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« Social security, inequality and growth »

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Abstract

In most industrial countries, public unfunded pension schemes are little, and even not at all, redistributive (from the rich towards the poor). This article presents an OLG model in which endogenous growth arises from the proportion of skilled people and the time they have spent to increase their training level. In such a framework, we show that a pension scheme which is not redistributive can nevertheless reduce inequalities by additional growth.

Keywords: social security, human capital, inequality

JEL classification: H55, J31, D63

1 Introduction

For several years, industrial countries have simultaneously experienced an increase in the lifespan and weak fertility. These two tendencies have induced an aging of their population. Under unchanged retirement starting conditions, the ratio between the number of retired people and that of workers, called the dependency rate, should reach in France, for example, 70.1% in 2040 whereas it was estimated at 35.8% in 1990. However, the debate on the financing of our public retirement systems is closely related to this rate. These systems are operated on a pay-as-you-go (PAYG) basis, i.e. contributions collected from workers are used directly to pay the benefits of current retirees. Hence, these retirement systems must cope with the increasingly larger number of pensioners compared to the number of contributors. Changes are therefore necessary. If we want to continue to guarantee in a near future the current level of benefits within the same system, it will be necessary to increase either the level of contributions, or the duration of contribution (by delaying the starting age of retirement). However, this financing problem calls into question the role of PAYG retirement schemes in our societies. For instance, by evaluating the real pre-tax return on non-financial corporate capital at 9.3%[1] and the growth rate over the same period (1960 to 1995) at 2.6%, Feldstein unequivocally advocates the privatisation of retirement schemes and the change to fully funded schemes. He thus assesses the potential present-value gain to nearly $20 trillion for the

[1] This return combines profits before all federal, state, and local taxes with the net interest paid. The method of calculation is described in Feldstein, Poterba and Dicks-Mireaux (1983).

United States. For others, like Greenspan, president of the FED, the retirement system must be privatized to support the private saving particularly low in the United States and therefore the capital accumulation.

Faced with these financial arguments, it is often argued that the PAYG retirement system is one of the essential instruments of the income redistribution towards the poor. However, the privatization of retirement systems would mean the end of this redistribution. A simple analysis of the calculation formula of pension benefits is often used to highlight the redistributive feature of these systems. This formula is of course specific to each country, and often very complex. However, it can be summarized by assuming that it associates for each individual a level of benefits proportional to his wages $\overline{\theta}W$ and a lump sum level $\overline{p}$. Let Θ be the replacement rate of the retirement system, we have $\Theta = \overline{\theta} + \frac{\overline{p}}{W}$. The fact that this formula is progressive is straightforward: the marginal replacement rate is negative, i.e. the replacement rate decreases with the individual wage level. It thus appears logical to think, after analyzing this formula, that retirement systems carry out transfers from high income people towards low income ones, and can reduce inequalities.

However, empirical studies focusing on the redistributive aspect of retirement systems (Burkhauser and Walick, 1981, and Garrett, 1995, Gustman and Steinmeier, 2001, Coronado and *alii*, 1999, 2000, on American data) all stressed that these systems are little, and even not at all, redistributive (from the rich towards the poor). Several elements enable us to explain this apparent paradox between this established fact and the progressiveness of pension benefits as exhibited by the above mentioned formula. First, the redistribution is primarily carried out from men towards women. Second, the redistribution is favorable to people who live long. However, we observe (e.g. Deaton and Paxton, 1998, 1999) that the differences in life expectancies are strongly related to social inequalities: high income people live longer than low income people. Taking into account these two elements, Gustman and Steinmeier (2001) show that the retirement system returns are almost identical whatever the household earnings.

The purpose of this article consists in examining what remains of the argument which supports that public pay-as-you-go retirement systems reduce inequalities, whereas they do not seem to make (or very weakly) any income redistribution, within the same generation, from the rich towards the poor. To achieve this objective, we first need to discuss the sources and the main features of income inequalities in order to construct a relevant framework. Since earnings are strongly related to human capital, as described in an abundant literature (e.g. Mincer, 1997, Neal and Rosen, 2000), this question is closely related to disparities in human capital.

It is quite admitted in the literature on human capital that learning activities and human capital formation are concentrated at young ages. Thus, a career can be summarized by a first period of learning, including schooling and job training, characterized by low wages, and a second when the human capital investment results in high wages. Wage profiles are thus increasing, except for dropouts whose wage profiles are almost flat (e.g. Andolfatto et alii, 2000). Lilliard (1977) then highlighted that the wage profile of a worker is increasing with the time spent to be formed and his ability to learn. In order to replicate these facts, the Ben-Porath (1967) human capital model has been widely used (e.g. Mincer, 1997, Neal and Rosen, 2000). In this model, individuals maximize the present value of their lifetime earnings by allocating their time between training activities and work. It predicts accurately that more able individuals will invest more in human capital and therefore will have steeper wage profiles than the less able ones.

In our version of this model, an individual is endowed at birth with some innate ability to learn and a human capital stock. The dispersion of the wage profiles can thus either come from heterogeneity among learning abilities, or among inherited stocks. However, Huggett and alii (2004) showed that the relevant variable allowing to replicate the observed dispersion in US data is the innate ability. Consequently, we suppose that in our model this variable is the only source of heterogeneity among individuals, and that all of them inherit an identical stock of human capital which we suppose to be the average stock of the former generation. As a result, only a proportion of individuals characterized by a strong learning ability, represented by a low cost to learn, will undertake studies and will increase their stock of human capital. We thus obtain a simplified but relevant representation of wage profiles and their dispersion. Individuals are divided into two worker groups: the skilled ones who invest in their human capital and then have increasing earnings and the nonskilled ones (the dropouts) who are satisfied with their initial human capital and thus have a flat earning profile. A wage profile is thus all the more increasing given that the ability is strong. Moreover, as future wages depend on the human capital stock which depends on the training duration, a wage profile is all the more increasing given that the training duration is longer.

In section 2, we present the model. We stress in this section the existence of a retirement system which, although being characterized by a progressive calculation formula of the pension benefits, formally does not redistribute from skilled workers towards unskilled ones. However, we show in a third section that this system can reduce inequalities. This reduction of inequalities is not due to fiscal redistribution but to additional growth which is favorable to unskilled workers. We then underline the dilemma inequality vs growth related to retirement systems. The calculation of pension benefits splits into two parts: a lump sum component (identical for all individuals) and a variable component which depends on a representative average wage. We show that an increase in the lump sum component of the pension allows stronger reduction in inequalities, but is harmful for growth. We then study in the last section the optimal degree of redistribution according to an utilitarian criterion.

2 The Model

The model is a version of the Ben-Porath model (1967).

Each period, population is growing at a constant rate n:

$$N_t = (1+n)\, N_{t-1} \tag{1}$$

Individuals, non altruistics, live two periods: they are respectively young and old. Their preferences are described by the following utility function:

$$U = \ln c_t + \beta \ln d_{t+1} + \varepsilon \ln (1 - x_t) \tag{2}$$

where c_t and d_{t+1} are, respectively, the consumption of an individual born in period t when young and when old, and x_t represents the cost of training.

Individuals born in t inherit the average human capital of the former generation $\overline{Z}_{t-1}$. These individuals can then choose to be trained during h_t in order to increase their level of human capital. During this period, they work only one share $\lambda < 1$ of their time, the remainder being devoted to the training[2]. At the end of their training period, they become skilled and are supplied with the following human capital:

$$Z_t^s = B h_t^{\delta} \overline{Z}_{t-1},\ B > 0,\ \delta > 0 \tag{3}$$

[2] If $\lambda = 0$, training is a full time activity and corresponds then to schooling. It is qualitatively similar since persons with more schooling tend to invest more in job training (Lillard and Tan, 1986, Mincer, 1993, 1997).

Skilled workers are thus characterized by a period h_t with low wages $\lambda \overline{Z}_{t-1} w_t$ and a period $1 - h_t$ when they benefit from their investment in human capital. Over this latter period, their wages are thus $Z_t^s w_t$. Income of skilled workers over their whole working period W_t^s is thus equal to $h_t \lambda \overline{Z}_{t-1} w_t + (1 - h_t) Z_t^s w_t$. Nevertheless, the training involves an extra work. It is thus associated with a cost evaluated by $\varepsilon \ln (1 - x_t)$, where x_t is uniformly distributed through the population, on the support $[0, 1]$. Consequently, some individuals will decide not to invest in human capital because training is too costly compared to the monetary profit. These dropouts are then characterized by the following level of unskilled human capital:

$$Z_t^u = \overline{Z}_{t-1} \tag{4}$$

Their earnings over the working period is constant and then defined by $W_t^u = Z_t^u w_t$.

During the first life period, individuals consume a part of their disposable income, and save:

$$c_t^i + s_t^i = W_t^i (1 - \tau), \; i = s, \; u \tag{5}$$

where s_t^i is the saving of an individual of type i born in t, and τ the public pension scheme payroll tax.

In the second life period, individuals get back the saving lent to firms with interest, receive their pension from the public retirement scheme and consume in order to use up their wealth. The budget constraint is then:

$$d_{t+1}^{i} = R_{t+1}s_{t}^{i} + p_{t+1}^{i},\ i = s,\ u \tag{6}$$

where R_{t+1} is the real interest factor, and p_{t+1}^{i} the pension benefits of an individual of type i.

The calculation of pension benefits splits into two parts: an identical lump sum part $\overline{p}_{t+1}$ for all, and a part related to a representative average wage. This latter is generally not calculated on the whole working period. In France, for example, before the Balladur reform of 1993, it corresponded to the average wage over the ten best years, then gradually over the 25 best years after the reform. In the United States, the period is longer and corresponds to 35 years. Let us consider two opposite cases: either the representative average wage is calculated over the whole active period, or over a duration only taking the skilled wages into account. The representative average wage of unskilled workers $\tilde{W}_{t}^{u}$ is then always equal to W_{t}^{u}, and that of skilled ones is:

$$\tilde{W}_{t}^{s} = \mu Z_{t}^{s} w_{t} + (1-\mu)\, W_{t}^{s} \tag{7}$$

where $\mu = 0$ if the representative average wage is calculated over the whole working period, and $\mu = 1$ in the opposite case.

The calculation of pension benefits for a worker is then given by:

$$p_{t+1}^{i} = \theta_{t+1}\tilde{W}_{t}^{i} + \overline{p}_{t+1} = \theta_{t+1}\tilde{W}_{t}^{i} + \nu_{t+1}\overline{\tilde{W}}_{t} \tag{8}$$

where $\overline{\tilde{W}}_t$ is the average reference wage for the whole population.

Let $\Omega_t^s = W_t^s (1-\tau) + \frac{\theta_{t+1}\tilde{W}_t^i + \overline{p}_{t+1}}{R_{t+1}}$ be the life cycle income of skilled workers, where $\tilde{W}_t^s$ is defined by (7), the maximization of this income leads to the relation defining the training duration:

$$h_t - \frac{\lambda h_t^{1-\delta}}{B(1+\delta)} = \frac{\delta}{1+\delta} + \frac{\delta}{1+\delta}\frac{\mu}{1-\tau+(1-\mu)\frac{\theta_{t+1}}{R_{t+1}}}\frac{\theta_{t+1}}{R_{t+1}} \tag{9}$$

Let h^0 be the training duration chosen in the absence of retirement system, the analysis of (9) gives us:

Proposition 1 *If* $\frac{\lambda}{B} < 1$, $\tilde{W}_t^s > W_t^s$ *and* $\theta_{t+1} > 0 \Leftrightarrow h_t > h^0$.

Proof. $\frac{\lambda}{B} < 1 \Rightarrow h^0 < 1$.

If $\tilde{W}_t^s = W_t^s$ $(\mu = 0)$ or $\theta_{t+1} = 0$, we have straightforward according (9) that $h_t = h^0$.

For $\frac{\lambda}{B} < 1$ and $\mu = 1$, we have $\frac{\delta}{1+\delta}\frac{\mu}{1-\tau+(1-\mu)\frac{\theta_{t+1}}{R_{t+1}}}\frac{\theta_{t+1}}{R_{t+1}} > 0 \Rightarrow h_t > h^0$. The rest of the proposition, $\forall\mu > 0$, can then be obtained by continuity. ■

The presence of a retirement system which calculates pension benefits according to reference wages without taking the whole career into account thus generates an incentive for a longer training (compared to the situation without retirement system). Initially, this lengthening of the training has a negative effect. Indeed, individuals must accept a low wage longer. But then, they profit as skilled workers from higher wages. Since the last wages weight more in the calculation of the reference wage, they also benefit, all things being equal, from an increase of their pension benefits. On the whole, we thus show that individuals who decide to train themselves may find profitable to have a supplement of training as an *investment* in the retirement system. This incentive disappears completely if the whole wages are taken into account in the calculation of pension benefits ($\tilde{W}_t^s = W_t^s$), or if the system is totally lump sum ($\theta_{t+1} = 0$).

To summerize, we can say that the incentive to be trained longer generated by the retirement system is due to the interaction of three factors:

- the last wages weight more in the calculation of the reference wage,
- the growth of wages which is higher for skilled workers during their career,
- the positive impact of the training level on the difference between first and last wages.

We can also notice that this incentive will be all the weaker as the interest rate will be higher. Indeed, the higher the interest rate is, the lower is the present actuarial value of pension benefits.

We suppose now that $\mu = 1$, and to simplify, we take $\lambda = 0$[3].

The utility maximization of a i type agent subject to his budgetary constraints leads to the following saving function:

[3] The objective is here the analysis of macroeconomic impact of a PAYG retirement system which have the main features of the existing systems. $\mu > 0$ is thus needed, and it is then natural to retain the case $\mu = 1$. Moreover, the argumentation being articulated around the steeper wage profile of the skilled workers, to choose $\lambda = 0$ does not alter the generality of the results. With $\mu = 1$ and $\lambda = 0$, we have $\tilde{W}_t^s = Z_t^s w_t$ and $W_t^s = (1 - h_t)\, Z_t^s w_t$; we thus verify in this case that $\tilde{W}_t^s > W_t^s$.

$$s_t^i = \frac{\beta}{1+\beta} W_t^i (1-\tau) - \frac{1}{1+\beta} \frac{p_{t+1}^i}{R_{t+1}} \tag{10}$$

By reducing at the same time the disposable income and the need for future income, we see that the private saving is reduced by the existence of the retirement system, and this, whatever the calculation of pension benefits is.

Lastly, an individual decides to qualify himself if his monetary profit is higher than the utility cost associated with the additional training, i.e. $(1+\beta)\ln\Omega_t^s + \varepsilon\ln(1-x_t) \geq (1+\beta)\ln\Omega_t^u$. Given the uniform distribution of the parameter x_t in the population, the proportion of individuals x_t^* who decide to be trained is defined by:

$$x_t^* = 1 - \left(\frac{\Omega_t^u}{\Omega_t^s}\right)^{\frac{1+\beta}{\varepsilon}} \tag{11}$$

The structure of skills in the economy is thus determined by the life cycle income differential between skilled workers and unskilled ones. The higher this differential is, the larger is the proportion of individuals encouraged to be formed.

Retirement systems have pay-as-you-go features, i.e. within one period, pensions are financed by contributions of workers of the same period. In other words, retirement systems transfer workers' income towards pensioners. The amount of contributions $\tau\left[L_t^u W_t^u + L_t^s W_t^s\right]$ must thus equalize the amount of pension benefits $N_{t-1}^u p_t^u + N_{t-1}^s p_t^s$. Knowing that at the date t there is a proportion x_t^* of skilled workers and that the flexibility of prices ensures the total use of production factors, the budget balance of the pension scheme, obtained using (1), (3), (4) et (8), is defined by:

$$\theta_t + \nu_t = (1+n)\,\tau\left[x_t^*\,(1-h_t)\,Bh_t^{\delta} + 1 - x_t^*\right]\frac{w_t}{w_{t-1}} \tag{12}$$

As noted in the introduction, existing retirement systems of industrialized economies do not redistribute from high income people towards low income ones. In terms of the retirement system implicit return, i.e. the ratio between the pension received by an individual and the amount of his contributions, this means that $\frac{p_t^u}{\tau W_t^u} = \frac{p_t^s}{\tau W_t^s}$. We verify such a condition is satisfied for:

$$\nu_t = \widetilde{\nu}_t = \frac{(1+n)\,Bh_{t-1}^{1+\delta}}{Bh_{t-1}^{\delta}-1}\tau\frac{x_t^*\,(1-h_t)\,Bh_t^{\delta}+1-x_t^*}{x_{t-1}^*\,(1-h_{t-1})\,Bh_{t-1}^{\delta}+1-x_{t-1}^*}\frac{w_t}{w_{t-1}} \tag{13}$$

If $\nu_t > \widetilde{\nu}_t$, then the retirement system is fiscally favorable to low income people $\frac{p_t^u}{\tau W_t^u} > \frac{p_t^s}{\tau W_t^s}$. We will say that it is redistributive. In the opposite case, $\nu_t < \widetilde{\nu}_t$, we will say it is reverse-redistributive. This characteristic is easily comprehensible. Indeed, the lump sum component is fiscally favorable to low income people since they receive as much as high income people whereas they have contributed less. A pure lump sum system will be thus redistributive. Conversely, the component which is proportional to the reference wage, characterized by θ_t, is favorable to high income people. Indeed, these ones have a rising wage profile which is favorable to them since reference wages are calculated over a period less long than the entire working period. A pure proportional system is thus reverse-redistributive. There is only one combination of the lump sum and the proportional components, defined by (13), that allows us to characterize a system which does not redistribute within a generation.

We consider a competitive sector characterized by a representative firm which produces the single commodity according to a Cobb-Douglas technology with constant return to scale:

$$Y_t = F\left(K_t, L_t^u, L_t^s\right) = AK_t^{\alpha}\left(Z_t^u L_t^u + (1-h_t) Z_t^s L_t^s\right)^{1-\alpha},\ 0 < \alpha < 1, \tag{14}$$

where Y_t is the output level, K_t the physical stock of capital and $L_t = Z_t^u L_t^u + (1-h_t) Z_t^s L_t^s$ the quantity of labor.

Let $k_t = \frac{K_t}{L_t}$ be the capital per work unit, under the assumption of a total capital depreciation, the optimality conditions resulting from the maximization of the profit are:

$$R_t = A\alpha k_t^{\alpha-1} \tag{15}$$

$$w_t = A(1-\alpha) k_t^{\alpha} \tag{16}$$

General equilibrium is then obtained by considering the simultaneous clearing in all the markets, that is:

in the unskilled labor market

$$L_t^u = (1 - x_t^*) N_t, \tag{17}$$

in the skilled labor market

$$L_t^s = x_t^* N_t, \tag{18}$$

and in the physical capital market

$$K_t = N_{t-1} \left[x_{t-1}^* s_{t-1}^s + \left(1 - x_{t-1}^*\right) s_{t-1}^u \right]. \tag{19}$$

For $\lambda = 0$ and $\mu = 1$, the skilled training spell, defined by (9), is rewritten as $h_t = h^0 + \frac{\delta}{(1+\delta)(1-\tau)} \frac{\theta_{t+1}}{R_{t+1}}$, where $h^0 = \frac{\delta}{1+\delta}$. Using (12), (13), (15) and (16), we show that the training spell is determined by:

$$h_t = h^0 + \frac{\delta\beta (1-\alpha) \tau}{(1+\delta) \left[\alpha (1+\beta) + \tau (1-\alpha)\right]} \left[\frac{B h_t^\delta (1-h_t) - 1}{B h_t^\delta - 1} \right] \tag{20}$$

where $\lim\limits_{h \to h^0} RHS > h^0$ and $\lim\limits_{h \to 1} RHS < h^0$. This equation thus defines a relation between the training spell and the contribution rate of the retirement system such as $h_t = h(\tau) < 1$.

With (9), (12) and (13), we obtain:

$$x_t^* = 1 - \frac{1}{\left[(1-h_t) B h_t^\delta\right]^{\frac{1+\beta}{\varepsilon}}} \tag{21}$$

By considering (21), we define a relation between the proportion of skilled workers and the contribution rate of the retirement system $x_t^* = x^*(\tau)$.

Lastly, with (1), (3), (4), (8), (10), (15), (16) and (19), we determine the physical capital accumulation dynamics:

$$k_{t+1}=\frac{A\alpha\beta\left(1-\alpha\right)\left(1-\tau\right)}{\left[\alpha\left(1+\beta\right)+\tau\left(1-\alpha\right)\right]\left(1+n\right)}\frac{1}{x^{*}\left(\tau\right)\left(1-h\left(\tau\right)\right)Bh^{\delta}\left(\tau\right)+1-x^{*}\left(\tau\right)}k_{t}^{\alpha} \tag{22}$$

Since $\alpha<1$, given $k_0>0$, the model has the good dynamic properties and converges towards the unique stationary state $h\left(\tau\right)$, $x^{*}\left(\tau\right)$ and
$k=\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[\alpha(1+\beta)+\tau(1-\alpha)](1+n)}\frac{1}{x^{*}(\tau)(1-h(\tau))Bh^{\delta}(\tau)+1-x^{*}(\tau)}\right]^{\frac{1}{1-\alpha}}$.

Let us now study the impact of the retirement system on growth and inequalities.

3 Social security, inequality and growth

In the balanced growth path, we derive from the (17), (18) and (19) market clearings as well as the (1), (3), (4) and (14) relations the growth rate g:

$$1+g=\frac{Y}{Y_{-1}}=\left(1+n\right)\frac{\bar{Z}}{\bar{Z}_{-1}}=\left(1+n\right)\left[x^{*}\left(\tau\right)Bh^{\delta}\left(\tau\right)+1-x^{*}\left(\tau\right)\right] \tag{23}$$

We must then define an aggregative index allowing us to evaluate a type of inequality, and its level. We define two sub-groups of mass m characterized, for the first, by the most favored individuals, i.e. characterized by $x\in[0;m]$, $m\leq 0.5$, and, for the second, by the least favored individuals , $x\in[1-m;1]$. We then define the index of inequality as being the difference between the average (indirect) utility of the most favored group and the average utility of the least favored one: $I=\int_{0}^{m}V\left(x\right)dx-\int_{1-m}^{1}V\left(x\right)dx$. If we now suppose that m is such that the most favored individuals always decide to train themselves whereas the least favored ones make the opposite choice[4], the index of inequality is equal to:

$$I = (1+\beta)\log\left(\frac{1}{1-x^{*}(\tau)}\right) + Cte \tag{24}$$

where $Cte = \varepsilon\int_0^m \log(1-x)\,dx$.

Proposition 2 *A social security which guarantees the equality of returns*

always reduces inequalities,

always increases economic growth (at least if the contribution rate is sufficiently low).

Proof. from (21) we have $\frac{dx^*}{d\tau} = \frac{1+\beta}{\varepsilon}\left[(1-h)Bh^{\delta}\right]^{-\frac{1+\beta+\varepsilon}{\varepsilon}}\frac{\partial\left[(1-h)Bh^{\delta}\right]}{\partial\tau}$. Since $h^0 = \arg\max\left\{(1-h)Bh^{\delta}\right\}$, $\left.\frac{\partial\left[(1-h)Bh^{\delta}\right]}{\partial\tau}\right|_{h\geq h^0} \leq 0$. We then derive from proposition 1: $\frac{dx^*}{d\tau} \leq 0 \Rightarrow \frac{dI}{d\tau} = \frac{1+\beta}{1-x^*}\frac{dx^*}{d\tau} \leq 0$.

$\frac{d(1+g)}{d\tau} = (1+n)\left[Bh^{\delta}-1\right]\frac{dx^*}{d\tau} + x^*\delta Bh^{\delta-1}\frac{dh}{d\tau}$. Close to $\tau = 0$, we have $\left.\frac{\partial\left[(1-h)Bh^{\delta}\right]}{\partial\tau}\right|_{\tau=0} = 0 \Rightarrow \left.\frac{dx^*}{d\tau}\right|_{\tau=0} = 0$.

We hence have $\left.\frac{d(1+g)}{d\tau}\right|_{\tau=0} = x^*\delta Bh^{\delta-1}\frac{dh}{d\tau}$. From (20), and since $h \geq h_0$, we show that $sign\left[\frac{dh}{d\tau}\right] = sign\left[\frac{\partial\frac{\tau}{[\alpha(1+\beta)+\tau(1-\alpha)]}}{\partial\tau}\right]$, where $\frac{\partial\frac{\tau}{[\alpha(1+\beta)+\tau(1-\alpha)]}}{\partial\tau} = \frac{\alpha(1+\beta)}{[\alpha(1+\beta)+\tau(1-\alpha)]^2}$.

We then have $\left.\frac{d(1+g)}{d\tau}\right|_{\tau=0} > 0$. ■

Considering inequalities, the result highlighted can first appear counter-intuitive. Indeed, we can observe that the reduction of inequalities follows the

[4] This is the case for $m = 0$. Indeed, the most favored individuals have in this case no cost to learn ($x = 0$) and always choose to learn. On the other hand the least favored individuals have the maximum learning cost and never choose to invest in human capital.

lengthening of the skilled workers training duration. We can note that the lengthening of the training duration raises the difference between the wages of the skilled workers and those of the unskilled ones. But, in an intertemporal prospect, in the absence of retirement system, individuals who decide to undertake a training choose the duration h^0 which maximizes their life cycle income, and thus which maximizes inequalities. A lengthening of the training duration thus raises inequalities when $h < h^0$. Conversely, when $h > h^0$, a lengthening of the training duration reduces inequalities because we move away from the individually optimal training length. However, even if the retirement system does not carry out transfers from the rich towards the poor, we observe that the earnings related pension benefits calculation formula has an incentive effect on the investment in human capital. This retirement system thus encourages skilled workers to train themselves more compared to their optimal level, and consequently, reduces inequalities (Cf. figure 1[5]). If we consider a redistributive system, we can observe that the incentive to train more is lower, which would seem to have a negative effect for inequalities. However, in this case, the system becomes explicitly favorable to unskilled workers by transferring income to them, and thus a redistributive system reduces inequalities more than

[5] This figure (and the following ones) is obtained for $\alpha = 0.3$, $\beta = 0.9$ et $n = 0$; δ is fixed to 0.1, which gives us a training spell without retirement system of approximately 9%; B and ε are then selected in order to obtain, in the economy without system of retirement, an annual growth rate of 2% (on the basis of a 40 years period) and a proportion of low skilled individuals of 30%, wich is roughly representative of the high-school dropout proportion in the United States and Canada. It gives us $B = 3.46$ and $\varepsilon = 0.98$.

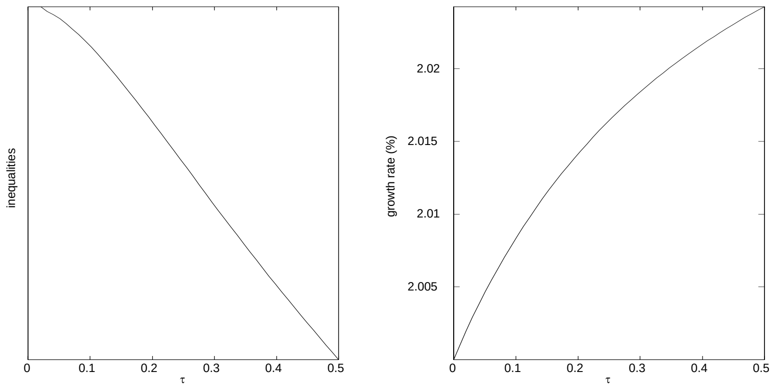

Figure 1: Inequalities and growth with the equality of returns

a non-redistributive one. In the extreme case of a pure lump sum system, the stationary equilibrium is characterized by a strong reduction in inequalities and a constant training duration.

Concerning growth, here its engine is human capital. This capital takes two distinct forms: the proportion of individuals who decide to undertake a training, and the length of this training. The retirement system which guarantees the equality of returns, by reducing inequalities, reduces the proportion of individuals who train themselves, which has a negative impact on growth. On the other hand, this system encourages skilled workers to be trained longer. This last effect dominates the former, at least for a sufficiently low size of the system, and we stress then, as Zhang (1995), Sala-i-Martin (1996) and Kemnitz and Wigger (2000), a positive correlation between the size of the retirement system and economic growth (cf figure 1). Besides, we can notice that this positive correlation is supported by the data (cf Barro, 1991, Heston and Summers, 1988, Perotti, 1996, and Mulligan and Sala-i-Martin, 1999).

Then, existing retirement systems seem to be favorable to low income people. Not by a direct fiscal redistribution, but by an additional growth which, as shown in the empirical studies, seems to be associated with a reduction in inequalities (cf for example Bénabou, 1996, Alesina and Rodrik, 1994, Clarke, 1992, Deiniger and Squire, 1995, Perotti, 1994, 1996, Persson and Tabellini, 1994). But are these systems and their features socially optimal? If we consider the view of the poorest, shouldn't we integrate a tax redistribution within retirement systems? But with which consequence for growth?

Proposition 3 *For a given payroll tax, compared with a social security which guarantees the equality of returns, a (reverse-)redistributive one exhibits a lower (higher) economic growth rate and a lower (higher) level of inequalities.*

Proof. Cf. appendix. ■

The contributory vs non-contributory structure thus fits in an inequality vs growth dilemma. In order to reduce (more) inequalities, we must accept more redistribution within the retirement system, given that it is harmful for growth. Indeed, an increase of the lump sum component, that implies a reduction of inequalities, also means a reduction of the contributory component and thus a reduction of the incentive to be trained longer, harmful for growth. Besides in the case of a pure lump sum system, it does not have any impact on the training spell (cf proposition 1) but reduces the proportion of skilled workers in the economy. Consequently, such a system implies a reduction of growth (cf figure 2). Conversely, we can observe that a pure proportional system is

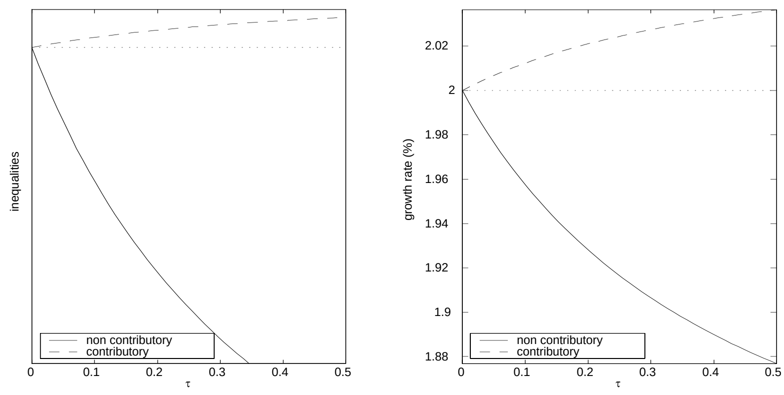

Figure 2: Inequalities and growth for pure contributory and non contributory systems

favorable for growth, but increases inequalities. The choice of the redistribution degree consists well in *more or fewer inequalities* and *more or fewer growth* trade-off.

4 Optimal Social Security and the degree of redistribution

We have seen that, although not redistributing formally from the rich towards the poor, retirement systems are integrated into the core of inequality vs growth issue. To study the optimality of the retirement system redistribution, as well as its size, we thus need a criterion which integrates these two dimensions. Concerning inequalities, we will only integrate, following Rawls, the (indirect) utility of low income people, i.e. V_t^u. In order to take the growth dimension into account, we will consider the utility of successive generations. Indeed, growth has a positive impact on the incomes of successive generations, therefore on their utility. A natural criterion which includes these two considerations is the intertemporal sum of the indirect utilities of unskilled workers, $S_t = \sum_{t=0}^{\infty} \gamma^t V_t^u$, $0 \leq \gamma < 1$, where γ will account for the relative importance of growth on inequalities. To see it, let us consider this criterion in the balanced growth path. In this case, the maximization of this criterion according to the variables of the retirement system (τ, ν, θ) is equivalent to the maximization of:

$$\begin{aligned} S(\tau, \nu) &= \frac{\alpha + (2\alpha - 1)\beta}{1 - \alpha} \log\left(\frac{1 - \tau}{\alpha(1 + \beta) + \tau(1 - \alpha)}\right) \\ &\quad + (1 + \beta)\log\left[1 - \tau + \frac{\tau(1 + g) - \tau x^* B h^{1+\delta} + \nu g}{1 + g} \frac{(1 - \tau)\beta(1 - \alpha)}{\alpha(1 + \beta) + \tau(1 - \alpha)}\right] \\ &\quad + \left(\frac{(1 + \beta)\gamma}{1 - \gamma} + \frac{(1 - 2\alpha)\beta - \alpha}{1 - \alpha}\right)\log(1 + g) \end{aligned} \tag{25}$$

We can then observe that, when $\gamma \rightarrow 1$, the impact of the retirement system on growth dominates any other consideration.

First, if we determine the optimal retirement system according to the criterion (25) subject to the equality of returns, we obtain, for our parameters of reference $\alpha = 0.3$, $\beta = 0.9$, $n = 0$, $\delta = 0.1$, $B = 3.46$ and $\varepsilon = 0.98$, the results reported in the figure 3. We note that the optimal size always increases according to the γ parameter. Indeed, we saw in the previous section (proposition 2) that a system which does not redistribute within a generation had a positive impact on economic growth. Consequently, a rise of the weighting of the growth increases obviously the size of the system. Moreover, for values of the

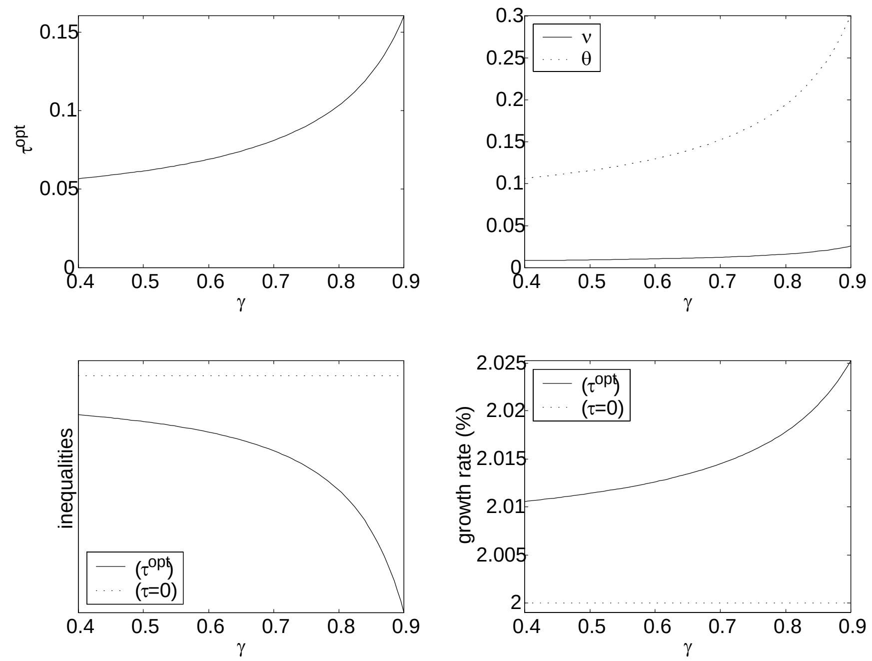

Figure 3: Optimal retirement system, conditional to $\nu = \tilde{\nu}$

contribution rate higher than 5%, we can note that $\frac{1+g}{R} = \frac{(1-\tau)\beta(1-\alpha)}{\alpha(1+\beta)+\tau(1-\alpha)} < 1$.

To return to our starting problem, we showed the possibility of the optimal existence of a PAYG retirement system characterized by a return (the economic growth rate) lower than the interest rate, and which does not redistribute from high income people towards low income ones. We observe that such a system can nevertheless reduce inequalities.

However, it is clear that this system is not necessarily the best according to our criterion of reference. For example, if $\gamma \to 1$, the criterion becomes the growth. We saw (proposition 3) that the lump sum component harmed growth. In this case, we must tend inevitably, according to the (25) criterion, towards a

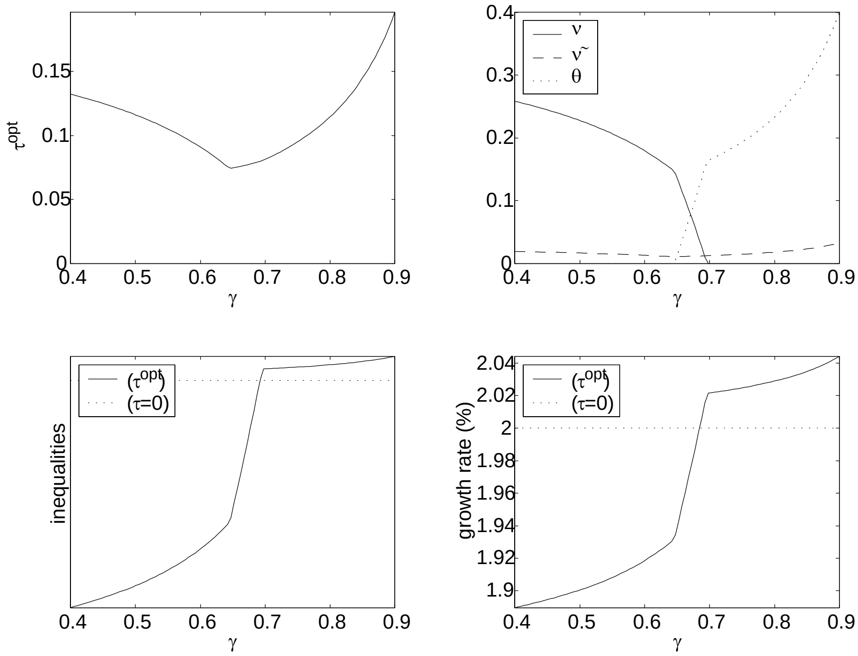

Figure 4: Optimal retirement system

pure proportional system (Cf. figure 4).

In a reciprocal way, if $\gamma \to 0$, the reduction in inequalities dominates any other objective and we tend towards a pure lump sum system, which is harmful for growth. We then explain easily the nonlinear shape of the optimal contribution rate as exhibited by the figure 4. For low values of $\gamma(<0.65)$, there is a pure lump sum system ($\theta = 0$). But this system being harmful for growth, the more γ increases, i.e. the more weighted the interest rate is, the more the size of the system decreases. For high values of $\gamma(>0.7)$, there is a pure proportional system ($\nu = 0$) favorable for growth. When γ increases, the size of the system thus increases. Lastly, for the intermediate values of γ, we observe an optimal coexistence of the lump sum and proportional components. Besides, for $\gamma = 0.69$, the equality of returns is an optimal feature of the retirement system.

5 Conclusion

Are retirement systems one of the essential instruments of the redistributive policy in our societies? Concerning the redistribution between generations, their pay-as-you-go structure contains in itself the answer. On the other hand, the answer is much more complex if we are interested in the redistribution of high income people towards low income ones. Indeed, to be strictly accurate, the existing retirement systems, for their large majority, do not redistribute *fiscally* (or very slightly) from the rich towards the poor. Thus, these systems seem not to be effective to reduce inequalities. Nevertheless, according to the argument developed in this article, they can be favorable to the most disadvantaged people by an additional growth. Accordingly, we have shown that their place in our societies has not to be called into question. However, in the prospect of a social security reform, it seems relevant to wonder about an increase of the retirement system redistribution degree in order to reduce inequalities. But we have to take into account the fact that it could lead to an economic growth reduction.

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Appendix: proof of proposition 3

For $\lambda = 0$ and $\mu = 1$, with eqs (9), (15), (16), (12) et (13), we show that the length of training is determined by:

$$h = h^0 + \frac{\delta}{(1+\delta)(1-\tau)} \frac{(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu}{R} \tag{26}$$

The proportion of skilled workers is determined by:

$$\begin{gathered} \frac{\varepsilon \log(1-x^*)}{1+\beta} = \\ \log\left\{1-\tau+\frac{(1+n)\tau\left[x^*(1-h)Bh^\delta+1-x^*\right]-\nu}{R}+\frac{\nu\left[x^*Bh^\delta+1-x^*\right]}{R}\right\} \\ -\log\left\{(1-h)Bh^\delta(1-\tau)+\frac{(1+n)\tau\left[x^*(1-h)Bh^\delta+1-x^*\right]-\nu}{R}Bh^\delta+\frac{\nu\left[x^*Bh^\delta+1-x^*\right]}{R}\right\} \end{gathered} \tag{27}$$

The interest rate is:

$$R = \frac{(1+\beta)\alpha + \tau(1-\alpha)}{\beta(1-\alpha)(1-\tau)}(1+n)\left[x^*Bh^\delta + 1 - x^*\right] \tag{28}$$

Equation (26) can be rewritten as:

$$\frac{(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu}{R} = \frac{(1+\delta)(1-\tau)}{\delta}\left(h - h^0\right) \tag{29}$$

By introducing this last equation in the skilled workers' equation (27), we obtain:

$$\frac{\varepsilon\log(1-x^*)}{1+\beta} =$$
$$\log\left\{1-\tau+\frac{(1+\delta)(1-\tau)}{\delta}\left(h-h^0\right)+\nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha+\tau(1-\alpha)](1+n)}\right\}$$
$$-\log\left\{(1-h)\,Bh^\delta\,(1-\tau)+\frac{(1+\delta)(1-\tau)}{\delta}\left(h-h^0\right)Bh^\delta+\nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha+\tau(1-\alpha)](1+n)}\right\} \tag{30}$$

Let be $\frac{\varepsilon\log(1-x^*)}{1+\beta} = \log\{Y\} - \log\{Z\}$,

where $Y = 1-\tau+\frac{(1+\delta)(1-\tau)}{\delta}\left(h-h^0\right)+\nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha+\tau(1-\alpha)](1+n)}$ et $Z = (1-h)\,Bh^\delta\,(1-\tau)+\frac{(1+\delta)(1-\tau)}{\delta}\left(h-h^0\right)Bh^\delta+\nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha+\tau(1-\alpha)](1+n)}$.

We then have:

$$\frac{-\varepsilon}{(1+\beta)\,(1-x^*)}dx^* = \frac{Y_h dh + Y_\nu d\nu}{Y} - \frac{Z_h dh + Z_\nu d\nu}{Z} \tag{31}$$

where $Y_h = \frac{(1+\delta)(1-\tau)}{\delta} \geq 0$, $Z_h = Y_h Bh^\delta = \frac{(1+\delta)(1-\tau)}{\delta}Bh^\delta \geq 0$ and $Y_\nu = Z_\nu = \frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha+\tau(1-\alpha)](1+n)} \geq 0$.

From an initial equilibrium $\nu = \tilde{\nu}$, $Z = (1-h)\,Bh^\delta Y$, we have:

- $\frac{Y_h}{Y} - \frac{Z_h}{Z} = \frac{Y_h}{Y}\left(1-\frac{1}{1-h}\right) \leq 0 \Rightarrow \left.\frac{\partial x^*}{\partial h}\right|_{\nu=\tilde{\nu}} \leq 0$
- $\frac{Y_\nu}{Y} - \frac{Z_\nu}{Z} = \frac{Y_\nu}{Y}\left(1-\frac{1}{(1-h)Bh^\delta}\right) \geq 0 \Rightarrow \left.\frac{\partial x^*}{\partial \nu}\right|_{\nu=\tilde{\nu}} \geq 0.$

The form of the differential (31) is then

$$dx^* = adh - bd\nu \tag{i}$$

where $a \geq 0$ and $b \geq 0$.

By replacing the interest rate equation (28) in the formation equation (26), we have:

$$h = h^0 + \frac{\beta(1-\alpha)\delta\left[(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu\right]}{(1+\delta)\left[(1+\beta)\alpha + \tau(1-\alpha)\right](1+n)\left[x^* Bh^\delta + 1 - x^*\right]} \tag{32}$$

Differenciating this equation, we get:

$$dh = -cdh - ed\nu + f\left[\nu - \tilde{\nu}\right]dx^* \tag{ii}$$

where $c \geq 0$, $e \geq 0$ and $f \geq 0$.

We then conclude, with (i) and (ii), for an initial equilibrium $\nu = \tilde{\nu}$, $\frac{dh}{d\nu} \leq 0$ and $\frac{dx^*}{d\nu} \leq 0$:

$$\frac{dg}{d\nu} \leq 0 \text{ et } \frac{dI}{d\nu} \leq 0$$