs

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<tr>
<td>1971-2000</td>
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<td>6.32</td>
<td>5.21</td>
<td>1.11</td>
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<td>0.10</td>
<td>0.01</td>
<td>0.08</td>
<td>0.01</td>
<td>0.29</td>
<td>0.06</td>
<td>0.22</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

Sources: authors’ calculations.

Notations: SR : sacrifice ratio; HYST = \( b_2 (\bar{U}_{t+1} - \bar{U}_{LT}) / (b_2 + b_3) \); ADJUST = (\( \text{Adjust} \)) / (b_2 + b_3); SHOCK = \( \bar{Z}_{LT} / (b_2 + b_3) \);
RES = \( \bar{e}_{p,t} / (b_2 + b_3) \) where \( e_{p,t} = e_{at} + e_{pc,t} + \dot{e}_{p,t+1} / \mu \).
2.2. The statistical weaknesses of the TV-NAIRU approach

For the United States, the reduced approach provides coherent results with the structural method and fairly stable estimates: the level and the evolutions of the TV-NAIRU depend to a very small extent on the presence or otherwise of certain variables in the reduced Phillips curve, on the estimation period or on the value of the SNR (graph 5). For France, the results are more disappointing. When unitary indexation is imposed, $\phi_1$ becomes insignificant. This leads to convergence problems of the Kalman filter. The estimations carried out over the 1978-2002 period are more stable. To be coherent, we chose this period for both countries (table 2).

<table>
<thead>
<tr>
<th>Table 2 - Estimations of TV-NAIRUs according to the SNR</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>$\hat{p}_{Ct-1}$</td>
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<tr>
<td>$U_t - U_{Ut}$</td>
</tr>
<tr>
<td>$\hat{U}$</td>
</tr>
<tr>
<td>$\hat{p}<em>{Mt} - \hat{p}</em>{Ct}$</td>
</tr>
<tr>
<td>$(\hat{r}<em>{t}^a - \hat{r}</em>{t-4}^a)$</td>
</tr>
<tr>
<td>$r_t - r_{t-4}$</td>
</tr>
<tr>
<td>$\sigma^2_{\hat{p}_{Ct}}$</td>
</tr>
<tr>
<td>$\sigma^2_{\hat{r}_{Ct}}$</td>
</tr>
<tr>
<td>SNR</td>
</tr>
<tr>
<td>Likelihood</td>
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<tr>
<td>Akaike</td>
</tr>
<tr>
<td>Schwarz</td>
</tr>
<tr>
<td>Hannan-Quinn</td>
</tr>
<tr>
<td>Final TV-NAIRU</td>
</tr>
</tbody>
</table>

The French TV-NAIRU estimations depend crucially on the choice of SNR. In theory, it can be estimated by the Kalman filter, but in practice, the results are disappointing. In many cases, the estimation does not converge. In others, it leads to a very low value, i.e. to an unwanted constant TV-NAIRU (table 2, column 5) described as a “pile-up problem” by Stock and Watson (1998). Lastly, the estimated SNR sometimes gives a highly erratic TV-NAIRU that is difficult to interpret from an economic point of view. In all these cases, the SNR is constrained in accordance with the Gordon (1997) smoothness criterion. However, this parameterisation is not very satisfactory because it can substantially influence the TV-NAIRU estimation. This is particularly true for France (graph 4).

**Graph 4 - French TV-NAIRU**

**Percent**

Sources: authors’ calculations, INSEE.
As the standard TV-NAIRU model gives disappointing results for France and more generally in the European case, several studies have tried other ad hoc stochastic specifications of the TV-NAIRU (Irac, 2000; Richardson et al., 2000; Laubach, 2001). In order to improve the explanatory power of the model, we preferred to test the effect of observable variables on the TV-NAIRU. The observable TV-NAIRU approach significantly increases the econometric performance of the model. The estimation is possible over all the sample period (table 2, equation 4) and the results are relatively insensitive to the value of the SNR. As in the structural approach a negative relation between the TV-NAIRU and the labour productivity is found. However, the elasticity is different: a 1% point decrease in the ERU needs a 1.77% increase in the annual growth rate of labour productivity against 0.77% with the structural approach. This decrease could also be achieved by a 0.95% decrease in the real interest rates.

Graph 6 presents the estimate of the observable TV-NAIRU and of its components. A HP filter was applied in order to consider trend values. The relative stability of the unobservable component (curve 1) is an interesting result compared to the unobservable TV-NAIRU model because the variations of unobservable do not provide further explanation for the rise in the French ERU. In
addition, the conclusions of this model differ significantly from those of the structural approach. Instead of more than 10%, the $ERU_{LT}$ would be around 8% in 2003. The coexistence of low inflation and a relatively low unemployment rate during the recent period has a permanent character because of the fall of the $ERU_{LT}$. The difference in diagnosis between the two approaches comes primarily from the effect of the real interest rates, which explains a significant part of the fall of the TV-NAIRU since the middle of the 1990’s (curve 3). This effect does not appear in the structural model because the real interest rates do not seem to influence the price setting.

**Graph 6 - France: observable TV-NAIRU**

*Sources: authors’ calculations, INSEE.*
Conclusion

The primary objective of this paper was to confront the two main approaches used to evaluate the ERU. It highlights the theoretical and empirical weaknesses of the TV-NAIRU approach. It appears preferable to regard the ERU rather as a theoretical construct than as an unobservable variable. Moreover, the distinction between the medium- and long-term ERUs based on a clear theoretical definition allowed us to separate the medium- from the long-term and the observable from the unobservable components of the ERU. Empirically, we have shown that the mathematical equivalence between the two approaches is not confirmed econometrically, especially in the French case. The French TV-NAIRU estimation suggests that further investigations about the influence of interest rates on the ERU may be promising. One way would consist in expanding the specification of our simple structural model by endogenising some key variables such as the labour productivity.
References


Irac D., 2000, “Estimation of a time varying NAIRU for France”, *Note d’études et de recherche de la Banque de France*, 75.


Appendix: Estimations of the structural model

### Table 3 - Wage equations

<table>
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<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Z / b (percent)</td>
<td>12.89 (15.85)</td>
<td>6.60 (12.55)</td>
<td>11.29 ( ^{1} ) (6.50) 13.73 ( ^{#} ) (21.15) ( \sigma = 0.43 ) ( \tau / \sigma = 82 : 2 )</td>
</tr>
<tr>
<td>( \hat{p}_c )</td>
<td>0.74 (15.36)</td>
<td>1 (( \sim ))</td>
<td>0.91 ( ^{1} ) (10.49)</td>
</tr>
<tr>
<td>( {0-2} )</td>
<td>[0-3]</td>
<td>[0-3]</td>
<td>0.54 ( ^{#} ) (3.71) ( \sigma = 0.43 ) ( \tau / \sigma = 82 : 2 ) [0-2]</td>
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<tr>
<td>( U_i )</td>
<td>0.19 (14.95)</td>
<td>0.13 (3.53)</td>
<td>0.18 (6.64)</td>
</tr>
<tr>
<td>( \dot{U}_i )</td>
<td>0.66 (4.29)</td>
<td>0.25 ( ^{***} ) (1.62)</td>
<td>0.59 (3.34)</td>
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<tr>
<td>( \tilde{x}_i )</td>
<td>( \sim ) 0.50 (4.37)</td>
<td>( \sim ) 0.50 (4.37)</td>
<td>( \sim ) 0.50 (4.37)</td>
</tr>
<tr>
<td>Dummies</td>
<td>81:2-82:2</td>
<td>( \sim ) 81:2-82:2</td>
<td>81:2-82:2</td>
</tr>
<tr>
<td>Centred R(^2)</td>
<td>0.94</td>
<td>0.45</td>
<td>0.95</td>
</tr>
<tr>
<td>SEE</td>
<td>0.32%</td>
<td>0.57%</td>
<td>0.30%</td>
</tr>
<tr>
<td>DW</td>
<td>1.01(^{1})</td>
<td>1.98</td>
<td>1.15</td>
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<td>553.68</td>
<td>492.52</td>
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<td>Schwarz</td>
<td>-8.41</td>
<td>-7.24</td>
<td>-8.39</td>
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</table>

\(^{1}\) The Cochrane-Orcutt correction shows that the residual auto-correlation has little impact on the value of the coefficients.

**Notations tables 3-5:** ***: not significant at 10%; DW: Durbin-Watson statistic; SEE: Standard Error of Estimate; Student statistic in brackets; [lags]; I: initial value; F: final value.
### Table 4 - Consumer price equations*

<table>
<thead>
<tr>
<th>Equation</th>
<th>Country</th>
<th>Period</th>
<th>Method</th>
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<th>(b)</th>
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<td></td>
<td></td>
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<td>OLS</td>
<td>OLS</td>
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<td>( \hat{p}_{10} )</td>
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<td>Centred ( R^2 )</td>
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</tr>
<tr>
<td>SEE</td>
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<td>-8.29</td>
<td>-9.18</td>
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</table>

* In both countries, the data validate the specification of (2) since the Wald test accepts the hypothesis according to which the sum of the coefficients of the import price and of the value-added price is equal to one.

### Table 5 – Value-added price equations

<table>
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<td>(4.81)</td>
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<td>( m_{1,-1} )</td>
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