Document de travail de l'OFCE

PRICE COMPETITION
ON NETWORKED DUOPOLISTIC MARKETS

N° 2011- 08
APRIL 2011

Zakaria Babutzidze

OFCE and SKEMA Business School

Price Competition
on Networked Duopolistic Markets

Zakaria Babutsidze[*]

OFCE and SKEMA Business School

Abstract

In standard Bertrand model duopolists compete on perfect markets. However, not many markets are perfect. In fact most of the markets have certain structure, and this structure is known to producers. We describe the market organization by modeling consumer-producer networks and demonstrate that if this structure is known to producers implications of the price competition depart substantially from the ones predicted by standard models. In particular we show that multiple pure strategy equilibria can emerge. We investigate the role of firm and consumer heterogeneity towards sustaining price dispersion as an equilibrium outcome. It turns out that if consumers are homogenous we need large dose of firm heterogeneity in order to sustain price dispersion in equilibrium. However, if consumers are heterogenous even small asymmetries across producers imply price dispersion. It is also predicted that larger consumer heterogeneity leads to higher prices in equilibrium.

Keywords: Price competition · networks · heterogeneity · price dispersion

JEL numbers: L12 · L13 · L14

1 Introduction

Price dispersion is a pronounced phenomenon. Several studies document it to be persistent across large number of products in many industries. Prominent examples include Janssen et al. (2004) on computer hardware, Baye et al. (2004) on large collection of consumer electronics, Barron et al. (2004) on gasoline and Wildenbeest (2009) on supermarket prices.

Price dispersion has two dimensions. Lateral, which means that prices differ across shops, and temporal, which means that prices differ across time. Economics has been careful in separating these two dimensions and has rightly modeled them

---

separately.[1] Current paper abstracts from the temporal dimensions of price dispersion and contributes to the understanding of why different firms might charge different prices in equilibrium for a homogenous good.

Usually price dispersion in equilibrium involves certain *ex ante* heterogeneity among agents. This heterogeneity can be on production side (for example in production costs, Reinganum, 1979) or on consumer side (for example in consumer search costs, Stahl, 1996). However, Burdett and Judd (1983) present a model where producers and consumers are ex ante homogenous but price dispersion can still arise. This is due to price search strategies employed by consumers that result into *ex post* heterogeneity of consumer information sets. This heterogeneity permits to sustain the price dispersion in equilibrium.

These models usually involve consumer search for information (Gastwirth, 1976). This search is costly and consumers are sampling producers at random. They always stop short of collecting full information, therefore can be tricked into paying higher than the minimal price available on the market. Therefore, these models require large number of producers. When we are on a duopolistic market, however, we can expect large number of consumers who will have complete information about all the prices available. However, there will be also consumers that will only observe only part of the prices, but sampling will not be random. For example people will observe prices quoted by the supermarket they go to, and will not observe the prices quoted by the supermarket they do not go to. And which supermarket to go to will not be drawn randomly every morning.

It is reasonable to expect that supermarket (shop/producer) choices have temporal stability. One reason to expect this is because of the commute to shops is costly and therefore people usually go to the shops that are located closer to where they live. As people do not change neighborhoods all that often, we would expect they go to the same shop(s) for extended periods of time. Therefore, it is more reasonable to describe the economic activity as buyer-seller networks. Early network models of market organization involved capacity constraints and bargaining between two connected nodes. The network in these models could have been exogenously given (Corominas-Bosch, 2004) or it could evolve endogenously over time (Kranton and Minehart, 2001).

More recent models are not concerned with bargaining, but rather model price setting sellers. These models use bipartite or even tripartite networks in order to model consumer-producer relationships. In case of bipartite networks consumers are

---

[1]For example, Reinganum (1979) is concerned by the lateral price dispersion while Varian (1980) is concerned by the temporal dispersion in prices.

linked directly to producers (Lever, 2011). In case of tripartite networks traders are incorporated into the market structure (Blume et al., 2009).

Few of these models, like Lever (2011) and Pasini et al. (2008) discuss the mixed strategy equilibria. This usually involves additional constraints in demand functions in order for the game to have a unique equilibrium, which usually is in randomized (mixed) strategies. This approach is justified in their case because they are concerned with the analysis of welfare. However, the usage of mixed strategy equilibrium is not appropriate to discuss the price dispersion. Randomization of prices imposes the price dispersion somewhat artificially. It combines lateral as well as temporal price dispersion and does not allow to separate the two effects in a clear way. Therefore, we follow the classical works in the discipline (Salop and Stiglitz, 1977, 1982; Stahl, 1996) and examine only pure strategy Nash equilibria of the pricing game.

We present a simple network reformulation of the Bertrand price competition model that has multiple pure strategy Nash equilibria and analyze implications of firm heterogeneity. Further, we extend the model in order to include more elaborate behavior by the consumers. We study the effects of consumer heterogeneity with respect to their risk attitudes and also the interaction between consumer and firm heterogeneity.

We find that in order for the model to sustain price dispersion in absence of consumer heterogeneity large dose of firm heterogeneity is necessary. However, when consumers are heterogenous even small asymmetries across producers imply price dispersion in equilibrium. In addition we find that more heterogenous consumers imply higher prices in equilibrium.

The paper is structured as follows. Section 2 presents a baseline model and its results. Section 3 extends the baseline model to include a sequential search by the consumers who visit more than one shops. It also presents the results with homogenous and heterogenous consumers. The last section concludes.

2   The Baseline Model

Consider the market where two firms ($i = 1, 2$) are engaging in price competition in order to attract the consumers. Firms use the same constant returns to scale technology and produce homogenous good. The unit/marginal cost of production is c, that can be payed after sales. The homogenous product is indivisible.

There are $\bar{S}$ consumers in the economy. Each of them is endowed with funds in amount of m. Consumers can only buy one unit of the product. They can spend all the money for a unit of product, but prefer spending as little as possible.

Realistically, we assume that price of the good is a discrete variable. There is a minimum unit of size ϵ that the price is a multiple of. This is realistic as each consumer buys at most one unit of the product. We can consider ϵ being equal to one cent.

Distinctive from the original Bertrand setup, we assume that each consumer can either go to one of the shops/producers, or go to both of them. Similar assumption has been used by Wilde and Schwartz (1979). They split the society in two: consumers that go only to one shop and the other part that go to several shops. However, in their setup the choice of shops for every agent are random. In our setup we relax the assumption of choosing the shop randomly. We assume that there is a structure to choices.

Denote part of the consumers that only go to the first shop by U_1 and the part that only goes to the second shop by U_2. We assume that consumers that belong to U_1 leave close to this shop and therefore go to this shop and do not choose randomly which of the two shops to visit. On the other hand, consumers belonging to U_2 leave close to the shop two. Remaining $S = \bar{S} - U_1 - U_2$ consumers go to both of the shops. Therefore, only these consumers can buy the cheapest product available on the market. We further assume that this market structure is known to duopolists and that it is the major consideration when considering the pricing of the product.

Naturally, the aim of each of the firms is to maximize its profits.

2.1 Analysis

Due to the fact that there are consumers that only go to one of the shops the duipolistic situation is somewhat non-standard. Technically, in principle, there are three distinct sub-markets and both of the firms have certain freedom to exercise their monopolistic power in one of the sub-markets. But they compete duopolistically only on common/shared sub-market. The problem that they are facing is that they cannot discriminate and thus have to charge one price for both sub-markets they operate on.

Intuitively, due to the fact that they have incentive to exercise their monopolistic power to certain extent prices in equilibrium need not converge to marginal cost (c). And furthermore, they need not converge to each other. Thus, we can have scenarios where there is price dispersion in equilibrium and scenarios where prices converge. We analyze these two groups in turn.

2.1.1 Price dispersion in equilibrium

For presentation of results we assume that the expensive shop is always the shop 1.

Proposition 1. *If there is a price dispersion in equilibrium, then it must be true that $p_1 = m$ and $p_2 = m - \epsilon$.*

Proof. If there is a price dispersion in equilibrium, the shop that charges the lowest price attracts the whole S consumers. Therefore, the other shop has no incentive to charge lower price than m (as it sells only to the consumers unique to it). Then the cheaper shop does not have an incentive to charge any price lower than $m - \epsilon$. $\square$

Denote the monopolistic profit margin by $r \equiv m - c$ and the "unit margin" by $\nu \equiv \frac{m-c}{\epsilon}$. Also denote $u_i = \frac{U_i}{S}$, for $i = 1, 2$.

Lemma 1. *If parameters of the model are such, that $2(1 + u_2) \leq \nu \leq 1 + 2u_1$ is satisfied, than price dispersion is a Nash equilibrium.*

The proof of lemma 1 is given in the appendix A. Lemma 1 has an important consequence.

Proposition 2. *If $u_1 - u_2 < \frac{1}{2}$, there exists no equilibrium such that there is a price dispersion in equilibrium.*

Proof. If $2(1 + u_2) > 1 + 2u_1$, then there exists no such ν that can satisfy requirement (4). Rewriting this restriction, we get

$$u_1 - u_2 < \frac{1}{2}.$$

$\square$

In order to understand the intuition behind the result recall the dispersion in equilibrium means $p_1 = m$ and $p_2 = m - \epsilon$. Therefore, there are three types of incentives that can disequilibrate the situation: (1) for the second shop to increase the price to m, (2) for the first shop to decrease the price to $m - \epsilon$ and (3) for the first shop to undercut the second one by setting the price at $m - 2\epsilon$. As demonstrated in the proof of lemma 1, the first shop will never opt for the option (3) when option (2) is available. Therefore, we discuss only the first two options.

Consider when will firm 1 have the incentive to decrease the price. Recall that it is enjoying the maximal price, but selling only to the consumers unique to it. If it decreases the price it will gain half of the shared consumers, but will be selling the product at a lower price. Clearly this will be beneficial if the number of shared

consumers is relatively high compared to the number of consumers unique to the firm. In other words, when u_1 is small.

Now consider when will firm 2 have the incentive to increase its price and give up half of the shared consumers. With the logic similar to the one presented in the previous paragraph, this will be the case if the number of consumers unique to the second firm are relatively numerous compared to the shared consumers. In other words, when u_2 is large. In this case the second firm will extract higher profits by selling its products to less consumers at higher price.

When $u_1 - u_2$ is small, this means that either u_1 is too small giving the first firm the reason to decrease the price, or u_2 is too high tempting the second firm to increase the price. Therefore we need large firm heterogeneity to sustain the price dispersion in equilibrium.

2.1.2 No price dispersion in equilibrium

In this case we might have multiplicity of equilibria. We know that $p_1 = p_2 = p$. Therefore, the profits are $\pi_i = (\frac{1}{2}S + U_i)(p - c)$ for $i = 1, 2$.

Two obvious candidates for equilibrium are Bertrand ($p_1 = p_2 = c$) and monopolistic ($p_1 = p_2 = m$) outcomes. In this respect we have two results.

Proposition 3. $p_1 = p_2 = c$ *is never the equilibrium of the game as long as $\exists i$ such that $u_i > 0$.*

Proof of and intuition behind this proposition is extremely simple. To see why proposition 3 is correct, note that if a firm charges price at the unit cost it brakes even. However, if he charges a monopolistic price it will extract a positive profit from the consumers unique to it. Therefore, as long as the producer has unique consumers it has an incentive to defect from the arrangement. Note, that once one of the shops has defected, the other also has an incentive to increase the price. This is in line with previous results by Stahl (1996), who finds that an equilibrium price is bound above marginal cost.

Proposition 4. *If $u_i \geq \frac{1}{2}\nu - 1 \ \forall i$, then $p_1 = p_2 = m$ is a Nash equilibrium of the game.*

Proof. For $p_1 = p_2 = m$ to be the Nash Equilibrium of the game we need none of the shops to have incentive to undercut the price. Thus we need

$$\left(\frac{1}{2}S + U_i\right)(m - c) \geq (S + U_i)(m - c - \epsilon)$$

6

to hold $\forall i$. This restriction directly results into $u_i \geq \frac{1}{2}\nu - 1$. $\qquad\square$

In order to understand the intuition behind this result it is convenient to discuss why monopolistic pricing will not be equilibrium if both u_is are low. Low u_i means fewer unique consumers for the firm relative to the shared consumers. When both firms have few unique consumers both of them are tempted to extract the profit from the shared consumers by lowering the price. Therefore, they engage in fierce Brtrand-type competition, which, clearly, disequilibrates the $p_1 = p_2 = m$ arrangement.

Besides marginal cost and monopolistic pricing current game can have equilibria in the interior of $(c; m)$ interval.

Lemma 2. *If there is a $p > c$ such that $\frac{2U_i m + Sc}{S + 2U_i} \leq p \leq (1 + u_i)\,\epsilon + c\ \forall i$, then $p_i = p_j = p$ is a Nash equilibrium of the pricing game.*

The proof of lemma 2 is given in the appendix B.

What lemma 2 says is that too low or high prices cannot be considered for equilibrium where both firms charge the same price. Intuition behind this result is as follows. If price that we consider for equilibrium is low firms make low profits (recall that profits go to zero as prices approach marginal cost). Therefore, in case of too low of a price firms will be tempted to give up their half of shared consumers and extract monopolistic profit from consumers unique to them. On the other hand, when prices are high firms make high profits. And each consumer is important for them. Therefore, when prices are too high firms will have incentive to undercut the opponent and sell the product to larger number of consumers at slightly lower (but still high) price.

All values of p that satisfy the restriction specified by lemma 2, correspond to Nash equilibria. Of course, higher price corresponds to higher profits to firms. In any case, in these situations both shops charge the same price, but this price is strictly higher than marginal cost.

Proposition 5. *If $\exists i$ such that $u_i < \frac{1}{2}\left(\nu - \frac{3}{2} + \sqrt{\left(\frac{1}{2} - \nu\right)^2 - 2\nu}\right)$, there exists no pure strategy Nash equilibrium in interval $(c; m)$ such that both firms charge the same price.*

The proof of proposition 5 is given in the appendix C. Intuition for this result can be easily derived from the intuition for lemma 2. Low u_i cannot maintain high price in equilibrium, because for firm i shared consumer base becomes overly important and tempts it to undercut the price. However, if the price becomes too low, due to the fast diminishing profits the firm that has larger number of unique consumers will be tempted to defect from the arrangement by charging monopolistic price.
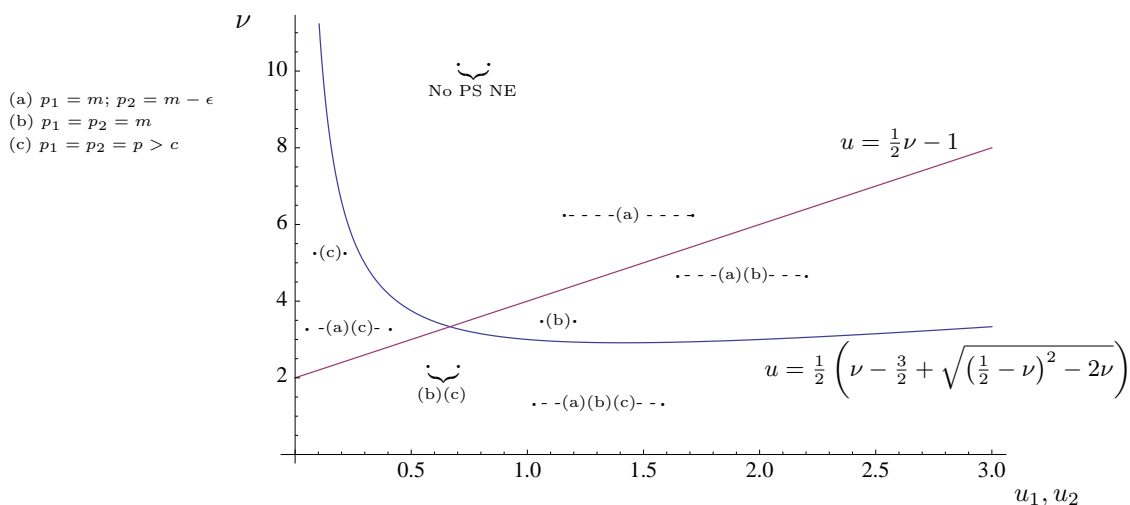
7

Figure 1: Summary of the results of the baseline model.

In the end we can summarize the results of the model on figure 1. On this figure ν (that is measured on ordinate) describes the income-cost relation, while u_1 and u_2 (that are measured on abscissa) describe the market structure. Therefore, the economy that we are considering can be depicted on the graph with two dots that are necessarily placed on the same (imaginary) horizontal line. The value for the dots on ordinate specifies ν, while values on abscissa specify u_1 and u_2. Possible pure strategy Nash equilibria for every type of industry is presented on the graph.[2] As one can see, in the model, depending on the constellation of parameters, we can have settings where all types of equilibria are possible, as well as situations where pricing game has no pure strategy Nash equilibrium. In this case prices are simply cycling. They gradually go down as firms undercut each other until they go so low that one of the firms increases the price to monopolize unique consumers. The competitor follows and sets the price high enough to engage the first firm into price cutting again. Similar result has been obtained by Salop and Stiglitz (1977).

3   Sequential trips to shops with no return option

In the baseline model discussed in section 2 consumers that go to both shops choose the cheapest option available. As asserted in introduction obtaining complete information about the prices in the economy might be costly. Consumers might have to engage in price search in order to identify the cheapest shop. In economics litera-

---

[2]We do not present the arrangements when firms are located in the different regions not to overcrowd the figure. Outcomes for these arrangements can be inferred from propositions 1 through 5.

ture typically there are two types of search rules are considered: sequential search (Benhabib and Bull, 1983) and fixed sample size non-sequential search (Manning and Morgan, 1982). However, several papers have identified search strategies that dominate these two.[3]

Our concern is with the sequential search. The seminal contribution in this area is due Stigler (1961). He has demonstrated that it is optimal for consumers to decide whether to continue the search for the best offer every time they receive the quote. Building on Stigler's work, Kohn and Shavell (1974) have demonstrated that sequential search would result in a switchpoint level of price. This means that consumer would terminate the search as soon as a price quote would fall below this threshold level. Kohn and Shavell (1974) have also demonstrated the uniqueness of this threshold.

In this paper we adopt this sequential threshold search approach. We assume consumers cannot obtain quotes without going to the shop in person. in original description when consumers decide to terminate their search they can choose to buy from the cheapest shop they have visited during the search process. However, in certain cases this is not very realistic.

Consider a long street which has two gas stations in either end of it. If a person wants to put a gas in his car he has to decide while being close to the street end. Some consumers live on the street. Therefore, going to work every morning they pass only one of the gas stations. Consumers who have to drive east to work can be U_1, while the consumers who have to drive west can be U_2. These consumers get one quote each and they have to buy from the respective gas stations, because driving to the other end of the street is simply too costly (imagine its a busy long street with traffic).

However, there are consumers who do not live on this street, but they have to pass it while going to work. Now, these are S consumers. They price quotes from both gas stations, but they can buy only from the one. And as driving from one gas station to the other is costly they are not free of choosing the cheapest shop even if they have complete information about the prices. Consider what decisions should these consumers make. The arrive on the street and pass the first gas station. They can stop and buy gas. However, if they decide not to do that they take on a gamble. In this case they have to buy the gas from the station at the other end of the street. Even if gas there is more expensive they cannot turn and drive back to the first gas station. Again, this is simply too costly.

---

[3]For the review of these works see Morgan and Manning (1985).

This behavior is not unique to gas stations. Many of us do our weekly shopping in two supermarkets. We only buy part of our shopping list in the first super market leaving the other part for the next shop. However, it is perhaps a negligible share of us who would go back to the first shop to buy a single item that she found to be more expensive in the second shop.

This is the consumer behavior we model in this section. We consider consumers following simple rule of thumb in their purchases. Each of them has an idiosyncratic thresholds $\hat{p}_s$, such that as soon as the price she sees in the first shop she enters $p \leq \hat{p}_s$, she buys the product. If the first shop charges higher price she takes a chance of buying the product in the second shop. Of course, this only applies to the consumers that go to both shops. The rest of the consumers go to only one shop and therefore purchase the product from there.

Notice that by assumption our consumers are risk lovers. Simply because they do not have a return option their behavior resembles a gamble. If the price in the first shop is too high to their taste, they gamble on the price in the second shop. This gamble might pay off, if the second shop they visit charges lower price. But they might have to pay even higher price in the second shop. Therefore, $\hat{p}_s$ is a measure of the risk attitude of consumer s. If $\hat{p}_s$ is low, consumer is more risk loving - she takes riskier gambles. This is the only characteristic of a consumer that we discuss in the remaining of the paper.

Consider $\hat{p}_s$ is distributed over S with a certain probability density function $f(\cdot)$. Then, corresponding cumulative density function at point at point p, $F(p)$, gives the share of shared consumers for which $\hat{p}_s < p$. Thus we can conclude that $F(p)$ share of consumers that will enter she given shop first will not buy the product from this shop and will gamble on the price charged by the second shop. This, in return implies that $1 - F(p)$ share these consumers will buy the product from this shop. In the case when profit functions for producers can be written as

$$\pi_i = \left[\frac{S}{2} \left(1 - F(p_i) + F(p_j) \right) + U_i \right] (p_i - c), \tag{1}$$

where $i = 1, 2$ and $j = 2, 1$.

In order to discuss the implications of this kind of behavior, we discuss two examples. One where consumer's are homogenous with regard to $\hat{p}_s$, or their risk aversion attitudes, the other with heterogenous consumers. In fact the former example is only a limiting case of the latter, but it helps to demonstrate few important implications of the model in simpler setup.

3.1 Homogenous consumers

We start off with the simple case where consumers are homogenous with respect to their risk attitudes. We assume that $\hat{p}$ is distributed according to Dirac's delta function that peak's at μ, where $c < \mu < m$. Effectively this means that $\hat{p}_s = \mu \; \forall s$. The value of μ measures how risk-taking is a society as a whole – the higher the μ, the less risk-taking is the society.

In order to characterize the equilibria of this game consider the following.

Remark 1. *No matter the price charged by the competitor and the market structure, it is never optimal to charge $p_i \in [c; \mu) \cup (\mu; m)$.*

If $S = 0$, we know that the optimal policy is to charge $p = m$. However, if $S > 0$ we have to discuss two components of the interval separately. We know that firms want to extract maximum possible profit from the unique consumers, therefore, for those consumers they would prefer charging as high prices as possible. For half of the shared consumers the shop i will be the first shop they visit. They will buy from shop i as long as $p_i \leq \mu$, they will not buy from the shop otherwise. On the other hand, shared consumers for whom shop i will be the second shop (and who have not bought the product in previous shop) will always buy the product no matter the price. Therefore, no matter the price charged by the competitor, shop i will have two possible strategies: either maintain the shared consumers entering the shop first or give them up. If a firm wants to maintain these shared consumers it is clearly suboptimal to charge any price lower than μ. If the firm wants to give them up for monopolistic profits, it is clearly suboptimal to charge the price lower than m.

Therefore, in homogenous consumer case with sequential trips and no return option we have only two possible prices in equilibrium – μ and m. Hence, we can have only tree kinds of pure strategy Nash equilibria: (i) where both firms charge m, (ii) where both firms charge μ and (iii) where one of the firms charges μ while the other charges m.

3.1.1 No price dispersion in equilibrium

First we discuss equilibria without price dispersion. In order to do that we denote $\mu - c = r'$. In consistency with earlier notations $m - \mu = r - r'$. Note that r' directly corresponds to μ, therefore r' is also a measure of the consumer risk attitudes.

Proposition 6. *If $r' \leq \frac{\frac{1}{2} + u_i}{1 + u_i} r$, $\forall i$, then $p_1 = p_2 = m$ is an equilibrium of the game.*

The proof of the proposition is straight forward as the incentive we have to exclude is any of the firms wanting to jump down to μ.
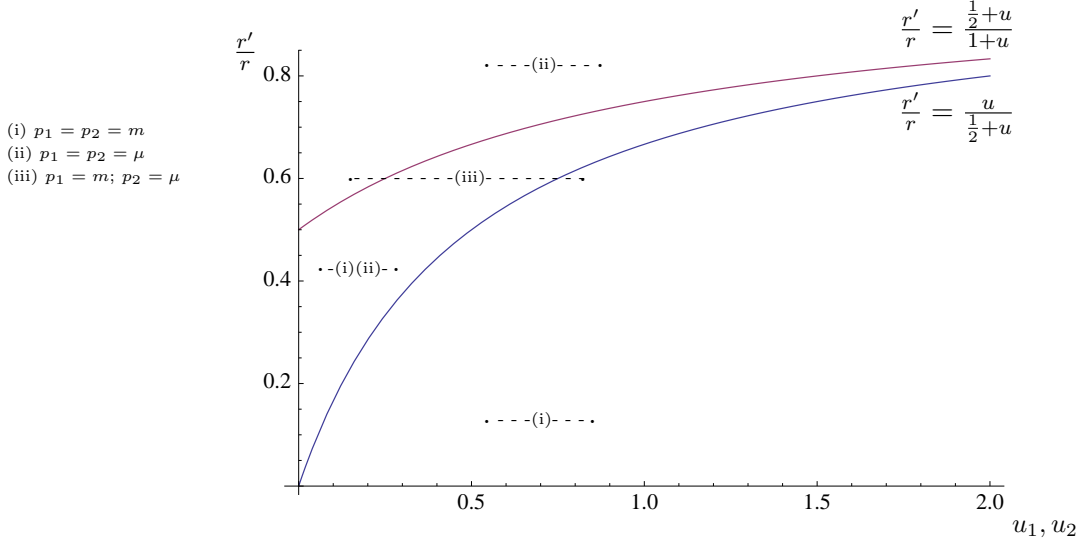
Figure 2: Summary of the results with homogenous consumers.

Proposition 7. *If $r' \geq \frac{u_i}{\frac{1}{2}+u_i}r$, $\forall i$, then $p_1 = p_2 = \mu$ is an equilibrium of the game.*

The proof of this proposition is similarly simple.

These results point to the fact that more risky consumers (lower r') imply higher equilibrium prices in the economy.

3.1.2   Price dispersion in equilibrium

As mentioned earlier in the setup of homogenous consumers the only equilibrium with price dispersion is $p_1 = m$ and $p_2 = \mu$.

Proposition 8. *If $\frac{\frac{1}{2}+u_2}{1+u_2}r \leq r' \leq \frac{u_1}{\frac{1}{2}+u_1}r$, then $p_1 = m$ and $p_2 = \mu$ is the equilibrium of the game.*

The proof of this proposition is also straight forward as it is sufficient to ensure that the firm i does not have incentive to change the price to μ and firm i not to have incentive to move the price to m.

Proposition 8 implies that the shop with larger number of unique consumers charges the higher price. This is intuitive as the shop with more unique consumers has to incur larger costs (in form of profits given up) for lowering the price. While for the firm with lower u, will be of greater importance, therefore it will be willing to fiercely hight for them by price cutting.

In the end the summary of the results with homogenous consumers can be seen on figure 2. In this figure $\frac{r'}{r}$ is measured on the ordinate and u_1 and u_2 are measured on abscissa. Similar to figure 1, in this space the market setup can be represented

12

by two points on the same horizontal line. Note that low values of $\frac{r'}{r}$ mean that the society is more risk-taking.

We have three regions on the figure. If the market setup is such that both firms are in the region below both curves – both firms are charging monopolistic price $p_1 = p_2 = m$, this is unique equilibrium. If market arrangement puts both firms above both curves – firms are changing $p_1 = p_2 = \mu$. If both firms are in the middle region both types of equilibria are possible. If market arrangement is such that only one of the firms is placed in the region between the two curves there exists no pure strategy Nash equilibrium. If one of the firm falls below both of the curves and the other above both of them then we have a unique equilibrium with a price dispersion: the firm above the two curves charges μ while the firm below the curves charges m. In order to understand why this is the case, note that being below both of the curves means that the firm has relatively high share of unique consumers, who in this case it exploits by charging m.

In general we can see that riskier consumers induce higher prices in the economy. We can also see that in order for the model to support the price dispersion in equilibrium we need two requirements. One is that μ has to be relatively high, more precisely $\frac{r'}{r} > \frac{1}{2}$. This requirement implies that on consumers should not be too risk-loving. The other requirement is that firms have to be sufficiently heterogenous. Meaning that one of them has to have sufficiently larger number of unique consumers. In case of consumer homogeneity, price dispersion is only sustainable with a large dose of firm heterogeneity.

3.2 Heterogenous consumers

In order to understand the effects of consumer heterogeneity on results of the game, we can assume thresholds are distributed normally $\hat{p}_s \sim \mathcal{N}(\mu, \sigma^2)$. In this case the mean and the standard deviation of threshold distribution have economic meanings. The mean of the distribution measures how risk-taking is the society on average. The standard deviation of the distribution measures the consumer heterogeneity with respect to risk attitudes. Larger σ implies higher heterogeneity. Note that as $\sigma \to 0$ consumers become homogenous and we revert to the situation discussed in section 3.1.

Remark 2. *If thresholds are distributed normally $\hat{p}_s \sim \mathcal{N}(\mu, \sigma^2)$, profit function for the producer i can be written as*

$$\pi_i = \frac{S}{2} \left[1 + 2u_i + \frac{1}{\sqrt{\pi}} \left(e^{\frac{\mu - p_i}{\sigma\sqrt{2}}} - e^{\frac{\mu - p_j}{\sigma\sqrt{2}}} \right) \right] (p_i - c). \tag{2}$$
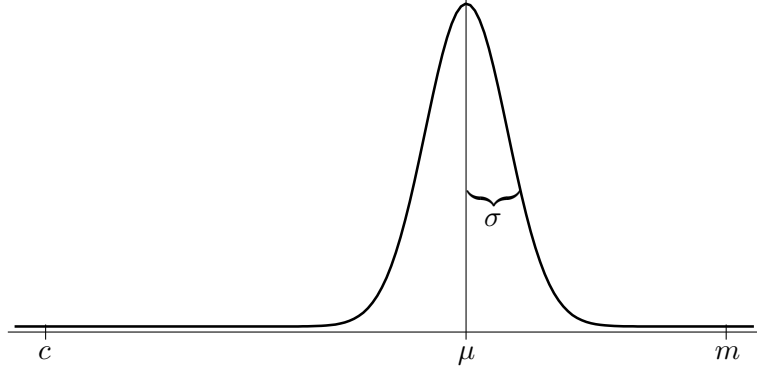
Figure 3: Distribution of $\hat{p}_s$ on the price range.

Demonstration of remark 2 is given in appendix D.

So, producer i has to maximize π_i given in equation (2) with respect to p_i on interval $[c; m]$. To analyze the Nash equilibria of the pricing game we need to derive the best response or reaction functions (RF) of consumers to each-other's prices on continuous interval $[c; m]$. Functional form of π_i in equation (2) results in tedious form of reaction function – the one that has a discontinuity. Therefore, we choose an alternative way to analyze the setup.

Before going further into details we have to note several facts about the profit function (2). First is the fact that the price that maximizes the profit of the producer i does not really depend on the number of consumers that is shared between the firms on common sub-market(S), but it rather depends on the ratio u_i.

The second, less straightforward fact is that what really matters for the reaction function on interval $[c; m]$ are not the exact values of c, m, μ or σ, bur rather their proportionate relationships. In order to understand the intuition behind the claim look at the figure 3. It is obvious from the figure that what matters is the location of μ on interval $[c; m]$ and the size of σ in terms of the size of the interval r.[4] Therefore, we can express μ and σ in terms of the size of the interval $[c; m]$ as follows $\mu = ar + c$ and $\sigma = br$. In this case a controls the value of μ proportionately with respect to the exact values of c and m; and b does the same for σ. As we know that prices have to also fall on interval $[c; m]$ we can also express them proportionately to the interval, so that $p_i = k_i r + c$. In this case k_i controls the changes in p_i proportionately to the interval under discussion. Therefore, every value can be proportionately adjusted to the size and the location of the feasible interval. This will have a very important implication.

---

[4]Recall we have denoted $r \equiv m - c$.

Proposition 9. *On an arbitrary interval $[c; m]$, reaction function implied by the profit function (3) has the same shape in terms of adjusted values.*

Proof. Reaction function is the best response to the price set by the competitor. The slope of the profit function in any arbitrary point p_i is

$$\frac{\partial \pi_i}{\partial p_i} = \frac{S}{2} \left(1 + 2u_i + \frac{1}{\sqrt{\pi}} \left[e^{\frac{\mu - p_i}{\sigma \sqrt{2}}} - e^{\frac{\mu - p_j}{\sigma \sqrt{2}}} \right] - \frac{p_i - c}{\sigma \sqrt{2}} e^{\frac{\mu - p_i}{\sigma \sqrt{2}}} \right). \tag{3}$$

Adjusting values of μ, σ, p_i and p_j to the size and the location of the interval, we can rewrite the equation (3) as follows

$$\frac{\partial \pi_i}{\partial k_i} = \frac{S}{2} \left(1 + 2u_i + \frac{1}{\sqrt{\pi}} \left[e^{\frac{a - k_i}{b \sqrt{2}}} - e^{\frac{a - k_j}{b \sqrt{2}}} \right] - \frac{k_i}{b \sqrt{2}} e^{\frac{a - k_i}{b \sqrt{2}}} \right). \tag{4}$$

We can clearly see now that the slope of the profit function does not depend on values of c and m. Consequently neither does the shape of the reaction function. $\quad\square$

From equation (4) one can also see that the reaction function, which will be obtained by equating the derivative of the profit to zero, will not be depend on the value of S, just as we argued above.

Now as we have established that the shape of the reaction function on feasible interval does not depend on values of c, m, or S, and that we can discuss adjusted values of μ and σ (a and b respectively), we can analyze the implication of the model using graphical approach.

It is more convenient to report results in general form, while keeping parameters of the model μ and σ in adjusted forms.[5]

Figure 4 shows the general shape of the reaction function with producer homogeneity ($u_1 = u_2$). Reaction function of producer i to the price set by producer j is presented with the blue line, while the reaction function of the producer j is given by the red line. We can clearly see that the reaction function of a firm has a discontinuity. At a certain price of the opponent the best response suddenly jumps to the maximal feasible value.

Nash equilibria are given by the points where reaction functions intersect. There two pure strategy and one mixed strategy Nash equilibria. Due to the fact that we are discussing homogenous producers prices across shops are equal in all (pure strategy) equilibria. The lower and higher price equilibria are pure strategy equilibria. We denote these equilibria with PS_L and PS_H respectively. We also denote the

---

[5]The values of the mean and the standard deviation of the threshold distribution can be easily obtained from adjusted values a and b by giving numeric values to c and m and making the appropriate adjustment with $\mu = ar + c = a(m - c) + c$ and $\sigma = br = b(m - c)$.
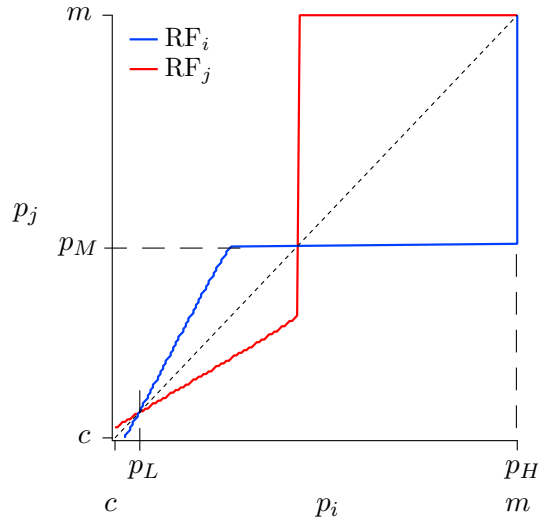
Figure 4: Reaction functions of duopolists with $a = 0.6$, $b = 0.15$ and $u_i = u_j = 0.5$.

prices implied by them with p_L and p_H. The one in the middle, the one that occurs at the intersection of discontinuous parts, is a mixed strategy Nash equilibrium. We denote this equilibrium by MS. Note that the location where the discontinuity occurs in the reaction function, which is given by the (opponent's) price, uniquely defines the location of the MS. Therefore, we denote this price with p_M.

An important fact to note about the figure 4 is that $p_H = m$, while $p_L > c$. The p_L occurs when continuous parts of reaction functions intersect. Actually we can analytically solve for the continuous portion of the reaction function (before at a certain price of the competitor it jumps to the monopolistic response) and therefore can analyze how the low price equilibrium changes with the parameters of the model. This analysis for the homogenous producers is presented in the appendix E.

Although we have elaborated in introduction that mixed strategy Nash equilibria are not suitable for the purposes of this paper, the MS equilibrium identified on figure 4 is still important. If you think of our game from the dynamical prospective as a game with sequential moves, the pure strategy equilibria are the only two stable equilibria of the game. The point where discontinuity occurs in the reaction functions (p_M) marks the border between the attraction basins of PS_L and PS_H. Therefore, the location of p_M lets us understand the equilibrium selection process starting from a random initial condition. Moving p_M closer to p_L increases the likelihood of p_H prevailing in equilibrium.

Further we discuss implications of changes in parameter values. Figure 5 demonstrates how reaction functions change with the increase of the degree of risk-taking.
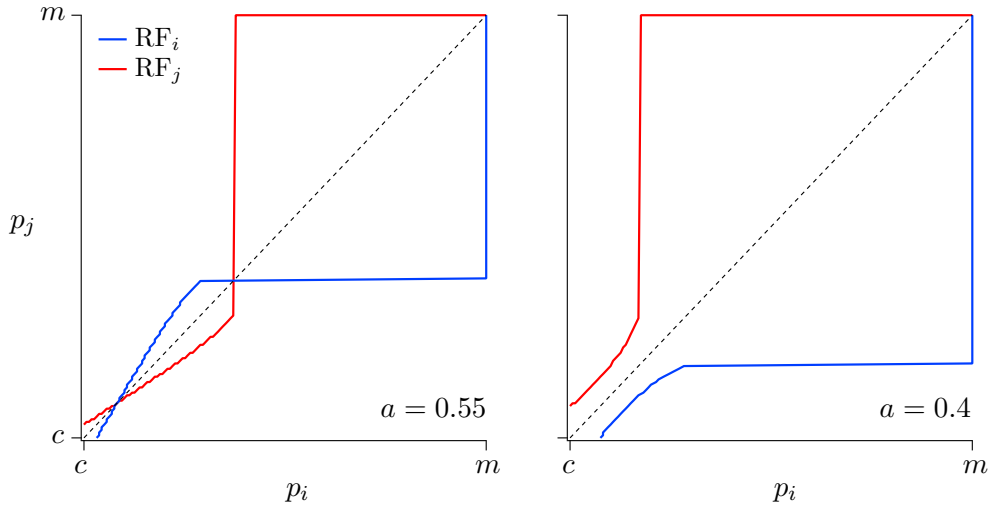
Figure 5: The effect of more risky consumers (lower μ or a).

As consumers become more risky producers can easily exploit them. There are two facts that we can note from the left panel of figure 5: (1) p_L increases (2) p_M decreases. The letter one is important as it implies that monopolistic prices will prevail in equilibrium for larger set of initial conditions.

Right panel of figure 5 demonstrates that as mean of the threshold distribution becomes too low, game reverts to having a unique equilibrium. And this equilibrium is monopolistic pricing. Intuition behind this result is that as consumers become too picky with respect to the price they see in the first shop, producers are tempted to abandon the strategy of selling product to the first entrants. They instead switch to charging maximal price to the second entrants that rejected the competitor's offer.

Figure 6 shows the effect of decrease in consumer heterogeneity. As standard deviation of the threshold distribution decreases, so does p_L. However, the effect is opposite on p_M. In order to understand the intuition behind these results we have to clearly distinguish the incentives in the baseline model discussed in section 2 and in this model with sequential trips and no return option. In the baseline model incentives are the same as in the standard Bertrand competition: firms undercut each other in order to sell to all the shared consumers, and if they have incentive to set the price above the competitor's they will always set it to the monopolistic price m.

In the current setup undercutting the competitor does not make much sense. Lowering the price just below p_j will not allow firm i to serve the whole shared market. It will change the number of consumers that buy from firm i only marginally. The gain will be the consumers with threshold values between the old price firm i
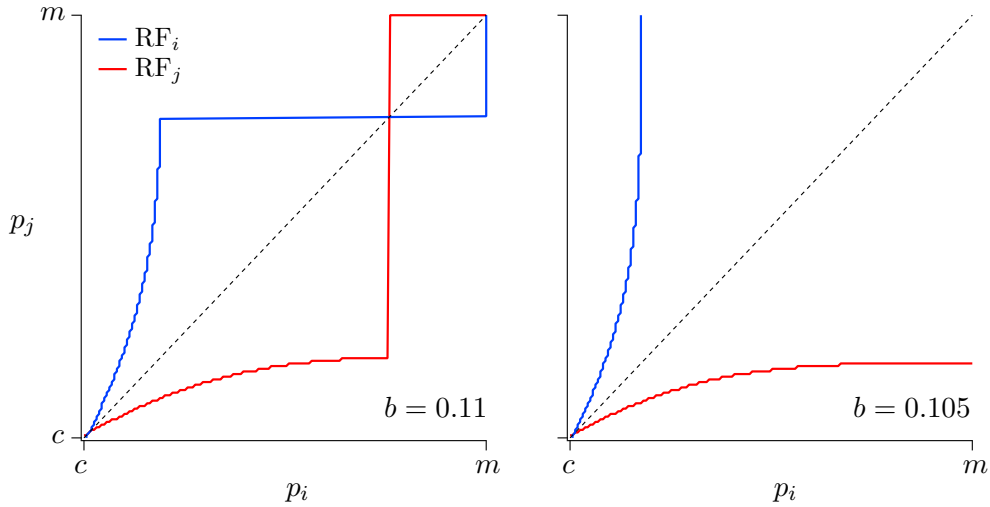
Figure 6: The effect of less consumer heterogeneity (lower σ or b).

was charging and the new price. The same is true for the marginal increase of price – it only decreases the number of consumers marginally. Therefore, undercutting or jumping to monopolistic price need not be optimal.

Now, consider the low price equilibrium first. When prices are low, clearly the mass of the $\hat{p}_s$ distribution to the left of p_j is small, thus setting the monopolistic price will not be optimal for the firm i. Therefore, we have to discuss the response of the firm i in the neighborhood of p_j. The reason for setting the price low is to retain as many first entrants as possible (this is measured by $F(p_i)$). The reason for setting the price high is to exploit the second entrants that have rejected the offer from j (measured by $1 - F(p_j)$). Consider how the magnitude of these incentives change when σ is higher. Higher σ implies that more mass of the distribution is concentrated to the left of the p_j and potential p_i.[6] This gives extra incentive to increase the price and take advantage of the (marginally lower number of) first entrants and large number or second entrants that have rejected the offer of the firm j. This is the reason why the larger consumer heterogeneity implies higher prices at low price equilibrium.

In order to understand the intuition behind the decrease in p_M recall again that this is the price where producer's reaction function has a discontinuity. From figure 4 it is clear that prices lower than p_M call for the response with even lower price, and prices above it call for the response with the monopolistic price. The reason for responding with the lower price is to capitalize on the quantity of the consumers, while the reason for responding with the monopolistic price is to capitalize on price.

---

[6]Note that low price equilibrium will usually be lower than the mean of the distribution.
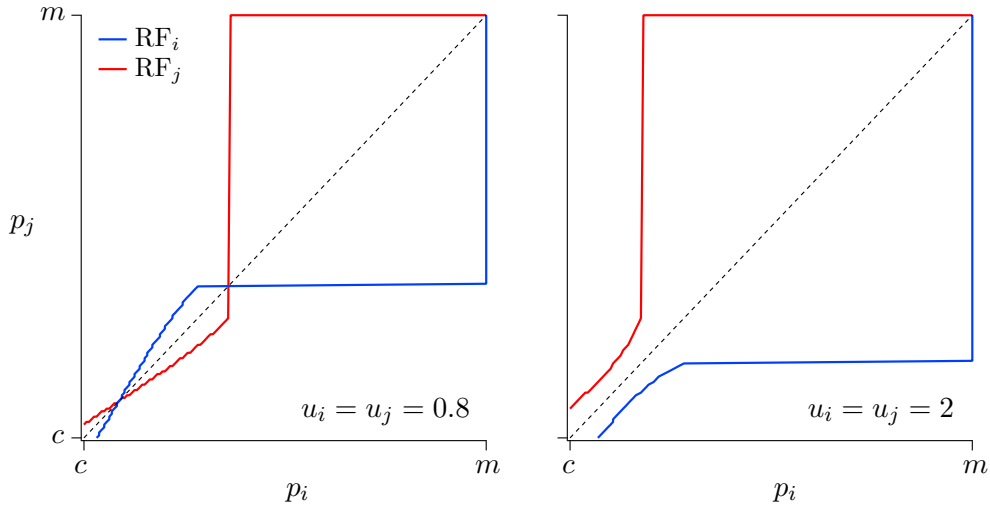
Figure 7: The effect of more unique consumers (higher $u_i = u_j$).

Because in our setup the response price m is always higher than μ, higher σ means that the larger mass of the $\hat{p}_s$ distribution is concentrated to the right of m. Which implies that profits at $p_i = m$ are higher. Therefore the $p_i = m$ becomes the best response to the lower values of p_j in environments with larger consumer heterogeneity.

To sum up turns out that increasing consumer heterogeneity affects both stable equilibria. It increases the price in the low price equilibrium and it also increases the attraction basin of the high price equilibrium. Therefore, we can conclude that higher consumer heterogeneity is likely to result in higher prices in equilibrium.

Figure 7 demonstrates the effect of increasing the number of unique consumers to both shops, or the decrease of the number of shared consumers. As we can see the result is similar to the increase of the risk-taking of consumers. This is intuitive as less shared consumers pushes the market structure from duopoly towards two local monopolies. As the number of shared consumers becomes too low the game reverts to the one with the unique equilibrium where both firms charge monopolistic price.

Figure 8 demonstrates the effect of the firm heterogeneity in presence of consumer heterogeneity. The left panel of the figure shows producer reaction functions in the setup where $u_i + u_j = 1$, which was also the case in figures 4, 5 and 6. But in this case we have the asymmetry in there distribution across the shops: now firm j has no unique consumers, while firm i has double of those it had in previous cases.

As we can observe introduction of firm heterogeneity in the model implies the price dispersion in PS_L. The left crossing of the reaction functions departs from the 45 degree line. In this particular case, we can observe that firm i would charge
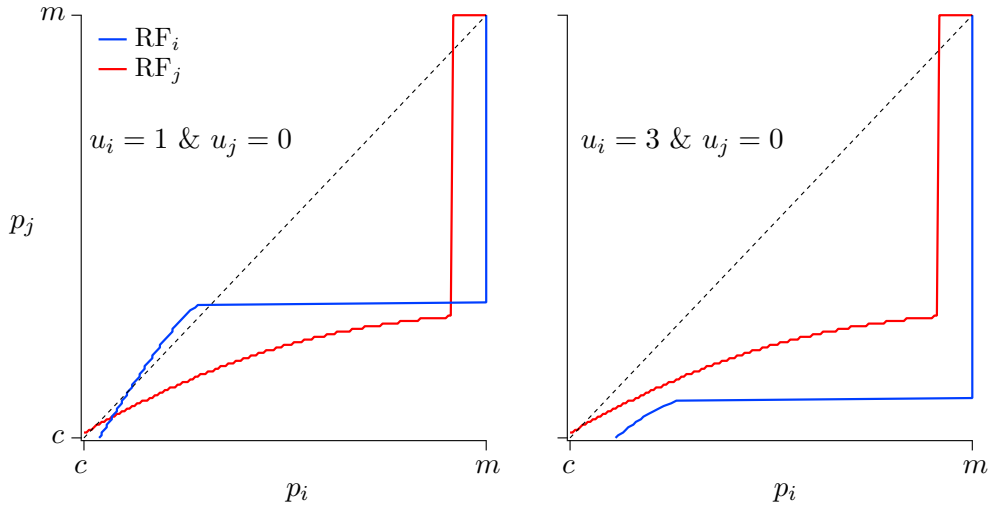
19

Figure 8: The effect of firm heterogeneity ($u_i \neq u_j$).

higher price compared to firm j in PS_L. This is intuitive as firm i has higher market power compared to firm j due to the higher number of unique consumers.

An important feature of the model to note here is how the left panel of figure 8 changes with decreasing level of risk aversion (increasing μ). Increasing μ or a pushes the point where RF_j has discontinuity to the right and with enough strength reverts the game to the one with the unique equilibrium at low prices. But in this case, due to the firm heterogeneity we have price dispersion in this unique equilibrium.

The right panel of the figure 8 also shows the effect of firm heterogeneity, but in this case in the arrangement that involves less shared consumers compared to the case in the left panel. As importance of unique consumers increase, the effects of firm heterogeneity amplify and the game reverts to the one with the unique equilibrium with monopolistic prices. At this equilibrium there is no price dispersion.

4 Conclusion

In this paper we have presented a model of price competition on networked duopolistic markets. We can explicitly modeled consumer-producer relationships as a bipartite network. We have allowed producers to have unique buyers as well as buyers for whom they have to compete with the competitor. We have further extended the framework to include the certain behavioral features of consumers.

In this framework we have discussed two types of heterogeneities. Producer heterogeneity with respect to the number of unique consumers they have, and consumer heterogeneity with respect to their risk attitudes. We have examined the importance

20

of these phenomena for maintaining price dispersion in pure strategy Nash equilibria of the pricing game.

The result is that firm heterogeneity is necessary to have price dispersion in equilibrium. But, consumer heterogeneity has a complementary role. If there is no consumer heterogeneity we need large dose of firm heterogeneity in order to maintain the price dispersion. However, once there is consumer heterogeneity even small portions of firm heterogeneity imply at least one equilibrium with price dispersion.

Another important result that has emerged from the models is that higher levels of consumer heterogeneity with respect to their risk attitudes implies higher prices in equilibrium. In particular it increases the price in one of the two stable equilibria and it increases the attraction basin of the monopoly (high) price equilibrium. This is due to the fact that increasing masses in extremes of the risk distribution makes it optimal for producers to exploit both extremes of the distribution. Cautious end makes it not profitable to get involved in undercutting business, so both firms decide to serve their unique markets and as a consequence the system winds up in a monopolistic equilibrium. While the riskier end of the consumers directly allows producers to rip them off.

References

BARRON, J. M., B. A. TAYLOR, AND J. R. UMBECK (2004): "Number of sellers, average prices, and price dispersion," *International Journal of Industrial Organization*, 22, 1041–1066.

BAYE, M. R., J. MORGAN, AND P. SCHOLTEN (2004): "Price Dispersion In The Small And In The Large: Evidence From An Internet Price Comparison Site," *Journal of Industrial Economics*, 52, 463–496.

BENHABIB, J., AND C. BULL (1983): "Job Search: The Choice of Intensity," *Journal of Political Economy*, 91, 747–764.

BLUME, L. E., D. EASLEY, J. KLEINBERG, AND E. TARDOS (2009): "Trading networks with price-setting agents," *Games and Economic Behavior*, 67, 36–50.

BURDETT, K., AND K. L. JUDD (1983): "Equilibrium Price Dispersion," *Econometrica*, 51, 955–969.

COROMINAS-BOSCH, M. (2004): "Bargaining in a network of buyers and sellers," *Journal of Economic Theory*, 115, 35–77.

GASTWIRTH, J. L. (1976): "On Probabilistic Models of Consumer Search for Information," *The Quarterly Journal of Economics*, 90, 38–50.

JANSSEN, M., J. L. MORAGA-GONZALEZ, AND M. WILDENBEEST (2004): "Consumer Search and Oligopolistic Pricing: An Empirical Investigation," Tinbergen institute discussion paper, Tinbergen Institute.

KOHN, M. G., AND S. SHAVELL (1974): "The theory of search," *Journal of Economic Theory*, 9, 93–123.

KRANTON, R. E., AND D. F. MINEHART (2001): "A Theory of Buyer-Seller Networks," *The American Economic Review*, 91, 485–508.

LEVER, C. (2011): "Price Competition on a Buyer-Seller Network," Discussion paper, Banco de Mexico.

MANNING, R., AND P. B. MORGAN (1982): "Search and Consumer Theory," *The Review of Economic Studies*, 49, 203–216.

MORGAN, P., AND R. MANNING (1985): "Optimal Search," *Econometrica*, 53, 923–944.

PASINI, G., P. PIN, AND S. WEIDENHOLZER (2008): "Network Model of Price Dispersion," Discussion paper, Fondazione Eni Enrico Mattei.

REINGANUM, J. F. (1979): "A Simple Model of Equilibrium Price Dispersion," *The Journal of Political Economy*, 87, 851–858.

SALOP, S., AND J. STIGLITZ (1977): "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," *The Review of Economic Studies*, 44, 493–510.

SALOP, S., AND J. STIGLITZ (1982): "The Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical Agents," *The American Economic Review*, 72, 1121–1130.

STAHL, D. O. (1996): "Oligopolistic pricing with heterogeneous consumer search," *International Journal of Industrial Organization*, 14, 243–268.

STIGLER, G. J. (1961): "The Economics of Information," *The Journal of Political Economy*, 69, 213–225.

VARIAN, H. R. (1980): "A Model of Sales," *The American Economic Review*, 70, 651–659.

WILDE, L. L., AND A. SCHWARTZ (1979): "Equilibrium Comparison Shopping," *The Review of Economic Studies*, 46, 543–553.

WILDENBEEST, M. R. (2009): "An Empirical Model of Search with Vertically Differentiated Products," Working Papers 2009-01, Indiana University, Kelley School of Business, Department of Business Economics and Public Policy.

Appendix

A Proof of Lemma 1.

Proof. In this case the profits of the expensive shop amount to $\pi_1 = U_1 r$, while of the cheap shop $\pi_2 = (S + U_2)(r - \epsilon)$.

For this situation to be a Nash Equilibrium, parameters have to satisfy three restrictions:

1. For the cheap shop not to have incentive to increase the price (in which case its profits will becomes $\pi_2 = (\frac{1}{2}S + U_2)r$), we need

$$r \geq 2\,(1 + u_2)\,\epsilon. \tag{5}$$

2. For the expensive shop not to decrease the price to the level of the cheap one (in which case its profits will be $\pi_1 = (\frac{1}{2}S + U_1)(r - \epsilon)$), we need

$$r \leq (1 + 2u_1)\,\epsilon. \tag{6}$$

3. For the expensive shop not to undercut the cheap shop (in which case its profits will be $\pi_1 = (S + U_1)(r - 2\epsilon)$), we need

$$r \leq 2\,(1 + u_1)\,\epsilon. \tag{7}$$

Requirement (7) becomes redundant as soon as requirement (6) is satisfied. Therefore, for the price dispersion to be a Nash Equilibrium, we need the following restriction on model parameters

$$2\,(1 + u_2) \leq \nu \leq 1 + 2u_1. \tag{8}$$

$\square$

B Proof of Lemma 2.

Proof. In this case we need only two requirements. For the firm not to have incentive to undercut the competitor. And for the firm not to have incentive to charge the maximum price of $p_i = m$ (as any price in interval $(p; m)$ does not make sense).

1. For the firm not to undercut the competitor (in which case its profits will be $\pi_i = (S + U_i)(p - c - \epsilon))$, we need

$$p \leq 2 \left(1 + u_i \right) \epsilon + c \tag{9}$$

2. For the firm not to increase the price to maximum (in which case its profits will be $\pi_i = U_i(m - c)$), we need

$$p \geq \frac{2U_i m + Sc}{S + 2U_i} \tag{10}$$

Therefore, any price that satisfies the following requirement

$$\frac{2U_i m + Sc}{S + 2U_i} \leq p \leq \left(1 + u_i \right) \epsilon + c \tag{11}$$

is a Nash equilibrium in which both shops charge the same price for the product. □

C   Proof of proposition 5.

Proof. If $\frac{2U_i m + Sc}{2U_i + S} > (1 + u_i)\,\epsilon + c$, then there is no way to satisfy the requirement (7). Expanding the restriction, we get

$$\frac{2\epsilon}{S}U_i^2 + (3\epsilon + 2c - 2m)U_i + S\epsilon < 0. \tag{12}$$

Roots for this quadratic are

$$U_i = \frac{S}{4\epsilon}\left((2m - 2c - 3\epsilon) \pm \sqrt{\epsilon^2 - 12\epsilon(m - c) + 4(m - c)^2}\right). \tag{13}$$

We can rewrite this as

$$u_i = \frac{1}{2}\left(\nu - \frac{3}{2} \pm \sqrt{\left(\frac{1}{2} - \nu\right)^2 - 2\nu}\right).$$

As we are looking for the negative part of the parabola (8), we need the values of u_i that fall between values of $\frac{1}{2}\left(\nu - \frac{3}{2} \pm \sqrt{\left(\frac{1}{2} - \nu\right)^2 - 2\nu}\right)$.[7] However, for reasonable values of ν, $\frac{1}{2}\left(\nu - \frac{3}{2} - \sqrt{\left(\frac{1}{2} - \nu\right)^2 - 2\nu}\right) < 0$. Therefore, we only need u_i to be less than $\frac{1}{2}\left(\nu - \frac{3}{2} + \sqrt{\left(\frac{1}{2} - \nu\right)^2 - 2\nu}\right)$. $\square$

---

[7]By the way, for the square root not to be complex we need $\nu > 2.9142$.

D Demonstration of remark 2.

According to equation (2) and taking into account the cumulative density function of the normal distribution the profit of the firm i is

$$\pi_i = \left[\frac{S}{2} \left(1 - \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{p_i - \mu}{\sqrt{2\sigma^2}} \right) \right] + \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{p_j - \mu}{\sqrt{2\sigma^2}} \right) \right] \right) + U_i \right] (p_i - c).$$

Rewriting and using the definition of the error function this can be written as

$$\pi_i = \frac{S}{2} \left[1 + 2\frac{U_i}{S} + \frac{1}{\sqrt{\pi}} \left(\int_0^{\frac{p_j - \mu}{\sqrt{2\sigma^2}}} e^{-t} dt - \int_0^{\frac{p_i - \mu}{\sqrt{2\sigma^2}}} e^{-t} dt \right) \right] (p_i - c),$$

that can be further reduced to

$$\pi_i = \frac{S}{2} \left(1 + 2\frac{U_i}{S} + \frac{1}{\sqrt{\pi}} \int_{\frac{p_i - \mu}{\sqrt{2\sigma^2}}}^{\frac{p_j - \mu}{\sqrt{2\sigma^2}}} e^{-t} dt \right) (p_i - c).$$

Integrating out the integral part gives us the equation (2).

E Investigation into the low price equilibrium of the model.

Here we are investigating only the arrangements with homogenous firms. Therefore in all the instances discussed here $u_i = u_j = u$ and as a consequence $p_i = p_j = p$.

In the text we have shown that the profit function of the firms is given by the equation (2). Deriving producer's reaction function on continuous interval is problematic as price of the competitor goes higher there will be a point when the firm would give up competing for shared consumers and would rather concentrate on extracting monopolistic profit from the consumers unique to it. Therefore, at that price the reaction function has a jump.

This discontinuity complicates the analysis of the system and this is the reason why we have used computational/graphical approach in the paper. However, at values of p_j lower than the critical threshold, reaction function of the of firm i is continuous and we can solve for it. The first order conditions of profit function (2) imply that the reaction function of the firm i is given by

$$p_i = \sigma\sqrt{2}\left\{1 - \Omega\left[\sqrt{\pi}\left(1 + 2u_i\right)\exp\left(\frac{p_j}{\sigma\sqrt{2}}\right) - \exp\left(\frac{\mu}{\sigma\sqrt{2}}\right) - \exp\left(1 - \frac{\mu - c + p_j}{\sigma\sqrt{2}}\right)\right]\right\} + c,$$

where $\Omega(\cdot)$ denotes the Omega (Lambert's W or product logarithm) function.

Assuming the firm homogeneity this reaction function can be solved for the low price equilibrium.

$$p_L = c - \Omega\left[-\sqrt{\pi}\exp\left(\frac{c - \mu}{\sigma\sqrt{2}}\right)\left(1 + 2u\right)\right]\sigma\sqrt{2}.$$

It can be easily verified that $\partial p_L/\partial\sigma > 0$, $\partial p_L/\partial\mu < 0$ and $\partial p_L/\partial u > 0$. The intuition behind these result are given in the text in section 3.2.

Some comparative static exercises with respect to the model parameters are shown on figures 9 through 11.
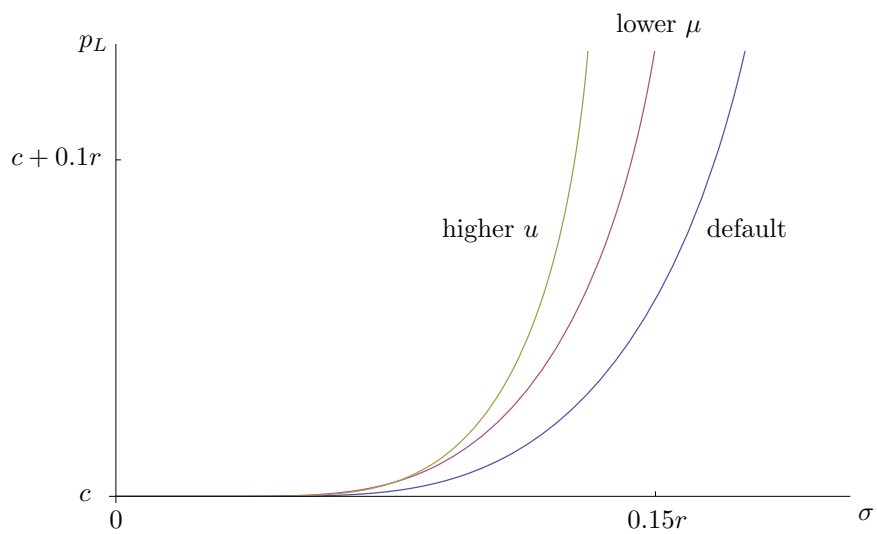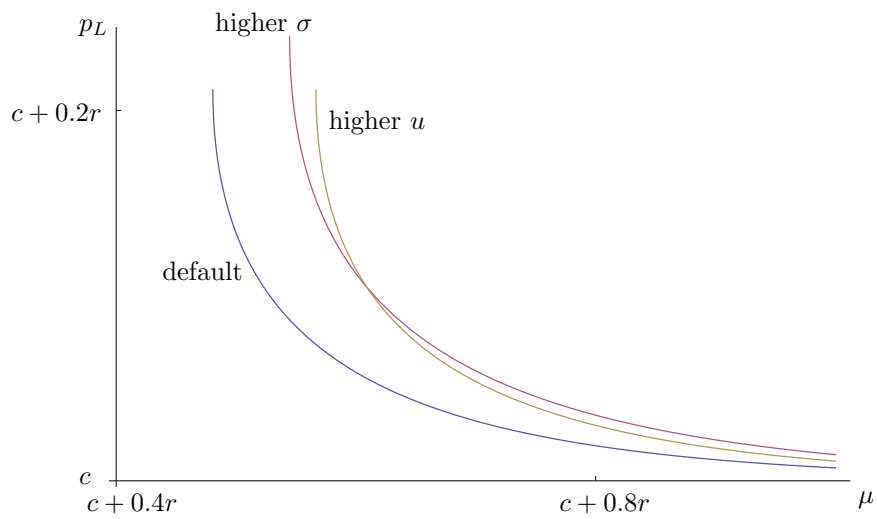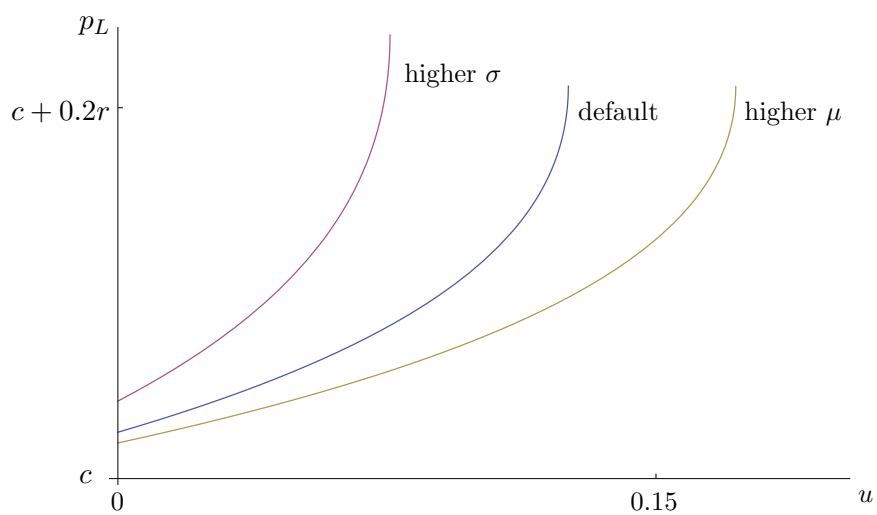
Figure 9: Comparative statics – σ.

Figure 10: Comparative statics – μ.

Figure 11: Comparative statics – u.