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Knowledge spillovers and the equilibrium location of vertically linked industries: the return of the black hole

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Abstract :

Using a generalised version of the Venables (1996) model, this paper explores the relative locations of two vertically linked sectors with knowledge spillovers. Analytical investigation shows that the dynamic properties of the Venables model are significantly affected by the presence of spillovers. In particular, the own-cost reduction effects at low transport costs can be so strong that runaway agglomeration phenomenon appears in a manner consistent with the "black hole" concept found in the literature. However, the assumption that because information decays over space means that these black hole dynamics are endogenous to the model and disappear when transport costs are high enough. Importantly, the location predictions obtained in simulations of the model are consistent with the empirical finding that industrials sector that benefit from spillovers are typically more agglomerated than sector that do not benefit from such spillovers.

Keywords : *knowledge spillovers, agglomeration, location of economic activity*

JEL Codes : *F12, R11, R12*

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1. Introduction

In the recent years, the concept of knowledge spillovers has become a central aspect of the urban economics and economic geography literature. Indeed, the existence of such spillovers is viewed as a possible micro-foundation for agglomeration, as emphasised by Duranton and Puga (2004) in their review of urban agglomeration models. Furthermore, the availability of large firm-level datasets and patent citations has also made this an important empirical issue, starting with the studies of Acs and Audretsch (1988) and Jaffe (1989). However, while this aspect of the field has generated a lot of interest, it is not clear that the location effects of knowledge spillovers have been made explicit. Audretsch and Feldman (2004) provide a good review of this literature, and emphasise that a lot of the research on knowledge spillovers focuses solely on explaining their link with the geography of innovation. Their conclusion is that “scholars have confirmed that knowledge spills over and that such knowledge spillovers matter in the formation of clusters and agglomeration. But to move beyond this insight much work remains to be done”. Similarly Gersbach and Schmutzler (1999), who attempt to link spillovers with the concept of optimal location, find no evidence of previous literature investigating this particular question.

A key stylised fact of location the knowledge spillover literature, visible for example in Audretsch and Feldman (1996) is that sectors that benefit from knowledge spillovers are more agglomerated than sectors that don’t. Another one, emphasised for example in Autant-Bernard (2001) is that technological proximity matters as well as geographical proximity in explaining the effect of knowledge spillovers. As was the case for Gersbach and Schmutzler (1999), however, this paper is less interested in the link between knowledge spillovers and innovation than the link between knowledge spillovers and the location of different industrial sectors in the first place. Of particular interest is the stylised fact that industries that benefit from spillovers are more concentrated than other industries. In this respect, while we will assume that knowledge spillovers are an externality that reduces the costs of production, there is no direct and explicit production of innovation or change in technology.

The aim of this paper is therefore to explore the location effect of knowledge spillovers within and between sectors on the location decisions of those sectors within the location framework of New Economic Geography (NEG)[1]. To do so, we compare the predictions of a NEG model with knowledge spillovers within and between sectors to a benchmark situation where knowledge spillovers are absent. Because it allows the interaction of sectors with different cost and demand characteristics, the benchmark spillover-free model that is used in this paper is the two-sector model developed in Venables (1996). The interpretation we make of the Venables model is that of an upstream sector that provides producer services to a downstream sector, which itself produces the final good. While this is of course a simplification, an illustration of this could be an R&D sector providing producer services to down-stream customers. Because the upstream sector in the Venables model only uses labour, we postulate that the existence of economically relevant knowledge can change productivity of labour in the upstream sector, and therefore the location decisions. The central finding is that the presence of knowledge of spillovers changes the analytical properties of the functions of the model compared to a benchmark situation where knowledge spillovers are absent. The economic interpretation of this analytical shift is that the explicit presence of cost-reducing knowledge spillovers modifies optimal location of sectors in several ways, including the appearance of endogenous "black holes".

[1] Gersbach and Schmutzler (1999) investigate the location aspects of spillovers and innovation in a game-theoretical model of competition between multi-plant firms, so their findings are not directly comparable.

It is important to explain from the start that for reasons of tractability, both the analytical and numerical work on the model is carried out in a partial equilibrium framework. The distribution of labour and wages over regions are assumed to be given and are not determined within the model. A way of picturing this would be to assume that the two vertically linked sectors are small with respect to the rest of the economy. This assumption is made because our central aim is to determine the effect of spillovers on the agglomeration and dispersion forces between the upstream and downstream sectors, and not to predict the overall geographical equilibrium of an economy.

The rest of the paper is structured as follows. Section 2 reviews the Venables (1996) model and evaluates its properties. Section 3 introduces knowledge spillovers and shows how the explicit presence of spillovers changes both the analytical and economic properties model. Section 4 briefly presents the numerical methodology used to get around the analytical problems created by the introduction of spillovers. The simulations of the model, generated using this methodology, are presented in section 5 and finally section 6 concludes.

2. Bijective property of the vertically linked Venables model

The Venables (1996) model uses two economic sectors, one of which is the upstream supplier of the other, which produces exclusively final goods. The main prediction of the model is that the existence of vertical linkages creates agglomeration forces that can explain agglomeration even when the inputs to production, such as labour, are immobile over space.

In terms of notation, throughout the rest of the paper, upper indices r and m are used to differentiate the two economic sectors in the model. Where vertical linkages are used, r denotes variables relating to the upstream “R&D” sector variables, and m the downstream "manufacturing" sector. A relation with a star as an upper index is meant to be valid for both sectors.

The modelling approach of this model uses the relative values of the model variables. In the partial equilibrium version these are the relative value of the sectors outputs v^*, the relative costs over regions ρ^*, and the relative expenditures η^* on those sectors. Relative wages ω are exogenously set to one in the partial equilibrium version, but can be made to be endogenous in a general equilibrium version of the model. The justification behind the use of such relative variables is that they simplify the analysis, as the evolution of the model variables over two regions is described by a single equation. This in turn simplifies the analysis of stability, as a phase diagram can be used, as seen in Figure 1. The relative value of output for both sectors is:[2]

$$v^* = \frac{\eta^*\left(\left(\rho^*\right)^{\sigma} - \tau^{\sigma-1}\right) + \left(\rho^*\right)^{\sigma} - \tau^{1-\sigma}}{\eta^*\left(\left(\rho^*\right)^{-\sigma} - \tau^{1-\sigma}\right) + \left(\rho^*\right)^{-\sigma} - \tau^{\sigma-1}}, \text{ or } \quad v^* = f^*\left(\eta^*, \rho^*, \tau\right) \tag{1}$$

It is important to note that restrictions have to be imposed on the relative costs ρ^*, for all sectors. As in Venables (1996), this is to ensure that the relative value function does not become negative, which would imply that one region has a negative number of firms. This restriction, which will be hold throughout the paper, is:

$$\tau^{\frac{1-\sigma}{\sigma}} < \rho^* < \tau^{\frac{\sigma-1}{\sigma}} \tag{2}$$

In order to close the model, and express the value of one sector as a function of the value of the other, the following general cost and demand linkages need to be defined.

$$\begin{cases} \rho^m = g^m\left(\omega, v^r, \tau\right) \\ \rho^r = g^r\left(\omega, v^m, \tau\right) \end{cases} \quad \text{and} \quad \begin{cases} \eta^m = h^m\left(\omega, v^r\right) \\ \eta^r = h^r\left(\omega, v^m\right) \end{cases}$$

[2] The equilibrium level equations are given in appendix, as well as the definitions of the relative variables. For a more complete derivation of the model, the reader is referred to Venables (1996)

As explained in the introduction, this is where the sectors will differ from each other, as different linkage structures can be assumed. In the specific case of the Venables model, the upstream sector *r* is the sole supplier of intermediate inputs to the downstream sector *m*, which itself is the sole supplier of final goods to the consumers. The downstream sector therefore only has a backward linkage and the upstream sector only a forward linkage. The specific linkages are:

$$\begin{cases} \rho^m = g^m\left(\omega, v^r, \tau\right) \\ \rho^r = \omega \end{cases} \quad \text{and} \quad \begin{cases} \eta^m = \omega \\ \eta^r = v^m \end{cases}$$

In the Venables model, the cost linkage g^m is given by:

$$\rho^m = \omega^{1-\mu}\left(\frac{v^r\left(\rho^r\right)^{-\sigma} + \tau^{1-\sigma}}{v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma} + 1}\right)^{\frac{\mu}{1-\sigma}} \tag{3}$$

However, more general cases could be imagined where both sectors use each other's output as intermediate inputs, as well as being consumed by final consumers. As is shown below, this does not change the bijective property of the model.

Figure 1 shows the model relationships for a range of transport costs. The full line gives the optimal location of the upstream sector v^r and the broken line the optimal location of the downstream sector v^m.[3] This graphical approach to the model gives an intuitive way of understanding why at least one of the functions needs to be invertible. Indeed, in Figure 1, given the choice of axes, the curve that gives v^r given v^m is actually that of the inverse function $\left(f^r\right)^{-1}$, as the dependant variable is on the horizontal axis, and the independent one on the vertical axis. Graphically, this is a trivial operation, as it simply involves symmetry about the 45° line. This serves, however, as an illustration of the analytical process used to solve for the equilibrium points. Once η^r, η^m, ρ^r and ρ^m are substituted into the relative

[3] This convention is used for all the simulation figures throughout this paper.

functions, finding the analytical solutions involves either equalising $v^r = f^r\left(\omega, v^m, \tau\right)$ and $v^r = \left(f^m\right)^{-1}\left(\omega, v^m, \tau\right)$, solving for v^m then v^r; or equalising $v^m = \left(f^r\right)^{-1}\left(\omega, v^r, \tau\right)$ and $v^m = f^m\left(\omega, v^r, \tau\right)$, solving for v^r then v^m. In either case, one inverse function needs to be defined. It is possible to show, however, that this condition always holds in the Venables (1996) model.

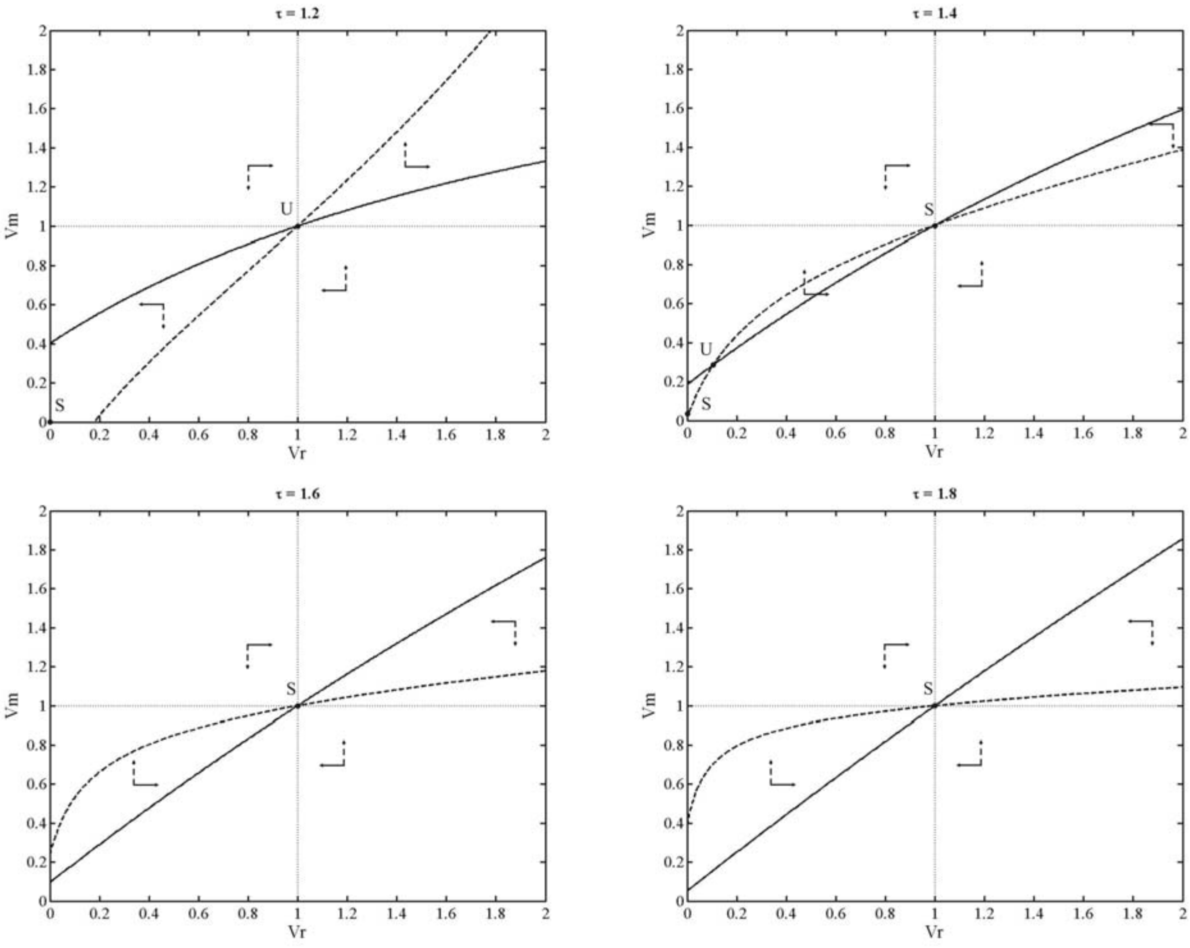


Figure 1: Relative values of the upstream and downstream sectors Benchmark Venables (1996) model, no spillovers

Proposition 1: *The functions* f^m *and* f^r *are both invertible for* $\tau > 1$

Proof: f^m and f^r are invertible if and only if they are bijective. A sufficient condition for functions f^m and f^r to be bijective is that they be strictly monotonic. Using the chain rule on (1) for both sectors, one finds the slopes of the relations as:

$$\frac{dv^m}{dv^r} = \frac{df^m\left(\eta^m, \rho^m, \tau\right)}{dv^r} = \frac{\partial f^m}{\partial \eta^m}\frac{\partial \eta^m}{\partial v^r} + \frac{\partial f^m}{\partial \rho^m}\frac{\partial \rho^m}{\partial v^r} \tag{4}$$

$$\frac{dv^r}{dv^m} = \frac{df^r\left(\eta^r, \rho^r, \tau\right)}{dv^m} = \frac{\partial f^r}{\partial \eta^r}\frac{\partial \eta^r}{\partial v^m} + \frac{\partial f^r}{\partial \rho^r}\frac{\partial \rho^r}{\partial v^m} \tag{5}$$

For the parameter values allowed in the model, the total differentials (4) and (5) are strictly positive for $\tau > 1$ and zero for $\tau = 1$.[4] For $\tau > 1$, both functions are therefore monotone-increasing over the domain, given the allowable parameter values. They are therefore bijective and invertible. □

This is also true for the specific case of the Venables model, where a clear separation between upstream and downstream sectors simplifies the total derivatives to:

$$\frac{df^m\left(\omega, \rho^m, \tau\right)}{dv^r} = \frac{\partial f^m}{\partial \rho^m}\frac{\partial \rho^m}{\partial v^r} \quad \text{and} \quad \frac{df^r\left(\omega, \eta^r, \tau\right)}{dv^m} = \frac{\partial f^r}{\partial \eta^r}\frac{\partial \eta^r}{\partial v^m}$$

This bijective property of the relative value functions has two main implications. The first, which is mentioned above, is of a purely technical nature. Within the framework above, it is always possible to invert the relative value functions and obtain analytical solutions for the partial equilibrium for two interdependent sectors in an economy. The second and more important implication, however, is of an economic nature. The bijective nature of the relations means that given a relative value of output for one sector, there is a single optimal relative value for the output of other sector. Furthermore, if one postulates an adjustment process for both sectors where the relative number of firms, and therefore v^*, responds positively to deviations in profits, deviations from these curves are always corrected dynamically.[5]

[4] The partial derivatives from (4) and (5) as well as their signs are detailed in the appendix.
[5] This point is shown in Figure 2 below.

3. Black hole dynamics in a vertically linked model with spillovers

Having explicited the structure of the Venables (1996) model, we now explore how this changes if knowledge spillovers are allowed within and between sectors. Importantly, in order to simplify the effect of spillovers, the linkages used in the spillover model are identical to the ones of the Venables model, in other words the upstream sector is a service provider to the downstream sector. Downstream firms then use labour and the upstream services to produce the final good. The linkage structure is therefore simpler than the more general cross-sectoral linkages analysed above. It is assumed as a further simplification that only the upstream sector, which provides producer services to the downstream sector, experiences cost-reductions through knowledge spillovers.[6] Assuming the existence of sector-specific economic knowledge K^m and K^r, the cost function for the upstream sector is changed to:

$$C_i^r = w_i \left(K_i^m\right)^{-\delta^m} \left(K_i^r\right)^{-\delta^r} \qquad (6)$$

It is assumed that the existence of relevant knowledge in a region reduces the unit cost of production of the upstream sector in that region. Two different types of knowledge spillovers are allowed. The first is intra-sectoral, where the spillovers originate from knowledge from other upstream sector firms. The second is inter-sectoral, and this here it is knowledge from downstream customers that reduces the production costs. The important aspect of these spillovers is that this cost reduction is an externality. The information is not paid for, and by its existence simply reduces the input requirements for a given level of output in the upstream sector, in other words it increases the productivity. Furthermore, the introduction of the two spillover channels is done using dummies δ^r and δ^m. These take value

[6] As mentioned in introduction, one can imagine that the upstream sector is a high tech R&D sector that produces an intermediate producer service for a downstream "manufacturing" sector. In this case, the knowledge spillovers are assumed to only benefit the R&D sector.

1 or 0 to determine the presence or absence of a particular spillover channel. If both are set to zero, then the model reverts to the one described on Venables (1996)

The amount of sectoral knowledge available in a region is assumed to be equal to the sum of the value of output in each region, multiplied by the transport cost between regions:

$$K_i^* = n_i^* p_i^* + n_j^* p_j^* \tau^{1-\sigma} \tag{7}$$

Although this specification is a simplification compared to more detailed knowledge production functions, it does capture the fact that economic knowledge is a positive function of the number of agents and of the value of their production, as well as the fact that economic knowledge decays with distance. Furthermore, this choice of specification means that relative knowledge across regions can be expressed in terms of the relative values v^*, thus closing the model in a straightforward way. The final benefit of this approach is that the relative costs do not depend on the proportion of knowledge that is actually useful. Even if generally only a proportion of the knowledge φK_i is useful in reducing costs in the upstream sector, this does not affect the cost function in relative terms, as the φ terms will cancel out. Indeed, in relative terms the upstream cost linkage is:

$$\rho^r = \omega \left(\kappa^m\right)^{-\delta^m} \left(\kappa^r\right)^{-\delta^r} \tag{8}$$

$$\text{With } \kappa^* = \frac{K_i^*}{K_j^*} = \frac{v^* + \tau^{1-\sigma}}{v^* \tau^{1-\sigma} + 1}$$

The downstream cost linkage ρ^m, as well as the two demand linkages η^r and η^m are unchanged from the Venables model in Section 2. The downstream cost ρ^m now indirectly depends, however, on the spillovers, as it is a function of the downstream cost ρ^r. One can see again here that if the two dummies δ^r and δ^m are set to zero, in other words if none of the spillover channels are active, the model reverts to the Venables case studied above with $\rho^r = \omega$. This means that the diagrams in Figure 1 serve as the benchmark case when both the dummies are zero.

Although this transformation of the Venables model to include spillovers in the upstream sector is not a large one, it is possible to show that this changes the model properties significantly. In particular, explicitly including knowledge spillovers in a model significantly modifies the dynamic adjustment of the location decision

Proposition 2: *If internal spillovers exist in at least one cost function* ρ^* *the functions* f^r *and* f^m *are not generally invertible over their domains.*

Proof: As for the approach used for the proof of Proposition 1, this involves determining the sign of the total derivatives of the relative value functions, in order to evaluate their monotonicity and bijective nature. With spillovers present in sector *r*, the upstream and downstream cost linkages are now functions of both upstream and downstream relative values:

$$\begin{cases} \rho^m = g^m\left(\omega, v^r, v^m, \tau\right) \\ \rho^r = g^r\left(\omega, v^r, v^m, \tau\right) \end{cases}$$

This in turn means that f^m and f^r are functions of both v^m and v^r, which complicates the determination of the derivatives. The simplest way of finding the total derivatives without having to substitute and re-arrange is to define the following implicit function for both sectors:

$$F^*\left(v^*, \eta^*, \rho^*, \tau\right) = 0 \qquad \text{as} \qquad v^* - \frac{\eta^*\left(\left(\rho^*\right)^{\sigma} - \tau^{\sigma-1}\right) + \left(\rho^*\right)^{\sigma} - \tau^{1-\sigma}}{\eta^*\left(\left(\rho^*\right)^{-\sigma} - \tau^{1-\sigma}\right) + \left(\rho^*\right)^{-\sigma} - \tau^{\sigma-1}} = 0 \tag{9}$$

Using implicit function differentiation and the chain rule one can recover derivatives of f^m and f^r that take into account their dependence on both v^m and v^r:

$$\frac{dv^m}{dv^r}=\frac{-\frac{\partial F^m}{\partial \eta^m}\frac{\partial \eta^m}{\partial v^r}-\frac{\partial F^m}{\partial \rho^m}\frac{\partial \rho^m}{\partial v^r}}{\frac{\partial F^m}{\partial v^m}+\frac{\partial F^m}{\partial \eta^m}\frac{\partial \eta^m}{\partial v^m}+\frac{\partial F^m}{\partial \rho^m}\frac{\partial \rho^m}{\partial v^m}} \tag{10}$$

$$\frac{dv^r}{dv^m}=\frac{-\frac{\partial F^r}{\partial \eta^r}\frac{\partial \eta^r}{\partial v^m}-\frac{\partial F^r}{\partial \rho^r}\frac{\partial \rho^r}{\partial v^m}}{\frac{\partial F^r}{\partial v^r}+\frac{\partial F^r}{\partial \eta^r}\frac{\partial \eta^r}{\partial v^r}+\frac{\partial F^r}{\partial \rho^r}\frac{\partial \rho^r}{\partial v^r}} \tag{11}$$

These expressions can be simplified. Indeed, one can see from the structure of the implicit functions F^* given by (9) that for both sectors:

$$\frac{\partial F^*}{\partial \eta^*}=-\frac{\partial f^*}{\partial \eta^*},\quad \frac{\partial F^*}{\partial \rho^*}=-\frac{\partial f^*}{\partial \rho^*},\ \text{and}\ \frac{\partial F^*}{\partial v^*}=1$$

Furthermore, given that as a simplification the demand and cost linkages are assumed to be the same as in Venables (1996), we have:

$$\frac{\partial \rho^r}{\partial v^m}=0,\ \frac{\partial \eta^m}{\partial v^r}=0\ \text{and}\ \frac{\partial \eta^*}{\partial v^*}=0$$

The total derivatives of the two functions f^m and f^r are therefore given by:

$$\frac{dv^m}{dv^r}=\frac{\frac{\partial f^m}{\partial \rho^m}\frac{\partial \rho^m}{\partial v^r}}{1-\frac{\partial f^m}{\partial \rho^m}\frac{\partial \rho^m}{\partial v^m}} \tag{12}$$

$$\frac{dv^r}{dv^m}=\frac{\frac{\partial f^r}{\partial \eta^r}\frac{\partial \eta^r}{\partial v^m}}{1-\frac{\partial f^r}{\partial \rho^r}\frac{\partial \rho^r}{\partial v^r}} \tag{13}$$

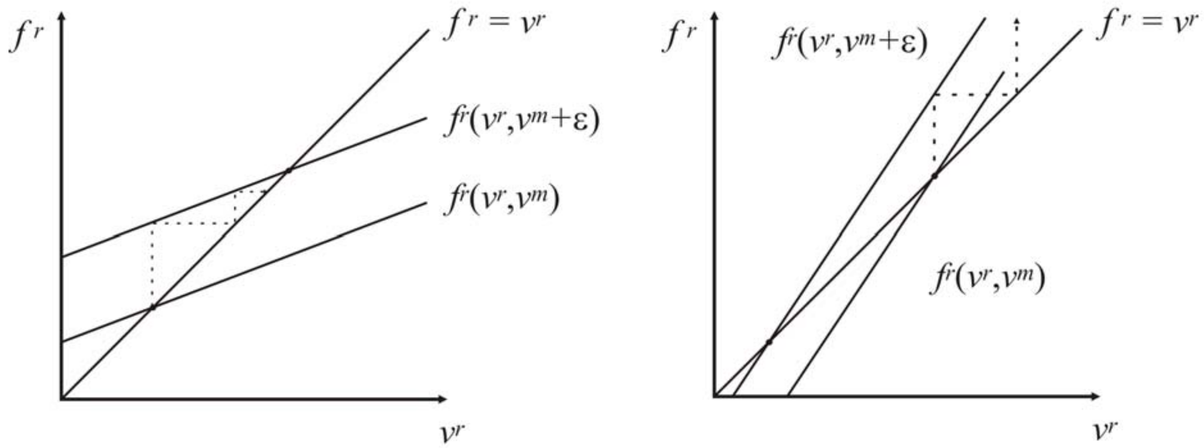
Because the demand linkages and the functions f^m and f^r are still the same as the ones in the Venables model, the partials $\partial f^m / \partial \rho^m$, $\partial f^r / \partial \eta^r$ and $\partial \eta^r / \partial v^m$ are the same as in the previous section and therefore have the same signs. Furthermore, as one would expect intuitively, $\partial \rho^m / \partial v^r$ stays negative in the presence of spillovers which means the numerators of the two derivatives are strictly positive for $\tau > 1$, and the sign of the total derivatives depends on the sign of the denominator. However, $\partial f^* / \partial \rho^* < 0$ and $\partial \rho^* / \partial v^* < 0$,[7] so the sign of these denominators of are indeterminate. The relative value functions are therefore not generally bijective, and the inverse functions are no longer defined over the domains of the relative value functions. □

Compared with the previous case, these total differentials take into account the existence of a positive own-cost linkage $\partial f^* / \partial v^*$, which acts as a multiplier to the derivatives found in the previous section for the Venables model. If no spillovers are present, then for both upstream and downstream sector, $\partial \rho^* / \partial v^* = 0$ and as expected the total derivatives reduce to the ones found in Section 2. Depending on the strength of the spillovers several situations are possible. If $\partial f^* / \partial v^* < 1$, then the denominator and the overall slope of the function remains positive, and the qualitative properties of the model are not affected. As one approaches the critical points where $\partial f^* / \partial v^* = 1$ and the denominator tends to zero, the slope becomes infinite, and is not defined at all at the critical point itself.

As the simulations in Section 5 will illustrate, the situation of interest appears when the own-cost effect $\partial f^* / \partial v^*$ is larger than one. Indeed, at this point, the slopes of the relative value function (12) and (13) will be negative. This appears inconsistent with the intuitive notion that the location choices of the vertically linked sectors are reinforced by knowledge spillovers within and between sectors. This apparent inconsistency can be rationalised by examining the dynamic adjustment of the phase diagram with respect to the downward sloping curve. Figure 2 presents an intuitive decomposition of the implicit value function F^r in (9), where the equilibrium point for a given value of is v^m defined as the intersection of f^r with the 45° line.[8]

[7] The details of these derivations are given in the appendix.
[8] Figure 2 shows a linear f^r This is simply for purposes of illustration, as of course f^r is non-linear in v^r.

Figure 2: Dynamic adjustment of the spillover model



If $\partial f^* / \partial v^* < 1$ If $\partial f^* / \partial v^* > 1$

As in Venables (1996), we postulate a dynamic adjustment process whereby the relative value v^r in a region increases through firm entry if v^r is below the optimal value given by f^r. For the case where $\partial f^* / \partial v^* < 1$, a positive shock to v^m increases the equilibrium value of v^r. This implies an upward sloping value function. Furthermore, following this shock the adjustment process brings the system back to equilibrium. If $\partial f^* / \partial v^* > 1$ however, a positive shock to v^m will decrease the equilibrium value of v^r. This is consistent with the negative slope of the value functions mentioned above. However, the adjustment process shows that in this case the shock will be dynamically amplified and not corrected.

The negative slopes seen in the simulations below therefore critically depend on $\partial f^* / \partial v^* > 1$. This means that when the slopes changes sign, so does the direction of dynamic adjustment. While a downward sloping relative value curve still describes the optimal zero-profit location of a sector, this location becomes a knife edge. Any disturbance to the relative value of that sector's output leads to a more than proportional increase in the optimal relative value predicted by the function f^*.

This situation is in fact an illustration of a "black hole", as defined in Fujita *et al* (1999). Indeed, Ishiguro (2005) shows that in general in NEG models, the "no black hole condition" requires profits and real wages in a region to decrease in response to increases in the number of firms and workers respectively. While the partial equilibrium assumptions on wages means that the labour market aspect cannot be verified, Figure 2 reveals that the $\partial f^* / \partial v^* > 1$ case clearly violates this condition, as increases in the relative value, through increases in firm mass, bring more than proportional increases in profits. This effect depends, however, on the spilling over of knowledge that decays over space and not on specific model parameters such as σ or μ. Therefore, contrary to the "black hole" situations of other NEG models, the black hole dynamics are endogenous to the model, and not present for all transport costs.

4. Numerical methodology

In order to provide an illustration of the effects discussed in the previous section, a simulation of the vertically linked model with spillovers was carried out, allowing for the various channels through which spillovers might occur. In order to get around the potential lack of invertibility, the value relative functions f^* for both sectors are determined numerically by calculating the size of the implicit functions F^* for all the possible values of the relative values v^*, and identifying the points that satisfy the $F^* = 0$ condition.

In practical terms, the first step involves constructing a discrete state vector for the relative value variables v^*, by choosing an upper bound for the variables v^* as well as a grid resolution n which gives the size of the vector. From this, a square $n \times n$ matrix V is constructed, where the n rows are all identical and equal to the v^* vector. *V'* is the transpose of this matrix, and therefore it is the n columns that are identical and equal to the v^* vector. V is used here to represent all the possible values that v^r can take, and *V'* all the values of v^m.

Using these two matrices, it is possible to calculate the space of cost linkages P, which are themselves square $n \times n$ matrices containing all the possible levels of relative costs within the bounds of the vector space:

$$\mathrm{P}_{i,j}^{r} = \omega \left(\frac{V'_{i,j} + \tau^{1-\sigma}}{V'_{i,j}\tau^{1-\sigma} + 1} \right)^{-\delta^{m}} \left(\frac{V_{i,j} + \tau^{1-\sigma}}{V_{i,j}\tau^{1-\sigma} + 1} \right)^{-\delta^{r}} \tag{14}$$

$$\mathrm{P}_{i,j}^{m} = \omega^{1-\mu} \left(\frac{V_{i,j}\left(\mathrm{P}_{i,j}^{r}\right)^{-\sigma} + \tau^{1-\sigma}}{V_{i,j}\left(\mathrm{P}_{i,j}^{r}\right)^{-\sigma}\tau^{1-\sigma} + 1} \right)^{\frac{\mu}{1-\sigma}} \tag{15}$$

All the possible values of the implicit functions F^{*} can be calculated using these cost linkages and the V and V' matrices. Keeping in mind the demand linkage assumptions that $\eta^{m} = \omega = 1$ and $\eta^{r} = v^{m}$, these are:

$$F_{i,j}^{r} = V_{i,j} - \frac{V'_{i,j}\left(\left(\mathrm{P}_{i,j}^{r}\right)^{\sigma} - \tau^{\sigma-1}\right) + \left(\mathrm{P}_{i,j}^{r}\right)^{\sigma} - \tau^{1-\sigma}}{V'_{i,j}\left(\left(\mathrm{P}_{i,j}^{r}\right)^{-\sigma} - \tau^{1-\sigma}\right) + \left(\mathrm{P}_{i,j}^{r}\right)^{-\sigma} - \tau^{\sigma-1}} \tag{16}$$

$$F_{i,j}^{m} = V'_{i,j} - \frac{\left(\left(\mathrm{P}_{i,j}^{m}\right)^{\sigma} - \tau^{\sigma-1}\right) + \left(\mathrm{P}_{i,j}^{m}\right)^{\sigma} - \tau^{1-\sigma}}{\left(\left(\mathrm{P}_{i,j}^{m}\right)^{-\sigma} - \tau^{1-\sigma}\right) + \left(\mathrm{P}_{i,j}^{m}\right)^{-\sigma} - \tau^{\sigma-1}} \tag{17}$$

Within the $n \times n$ state space, the values of interest are the ones where original implicit function condition is satisfied and hence $F_{i,j}^{*} = 0$. For each sector, the locations of the zero points of the implicit functions are therefore given by $\min\left|F^{*}\right|$.

5. Simulation predictions

Figures 3 to 5 below show the predictions of the model for all the three possible configurations of the possible spillover channels. As mentioned in Section 3, the main feature of these figures compared to the benchmark case in Figure 1 is that when a sector's knowledge spills over, the relative value function for that sector tends to be downward-sloping for low levels of the transport cost. The important aspect, however, is that in that case the dynamic adjustment amplifies deviations from the relative value curve instead of reducing them.

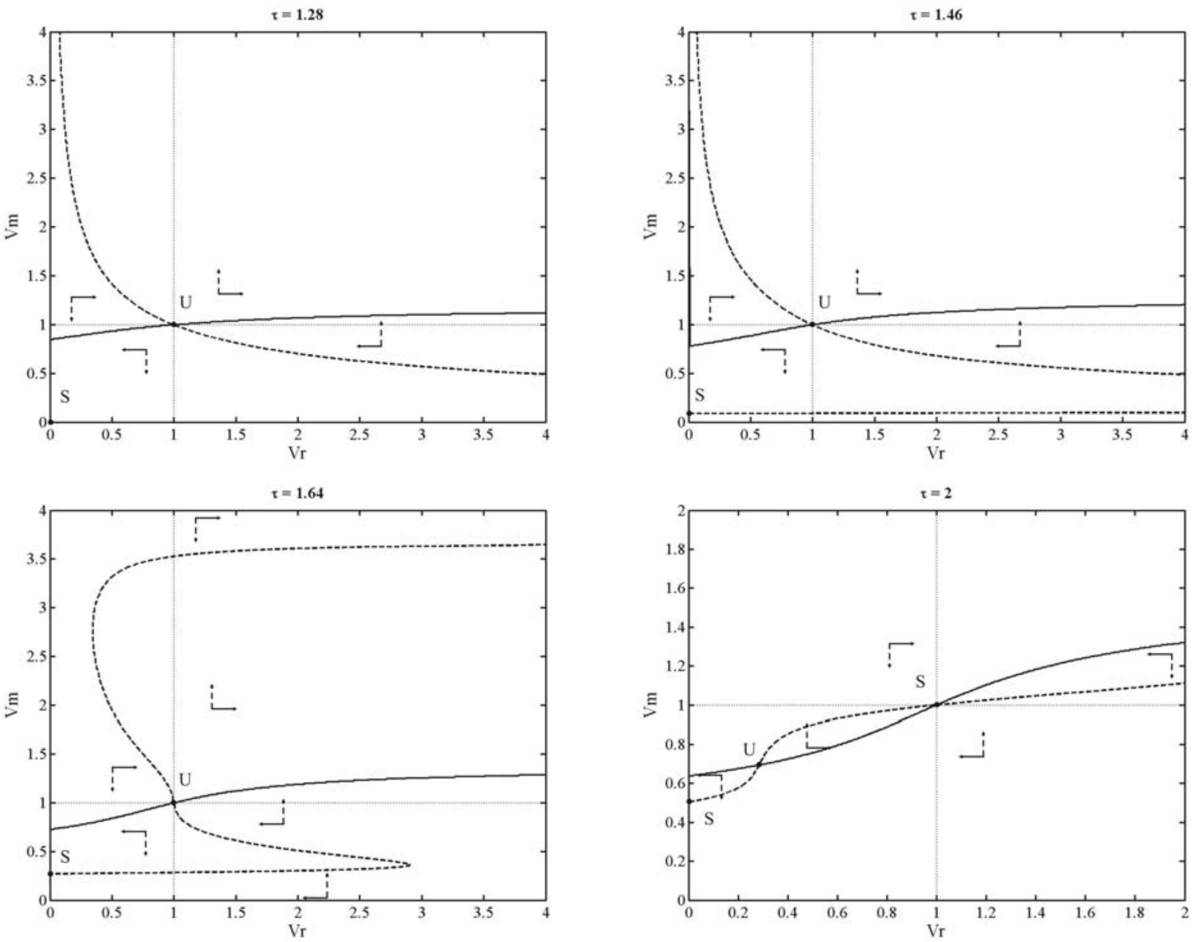


Figure 3: Relative values of the upstream and downstream sectors
Only downstream knowledge spills over ($\delta^m = 1$, $\delta^r = 0$)

Figure 3, first of all, presents the simulations for the case where downstream knowledge spills over to the upstream sector, resulting from interactions with downstream customers. For low transport costs, the value function for the downstream sector is strictly downward-sloping. While the spillovers are located in the upstream sector, this occurs because the downstream sector does benefit indirectly from these spillovers through cheaper intermediate inputs. As explained previously, this situation is an illustration of the black hole effect: the symmetric equilibrium is unstable, and the only other stable equilibrium is a full agglomeration of both sectors in a single region. For high values of the transport cost, however, the downstream value function is increasing. Both the symmetric equilibrium and an agglomerated one are stable, with two interior equilibria that are unstable.

Indeed, at intermediate values of the transport cost, the downstream relative value function flips over, and becomes again an increasing function. In doing so, however, it produces an illustration of the invertibility problem mentioned in the previous section. Clearly the function $v^m = f\left(v^r, v^m\right)$ ceases to be bijective, and several values of v^m can be optimal for a given v^r. As mentioned in the analytical discussion above, one can also see the existence of several branching points for which the slope of the curve is infinite, and in between these points the direction of the dynamic adjustment will depend on the slope of the curve itself. Interestingly, this branching creates a situation where the only stable equilibrium is one where the upstream sector, which benefits from spillovers, is completely agglomerated while the downstream sector, while still very concentrated, is more equally spread over regions.

Figure 4 shows the predictions for the opposite case, where knowledge spills over only within the upstream sector. Compared to predictions for inter-sectoral spillovers in Figure 3, this time it is the upstream relative value function that is downward-sloping for low levels of the transport cost and causes a black hole effect. As for Figure 3, the relation becomes upward-sloping again at high levels of transport costs. In terms of stability, the predictions are also similar. The dispersed, symmetric equilibrium is unstable for low levels of transport costs, and full agglomeration occurs. At high levels of transport costs, the symmetric equilibrium and an agglomerated equilibrium are stable. Two intermediate equilibria also exist at this point due to the branching of the upstream value function. Given the downward sloping nature of the upstream value function around these equilibria, they are unstable. This was also the case for the partially agglomerated equilibria in Figure 2.

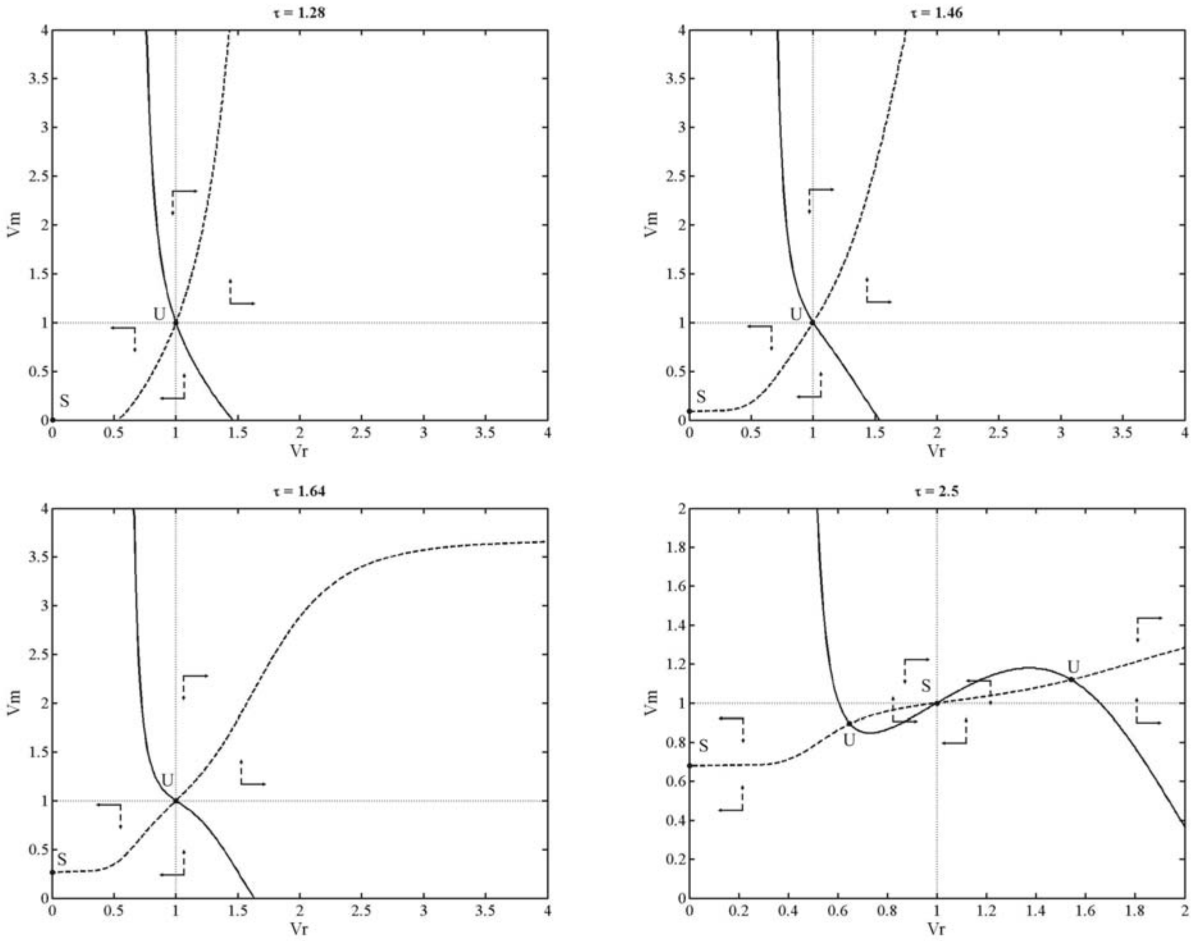


Figure 4: Relative values of the upstream and downstream sectors
Only upstream knowledge spills over ($\delta^m = 1$, $\delta^r = 0$)

Again, this transition is accompanied by a multi-valued branching of the upstream value function. The difference is that this branching of the f^r function occurs at a much higher level of transport costs than that of the f^m function in Figure 3. This is because of the type of vertical linkages we have assumed. In the case of manufacturing spillovers seen in Figure 3, the manufacturing firms effectively incur the transport costs twice when they are separated from R&D firms: the first time when the manufacturing knowledge decays as it reaches the R&D firms in the other region, the second time when they have to import the R&D inputs from the other region. With inter-sectoral spillovers, the location of manufacturing relative to R&D is sensitive to the level of transport costs. In Figure 3, however, because knowledge spillovers are intrasectoral and R&D firms are assumed not to use any intermediate inputs, their location decisions are therefore less sensitive to transport costs. Compared to Figure 3, while the qualitative properties are similar, it takes a much higher level of transport costs to eliminate the black hole effect of spillovers. Importantly, as for Figure 3 one also finds here the situation where the upstream sector is completely agglomerated while the downstream sector is more spread out for intermediate values of the transport costs.

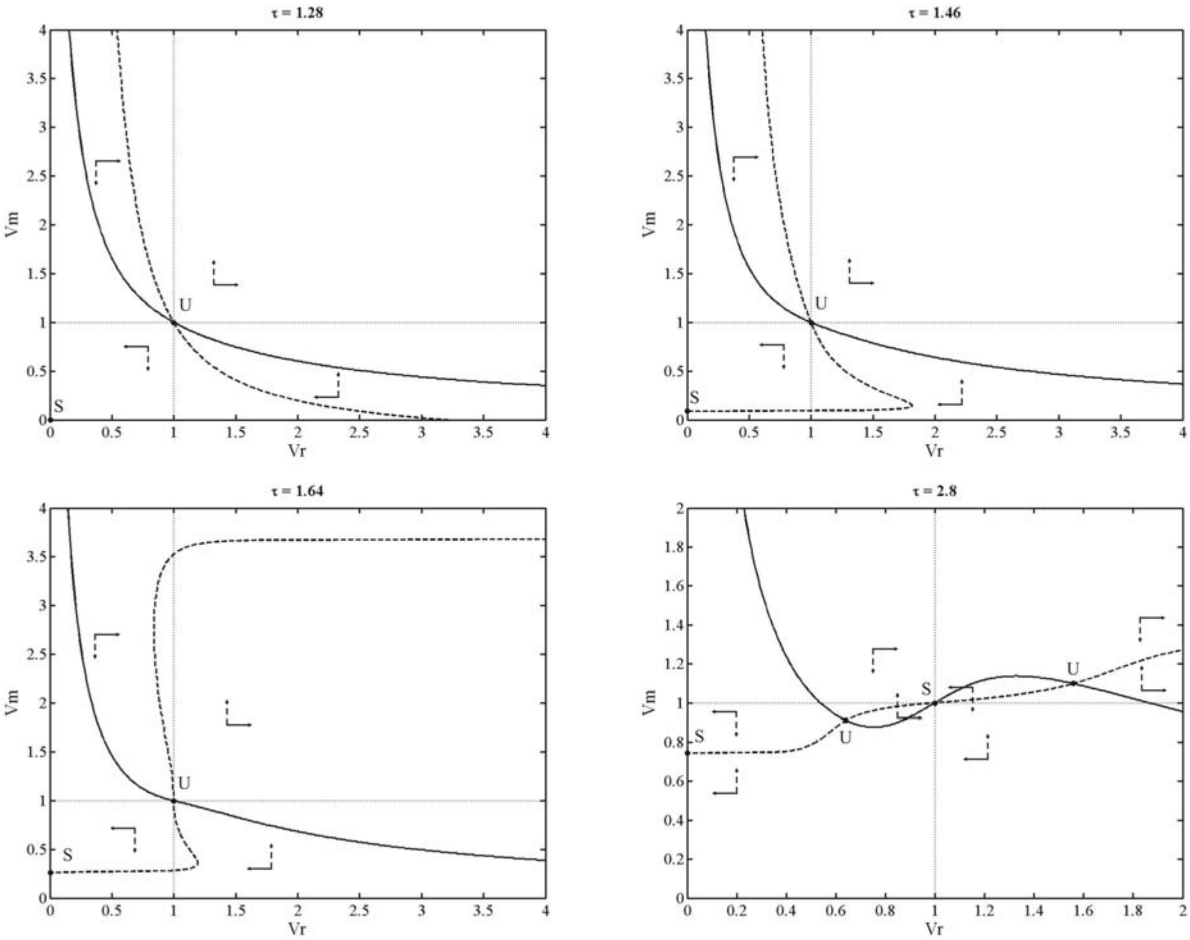


Figure 5: Relative values of the upstream and downstream sectors
Upstream and downstream spillovers ($\delta^m = 1$, $\delta^r = 1$)

Finally, Figure 5 shows the predictions for the relative value functions when both upstream and downstream spillover channels are active. As is to be expected, the location predictions contain elements from both the purely upstream and downstream spillover cases shown above. The general behaviour of the f^m curve is similar to the predictions of Figure 3, whereas the predictions for f^r mirror Figure 4. Consistently with the previous findings, both the relative value curves are now decreasing for low levels of transport costs, and deviations away from both of the curves are amplified instead of corrected. As transport costs increase the f^{m} curve, followed by the f^{r} relation, become increasing around the symmetric equilibrium. As explained previously, because of the linkages we have assumed, the manufacturing sector is more sensitive to information decay over distances than the R&D sector. Unsurprisingly, for low levels of transport costs, the effect of the two decreasing relative value curves is a stable agglomeration of both sectors in a single region. Again, the symmetric equilibrium is stable at high values of the transport cost, there are two unstable interior equilibria. Consistent with the simulations shown in Figures 3 and 4, there is a wide range of transport costs for which the only stable equilibrium is one where the upstream sector is totally agglomerated and the downstream one is not.

At this point it is important to summarise and discuss these findings. Knowledge spillovers create a runaway agglomeration force between sectors for low levels of the transport costs. The analytical cause, the own value effect $\partial f^{*}/\partial v^{*}$ being larger than 1, and the dynamic adjustment away from the equilibrium value both correspond to the concept of "black hole" mentioned in the literature. However, the existence of this effect is conditioned on the strength of the knowledge spillovers, and therefore on the transport costs which govern the geographical decay of information. At high levels of transport costs, this phenomenon disappears as expected because the decay of knowledge and goods becomes too large to sustain the agglomerative force. This suggests that the explicit introduction of knowledge spillovers allows the creation of endogenous "black hole dynamics", and that compared with the previous literature, there is no need to assume them away.

The second important aspect of these findings is that whilst the multi-valued branching complicates somewhat the analytical and technical aspects of the problem, as seen in Sections 3 and 4, the simulations in this section reveal that the qualitative properties of the model remain similar to Venables (1996). Indeed, only the symmetric and agglomerated equilibria are stable, and although interior equilibria are created by the branching problem, they always seem to be unstable. This means that the model still retains the general properties of NEG models outlined in Robert-Nicoud (2004) and Ottaviano & Robert-Nicoud (2004).

A final aspect of these results is how they relate to the empirical literature mentioned in introduction. The main stylised fact that this model of location addresses is the higher level of agglomeration that is observed in sectors that benefit from spillovers, relative to other sectors. Indeed, what stands out throughout the simulations is that for wide ranges of transport costs, the only stable equilibrium is one where the sector that benefits from the spillovers (the upstream sector) agglomerates fully while the other sector, which does not benefit from cost-reducing spillovers, does not. Not only is this location prediction consistent with the observations, but the fact that similar equilibria are found in Figures 3, 4 and 5 suggest that this effect does not depend on the sector from which the knowledge spillovers originate, but on the destination of the spillovers, in this case the upstream sector.

A second suggestion in many studies is that spillovers between sectors depend more on technological proximity than on geographical proximity. The spillover model here is greatly simplified in order to establish some theoretical predictions, and it is therefore difficult to compare these stylised predictions with the more detailed empirical analyses. Nevertheless, within the framework used in this paper, the findings of the empirical literature on knowledge spillovers suggest that the theoretical case closest to the observations is the one shown in Figure 3, where upstream knowledge spills over but not downstream knowledge. In particular, this would account for the lower empirical dependency on geographical proximity.

It is important to remember, however, that these predictions are made in a partial equilibrium framework. The central aim of the study is to identify how integrating spillovers changes the location decisions of two linked sectors, keeping all other geographical aspects constant. In particular, it is assumed as a simplification that the relative wages are fixed over regions, and do not adjust as our two sectors move around. This is a strong assumption in location terms, first of all because this variable gives the relative demand for the final good, therefore influencing the downstream sector's location choice. Secondly, one can see from the cost linkages ρ^* that both sectors are competing for inputs, labour in this case. Assuming that both sectors are large economically, their agglomeration in a single region should be matched by higher wages, and therefore higher costs. The literature suggests that it is reasonable to believe that spillovers to occur between sectors when similar skills are used in both, implying that sectors experiencing spillovers between each other do compete for inputs. Combes and Duranton (2006), in particular show more specifically that if the source of knowledge spillovers is the firm-specific knowledge that workers posses, a firm may decide not to locate in the same local labour market as its competitors, simply to avoid poaching behaviour. Because the partial equilibrium assumption means that labour markets are "frozen", this potential dispersion force is ignored here.[9]

6. Conclusion

Using a generalised version of the Venables model, this paper shows that the relative value functions describing the relative locations of 2 vertically linked sectors are always bijective. This means that it is possible to explicitly solve the system for the equilibrium values. We show that this is modified when spillovers are allowed. In particular, the value functions cease to be bijective in general. From a purely technical point of view, this implies

[9] As pointed out previously, further simplifications of the model are the fact that both sectors share the same elasticity of demand and that the dummies are set to either 0 or 1. Sensitivity analyses carried out on these parameters show that while the curves retain the same properties, these are important in determining the sensitivity to transport costs, and therefore the level of those costs at which the black hole dynamics disappear.

an increase in the number of steps required to solve the model analytically, and calls for numerical analysis of the model, which is the approach used here.

From an economic point of view, this property has several consequences. The first is that the existence of spillovers in a vertically linked economy is important in determining the effect on the location of activity. The general result is that if a sector's knowledge spills over, then for low transport costs this creates own-cost reductions so strong that runaway agglomeration is the only outcome regardless of which sector actually benefits from the knowledge spillovers. This finding is shown to be consistent with the concept of a "black hole" mentioned in the literature. Importantly, however, because these black hole dynamics depend on knowledge spillovers they are endogenous to the model and disappear when transport costs are high enough. The benefit of this is that these effects do not need to be assumed away as is the case in most of the NEG literature.

The second economic aspect of this phenomenon is that the general properties of the Venables models are not modified by the presence of these black hole dynamics. Only equilibria that are symmetric or have full agglomeration of at least one sector are stable. If interior equilibria exist, they are unstable. This means that the location predictions remain consistent with the larger body of NEG models. Crucially, these location predictions are consistent with the empirical findings: for most intermediate values of transport costs, the stable equilibrium is one where the sector that benefits from spillovers experiences more agglomeration than the sector with does not.

There are limits, however, to the approach used here. The model is voluntarily restricted and simplified in order to focus on the changes to the dispersion and agglomeration forces. Clearly, a more complete integration of spillovers is needed, in particular with a flexible wage and more sectors with richer inter-linkages, in order to better understand the interaction between spillovers and vertical linkages. A further integration of new economic geography and the knowledge spillover literature in general would also have to involve the innovation and technological processes which have been purposely left aside here.

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Appendix 1: Model equations

Level and relative equilibrium equations:

For both sectors, the equilibrium $\pi_i^* = 0$ level of output of a variety produced in region *i* is:

$$x_i^* = \left(p_i^*\right)^{-\sigma} \left(P_i^*\right)^{\sigma-1} e_i^* + \left(p_i^*\right)^{-\sigma} \left(P_j^*\right)^{\sigma-1} e_j^* \tau^{1-\sigma} = 1$$

Where the price index of the CES aggregate is $P_i^* = \left(n_i^* \left(p_i^*\right)^{1-\sigma} + n_j^* \left(p_j^* \tau\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, p_i^* is the mill price of the variety and expenditure in the i^{th} region is e_i^*. At equilibrium, zero profits and an equilibrium output of 1 ensure that costs are equal to prices. Given the linkages specified in Venables (1996) and an elasticity of downstream output w.r.t upstream inputs of μ, the benchmark cost functions are given by:

$$C_i^r = p_i^r = w_i \text{ and } C_i^m = p_i^m = w_i^{1-\mu} \left(P_i^r\right)^{\mu}$$

For the spillover extension presented in the paper the equilibrium upstream cost function is changed to:

$$C_i^r = p_i^r = w_i \left(K_i^m\right)^{\delta^m} \left(K_i^r\right)^{\delta^r}$$

With the amount of sector-specific knowledge in a region given by $K_i^* = n_i^* p_i^* + n_j^* p_j^* \tau^{1-\sigma}$.

From these equilibrium equations, the following relative variables are defined:

$$v^* = \frac{n_i^* p_i^*}{n_j^* p_j^*}, \qquad \rho^* = \frac{p_i^*}{p_j^*} = \frac{C_i^*}{C_j^*} \qquad \text{and} \qquad \eta^* = \frac{e_i^*}{e_j^*}$$

Restrictions on relative costs:

The standard relative value function is given by:

$$\frac{n_i^* p_i^*}{n_j^* p_j^*} = \frac{\eta^r \left(\left(\rho^*\right)^{\sigma} - \tau^{\sigma-1}\right) + \left(\rho^*\right)^{\sigma} - \tau^{1-\sigma}}{\eta^r \left(\left(\rho^*\right)^{-\sigma} - \tau^{1-\sigma}\right) + \left(\rho^*\right)^{-\sigma} - \tau^{\sigma-1}}$$

If $\rho^* > \tau^{\frac{\sigma-1}{\sigma}}$, then the denominator is negative and the numerator is positive. This implies that one of the regions has a negative number of firms, which makes no sense economically. On the other hand, if $\rho^* < \tau^{\frac{1-\sigma}{\sigma}}$, then it is the numerator that is negative and the denominator positive, which means that the other region has a negative number of firms. As for the original Venables (1996) model, in order to restrict the relative value functions to economically meaningful values, the following restriction is imposed:

$$\tau^{\frac{1-\sigma}{\sigma}} < \rho^* < \tau^{\frac{\sigma-1}{\sigma}}$$

Furthermore, this condition technically precludes the case where $\tau = 1$.

Appendix 2: Elements relating to the proof of Proposition 1

Partial differentials of the demand linkages:

For both sectors:

$$\frac{\partial f^*}{\partial \eta^*} = \frac{\left(\tau^{\sigma-1} - \tau^{1-\sigma}\right)\left(\left(\tau^{\sigma-1} + \tau^{1-\sigma}\right) - \left(\left(\rho^*\right)^{\sigma} + \left(\rho^*\right)^{-\sigma}\right)\right)}{\left(\eta^*\left(\left(\rho^*\right)^{-\sigma} - \tau^{1-\sigma}\right) + \left(\rho^*\right)^{-\sigma} - \tau^{\sigma-1}\right)^2}$$

The sign of this partial differential depends only on the sign of the two numerator brackets. The first bracket is strictly positive for $\tau > 1$ and equal to zero if $\tau = 1$. Given the restrictions on ρ^* in appendix 1, the second numerator bracket is also strictly positive. The partial differential is therefore strictly positive for $\tau > 1$. As one would expect intuitively, the relative value of a sector's output in a region increases with the relative expenditure on that sector in the region. Furthermore, in the Venables model, if the output of a sector is demanded by the other sector, then $\partial \eta^r / \partial v^m = 1$ and $\partial \eta^m / \partial v^r = 1$ The combined demand linkages part of $\partial f^r / \partial v^m$ and $\partial f^m / \partial v^r$ are therefore strictly positive, for all $\tau > 1$.

Partials differentials of the cost linkage:

For both sectors:

$$\frac{\partial f^*}{\partial \rho^*}=\frac{\sigma\left(\eta^*+1\right)\left(\rho^*\right)^{-1}\left[2\left(\eta^*+1\right)-\tau^{\sigma-1}\left(\left(\rho^*\right)^{\sigma}+\eta^*\left(\rho^*\right)^{-\sigma}\right)-\tau^{1-\sigma}\left(\left(\rho^*\right)^{-\sigma}+\eta^*\left(\rho^*\right)^{\sigma}\right)\right]}{\left(\eta^*\left(\left(\rho^*\right)^{-\sigma}-\tau^{1-\sigma}\right)+\left(\rho^*\right)^{-\sigma}-\tau^{\sigma-1}\right)^2}$$

The sign of this partial differential depends on sign of the term in square brackets. As for the demand linkage, given the restrictions on relative costs in appendix 1, this is always negative, which is intuitive, as the larger the relative cost of production in a region, the lower the relative value of output will be. Furthermore, if a linkage exists between an upstream and a downstream sector, the partial differential is as follows. Swapping *m* and *r* indices gives the expression for the opposite linkage.

$$\frac{\partial \rho^m}{\partial v^r}=\frac{\mu\omega^{1-\mu}}{1-\sigma}\left(\frac{v^r\left(\rho^r\right)^{-\sigma}+\tau^{1-\sigma}}{v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma}+1}\right)^{\frac{\mu}{1-\sigma}-1}\frac{\left(\rho^r\right)^{-\sigma}\left(1-\tau^{2(1-\sigma)}\right)}{\left(v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma}+1\right)^2}$$

As one would expect, the sign of this partial is negative. Hence, the larger the value of a sector in a region, the lower the production costs of the downstream sectors in that region. This is given by the presence of *1-σ*, all the other terms being positive. Importantly, if $\tau=1$, $\partial\rho^m/\partial v^r$, as $1-\tau^{2(1-\sigma)}$ will be zero. As for the demand linkages, this means that the combined differential for the cost linkage part of $\partial f^r/\partial v^m$ and $\partial f^m/\partial v^r$ are positive for all $\tau>1$.

Appendix 3: Elements relating to the proof of Proposition 2

Partial differentials of the upstream cost function:

The partial differentials of the upstream cost function $\rho^r = \omega\left(\kappa^m\right)^{-\delta^m}\left(\kappa^r\right)^{-\delta^r}$ are :

$$\frac{\partial\rho^r}{\partial v^r} = -\delta^r \omega\left(\kappa^m\right)^{-\delta^m}\left(\kappa^r\right)^{-\delta^r-1}\frac{1-\tau^{2(1-\sigma)}}{\left(v^r\tau^{1-\sigma}+1\right)^2}$$

$$\frac{\partial\rho^r}{\partial v^m} = -\delta^m \omega\left(\kappa^m\right)^{-\delta^m-1}\left(\kappa^r\right)^{-\delta^r}\frac{1-\tau^{2(1-\sigma)}}{\left(v^m\tau^{1-\sigma}+1\right)^2}$$

As expected, they are both clearly negative given the signs and values of the model parameters. Importantly, if the spillover dummies δ^r and δ^m are set to zero and no spillovers occur, then the upstream cost function reverts to the Venables model and both of these partials become equal to zero

Partial differentials of the downstream cost function:

$$\frac{\partial\rho^m}{\partial v^r} = \frac{\mu\omega^{1-\mu}}{1-\sigma}\left(\frac{v^r\left(\rho^r\right)^{-\sigma}+\tau^{1-\sigma}}{v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma}+1}\right)^{\frac{\mu}{1-\sigma}-1}\frac{\left(-\sigma\left(\rho^r\right)^{-\sigma-1}v^r\frac{\partial\rho^r}{\partial v^r}+\left(\rho^r\right)^{-\sigma}\right)\left(1-\tau^{2(1-\sigma)}\right)}{\left(v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma}+1\right)^2}$$

$$\frac{\partial\rho^m}{\partial v^m} = \frac{\mu\omega^{1-\mu}}{1-\sigma}\left(\frac{v^r\left(\rho^r\right)^{-\sigma}+\tau^{1-\sigma}}{v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma}+1}\right)^{\frac{\mu}{1-\sigma}-1}\frac{-\sigma\left(\rho^r\right)^{-\sigma-1}v^r\frac{\partial\rho^r}{\partial v^m}\left(1-\tau^{2(1-\sigma)}\right)}{\left(v^r\left(\rho^r\right)^{-\sigma}\tau^{1-\sigma}+1\right)^2}$$

Compared to the Venables model, these downstream partial differentials are now more complex, as they take into account the fact that the presence of spillovers imply the upstream cost are modified by the presence of relative values v^r and v^m. Both are clearly negative, however. Their sign is given by *1-σ*, as all other terms are positive. If there are no spillovers then one can see from above that $\partial\rho^m/\partial v^m = 0$ and the effect $\partial\rho^m/\partial v^r$ is equal to the one found previously.