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SOCIAL SECURITY AND GROWTH IN AN AGING ECONOMY:
THE CASE OF ACTUARIAL FAIRNESS

Gilles Le Garrec
OFCE - Sciences Po

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Gilles Le Garrec

OFCE – Sciences Po

gilles.legarrec@ofce.sciences-po.fr

Abstract

In many European countries, due to population aging, the switch from conventional unfunded public pension systems to notional systems characterized by individual accounts is in debate. In this article, we develop an OLG model in which endogenous growth is based on an accumulation of knowledge driven by the proportion of skilled workers and the time they have spent to be trained. In such a framework, we show that conventional pension systems, contrary to notional systems, can enhance economic growth by linking benefits only to partial earnings history. Thus, considering economic growth, the optimal adjustment to aging could consist in increasing the size of existing retirement systems rather than switching to notional systems.

Keywords: social security, intertemporal choice, human capital

JEL classification: H55, D91, E24

1 Introduction

In 1950, life expectancy at birth in Western Europe was 68 years. Nowadays, it is 80 years and should reach 85 years in 2050 (United Nations, 2009). The downside of this trend is the serious threat that is hanging over the financing of our public retirement systems. Financed on a PAYG basis, i.e. pension benefits are paid through contributions of contemporary workers, they must cope with the increasingly larger number of pensioners compared to the number of contributors. With an unchanged average age of retirement, the ratio of pensioners to workers (the dependency ratio) should reach in France, for example, 70.1% in 2040, whereas it was 35.8% in 1990. Changes are unavoidable. If we want to guarantee in the near future the current level of benefits within the same system, it will be necessary either to increase the contribution rate or the length of contribution (by delaying the age of retirement).

This financing problem calls into question the role of PAYG retirement systems in our societies. For instance, by evaluating the real pre-tax return on non-financial corporate capital at 9.3%[1] and the growth rate over the same period (1960 to 1995) at 2.6%, Feldstein (1995a, 1995b, 1996) unequivocally advocates the privatisation of retirement systems and to opt for fully funded systems. He assesses the potential present-value gain to nearly $20 trillion for the United States. However, replacing conventional PAYG systems by financial -or funded- defined contribution (FDC) systems would certainly involve prohibitive social and political costs of transition. One generation will have to pay twice. Implementing such a reform in Western democracies ap-

[1] This return combines profits before all federal, state, and local taxes with the net interest paid. The method of calculation is described in Feldstein, Poterba and Dicks-Mireaux (1983).

pears then difficult. For that reason, in recent years a large focus has been put on non-financial -or notional- defined contribution (NDC) systems as legislated in Sweden in 1994. As described by Palmer (2006), NDC systems are PAYG systems which mimic FDC systems. Individual contributions are noted on individual accounts. Accounts are credited with a rate of return which reflects demographic and productivity changes. Obviously, replacing conventional PAYG systems by NDC systems does not adress the main concern of Feldstein (1995a, 1995b, 1996) that is the low return associated with PAYG financing method. However, supporters of the latter claim that the former, by linking pension benefits only partially to contributions, distort individual behaviors, inducing a reduced work effort or an earlier retirement. On this matter, they would defend actuarial fairness, versus progressivity, as a desirable feature.

On the basis of their pension benefit formulas, progressivity is one of the features most associated with conventional retirement systems. It is especially true in Anglo-Saxon countries where pensions are weakly related to earnings. However, going further into conventional systems reveals that pension benefit formulas cannot account for the true progressivity of retirement systems. Though the American system has one of the most progressive pension benefit formula (see OECD, 2007), all empirical studies stressed its low progressivity (Burkhauser and Walick, 1981; Garrett, 1995; Gustman and Steinmeier, 2001; Coronado et al., 1999, 2000; Brown et al., 2006). The pension benefit formula defines a poor index of progressivity because it does not take into account specificities related to gender, life expectancy or institutional features. First, the redistribution within the system is carried out from men towards women. Second, redistribution within the system is to the advantage of people who live longer and, as noted by Deaton and Paxton

(1998, 1999), differences in life expectancies are strongly related to social inequality: high-income earners live longer than low-income earners. Third, as argued by Lindbeck and Persson (2003) and Bozio and Piketty (2008), institutional features such as linking pensions to the best or last years tend to favor those with steep age-earnings profiles, i.e. again high-income earners. When considering these elements, Gustman and Steinmeier (2001) show that retirement system returns are almost identical whatever the household earnings. In the same line, Coronado et al. (2000) and Brown et al. (2006) show that the U.S. Social Security has no impact on the GINI index measuring income inequality. As the American pension benefit formula is one of the most progressive, most retirement systems in the industrial world appear, in fact, close to actuarial fairness[2] (see Stahlberg, 1990, for the Swedish system). As a consequence, we can not expect from NDC systems a significant decrease in the negative incentive effects associated with conventional systems.

In many respects, introducing a NDC system largely involves moving from a defined benefit to a defined contribution system to guarantee the stability of contributions in spite of aging populations. It may be pointed out that this objective can be achieved similarly within the scope of more conventional defined benefit systems, as seen in the "point system" in France or in Germany. In that case, the unit of pension rights is earnings points (not euros) and can be adjusted according to demographic and productivity changes as in a NDC system. As stressed by Börsch-Supan (2006), cleverly designed conventional retirement systems can often do the same job as NDC

[2] Strictly speaking, a retirement system is said actuarially fair if its return is equal to the interest rate (Lindbeck and Persson, 2003; Cigno, 2008). Considering that the economic growth rate, which is the retirement system return, is lower than the interest rate, retirement systems could be described more properly as quasi-actuarial fair as noted by Lindbeck and Persson (2003).

systems.

In this article, observing that most conventional retirement systems are actuarially fair, we then compare the latter with NDC systems. In particular, focusing on the age-earnings profile, we investigate the relation between pension benefits and earnings history. Indeed, as pointed out by Lindbeck and Persson (2003) and Bozio and Piketty (2008), the way pension benefits are calculated when considering heterogenous work histories and age-earnings profiles can have important consequences in terms of income redistribution, even when comparing actuarially fair systems. From this perspective, this article also relates to the literature studying the impact of retirement systems on the investment in human capital and on economic growth. Theoretically, if considering that economic growth is driven by physical capital accumulation, by reducing private savings, PAYG retirement systems are harmful for economic growth (Saint-Paul, 1992; Belan et al.[3], 1998). However, empirical findings from Sala-i-Martin (1996) and Zhang and Zhang (2004) tend to support a positive impact of retirement systems on economic growth through the human capital channel. In this line, Zhang (1995), Sala-i-Martin (1996), Kemnitz and Wigger (2000), Le Garrec (2001) and Zhang and Zhang (2003) have therefore shown that PAYG retirement systems could stimulate economic growth by stimulating investment in education. Interestingly, these results have been obtained in models with identical learning ability of individuals. By contrast, when considering heterogenous learning ability, Docquier and Paddison (2003) show that conventional retirement systems can not enhance economic growth even when economic growth is driven by investment in education. To explain these conflicting results, one can observe that in

[3] In that case, they show that the transition from PAYG to funded systems could be Pareto-improving.

Kemnitz and Wigger (2000) and Le Garrec (2001), the positive impact of conventional PAYG retirement systems on economic growth goes through the lengthening of training, while the negative impact in Docquier and Paddison (2003) corresponds to the decrease of the proportion of individuals who decide to train themselves when considering a fixed training length. By embeding both effects, Le Garrec (2012) then shows that the positive effect always dominate the negative one, at least for low contribution rates.

In this article, we extend the literature into two directions. First, following Le Garrec (2012), we consider investment in human capital both through the proportion of individuals who decide to invest and the time they invest. However, by not specifying a particular distribution of learning abilities, we can provide explicit and general conditions so that the positive effect associated with the lengthening of training may be dominated by the negative effects, i.e. the decrease of the proportion of educated individuals. We then show that economic growth may exhibit an inverse U-shaped pattern with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years, while a NDC system has no impact on economic growth. Second, we consider the aging process not through decreased fertility as is usual, but through increased longevity. It has important consequences. Indeed, as increased longevity raises the value of investments that pay over time, it also encourages investment in education as well documented in the literature[4]. Therefore, social security interacts with longevity in determining the individual investment in education. We then show that

[4]See de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Boucekkine et al. (2002), Cervellati and Sunde (2005, 2011), Soares (2005), Jayachandran and Lleras-Muney (2009). Challenging the conventional wisdom, Acemoglu and Johnson (2007) find no effect of life expectancy on schooling.

increased longevity may raise the size of a conventional retirement system rate which maximizes economic growth. This result suggests that the optimal adjustment to aging could consist in increasing the size of existing retirement systems rather than switching to notional systems.

The rest of the paper is organized as follows. In section 2, we present the basic assumptions related to the age-earnings profiles and the calculation of pension benefits. In section 3, we analyse optimal behaviors of individuals and firms considering the basic assumptions. We assume in particular that individuals differ in their learning abilities as in Docquier and Paddison (2003) and Le Garrec (2012). In section 4, we specify the equilibrium features whith actuarially fair retirement systems. In section 5, we then show that actuarially fair retirement systems, depending on their size and on the calculation of pension benefits, can enhance economic growth. In section 6, we then specify optimal adjusments regarding economic growth when longevity increases. In the last section, we briefly conclude.

2 Earnings profile and pension benefits: basic assumptions

The model is an extended version of the Ben-Porath model (1967) with uncertain lifetimes. Individuals live either for two or three periods: they are respectively young, adult, and old. Survival is complete through adulthood. Each adult has a probability $\rho \in (0, 1)$ to survive to old age. The size of the young generation is normalized to one at each date. Due to complete survivance, the size of the adult generation is also equal to one at each date, whereas the size of the retired generation is equal to ρ. Aging then occurs in the model through increased longevity.

2.1 Human capital and age-earnings profiles

When young, individuals go to school. During this period which corresponds to primary and secondary education (compulsory schooling), individuals born in $t-1$ learn basic knowledge represented by the average knowledge $\overline{Z}_{t-1}$ of the contemporary working generation. In addition, they can choose to make an effort e_{t-1} in learning (where $e_{t-1} = 0$ or 1) to pass the final secondary school examination, qualifying for university entrance. In the second period, those who have made the effort can then complement their basic knowledge by pursuing training during a period h_t instead of entering directly the labor market[5]. At the end of their complementary training, their human capital is characterized by:

$$Z_t^s = Bh_t^\delta \overline{Z}_{t-1},\ B > 0,\ \ \delta > 0 \tag{1}$$

where δ denotes the return to complementary training in terms of human capital.

Skilled workers, those who have completed their training before entering the labor market, are thus characterized by a first period h_t with no earnings. Afterwards, they earn $Z_t^s w_t$, where w_t is the wage rate per unit of effective labor. Earnings of skilled workers W_t^s over their whole active period are thus:

$$W_t^s = (1 - h_t)\, Z_t^s w_t \tag{2}$$

and are then characterized by a steep profile. By contrast, unskilled workers are characterized by the basic human capital during all their working period:

[5]In that case training is a full-time activity which can be assimilated to higher education. We could have assumed alternatively that training is a part-time activity without changing the qualitative results (see Le Garrec, 2005).

$$Z_t^u = \overline{Z}_{t-1} \tag{3}$$

and are then characterized by flat age-earnings profiles:

$$W_t^u = Z_t^u w_t \tag{4}$$

From eqs. (1)-(4), making sure that skilled workers earn more than unskilled workers during their whole active period requires that:

$$(1 - h_t)\, B h_t^{\delta} > 1 \tag{5}$$

In a simple way, the economy is then characterized in line with Lilliard (1977) and Andolfatto et al. (2000) by age-earnings profiles of workers increased with the time spent in training and by high-school dropouts with flat age-earnings profiles.

2.2 Pension benefits

In conventional systems, the calculation of pension benefits is specific to each country, and sometimes can be very complex. In the theoretical literature on social security[6], two different parts are generally distinguished: a redistributive part (the Beveridgean part) characterized by a basic flat-rate benefit, and an insurance part (the Bismarckian part) characterized by earnings-related benefits. The latter is not generally proportional to all contributions and then not based on full lifetime average earnings (see OECD, 2007). It is particularly the case in Greece and Spain where benefits are only linked to final salary. It also used to be the case in Sweden before the 1994 legislation

[6] See Casamatta et al. (2000), Docquier and Paddison (2003), Sommacal (2006), Cremer et al. (2007), Hachon (2010), Le Garrec (2011).

introducing NDC systems. In France, before the Balladur reform of 1993, earnings-related benefits were linked to the ten best years, then gradually to the 25 best years after the reform. In the United States, the 35 best years are considered to calculate the benefits, 20 in Norway.

Let us define $\tilde{W}_t^i$, $i = s, u$, as the representative earnings on which benefits are based in a conventional system. It does not matter which period is used to calculate the unskilled representative earnings because the age-earnings profile is consistently flat. It follows that:

$$\tilde{W}_t^u = W_t^u \tag{6}$$

For the skilled workers, as the reference earnings $\tilde{W}_t^s$ corresponds to the best or last years, it is specified as:

$$\tilde{W}_t^s = Z_t^s w_t \tag{7}$$

Assuming that the basic flat-rate benefit $\overline{p}_{t+1}$ is linked to the contemporary wage of unskilled workers[7], the calculation of pension benefits for any worker in t in a conventional system is then given by:

$$p_{CONV,t+1} = \theta_{t+1}\tilde{W}_t + \nu_{t+1}W_{t+1}^u \tag{8}$$

where ν_{t+1} represents the size of the flat-rate component of the pension benefits and θ_{t+1} the size of the earnings-related component.

As noted in the introduction, most conventional retirement systems of industrialized economies are close to actuarial fairness. In terms of the retirement system implicit return, i.e. the ratio between the expected pension benefits of an individual and the amount of his contributions, this means that:

[7] It is designed to ensure that pensioners achieve some minimum standard of living.

$$\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} \approx \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}} \tag{9}$$

where τ denotes the public pension system contribution rate, and p^i_t denotes the pension benefits in t of a worker of type i in $t-1$, $i = u, s$[8]. If $\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} > \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}}$, then the retirement system is fiscally favorable to low-income earners. In this case the system is progressive. In the opposite case, $\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} < \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}}$, it is regressive.

Consider alternatively a NDC system. In that case, individual contributions are noted on individual accounts which are credited with a factor of return ψ. By definition, such a pure contributory system whose pensions are calculated proportionally to all contributions is actuarially fair. Explicitly adjusted to life expectancy, pension benefits are then as follows:

$$p_{NDC,t+1} = \frac{\psi_{t+1}}{\rho} \tau W_t \tag{10}$$

For convenience, we will further note the calculation of pension benefits as:

$$p_{t+1} = \xi \left(p_{CONV,t+1}\right) + (1-\xi)\, p_{NDC,t+1} \tag{11}$$

where $\xi = 1$ for a conventional system, $\xi = 0$ for a NDC system.

[8] If we had considered socioeconomic inequalities in mortality, actuarial fairness would have been defined as $\frac{\rho^u p^u_{CONV,t}}{\tau W^u_{t-1}} \approx \frac{\rho^s p^s_{CONV,t}}{\tau W^s_{t-1}}$, where $\rho^s \geq \rho^u$.

3 Optimal behaviors

3.1 Individuals

As specified in the previous section, individuals live for three periods. They invest in education in the first and possibly in the second period, work in the second one and retire in the third one with probability ρ. Preferences of an individual of type x born in $t-1$ are described by the following utility function:

$$U_x = \ln c_t + \beta\rho \ln d_{t+1} - \sigma_x e_{t-1} \tag{12}$$

where c_t and d_{t+1} denote, respectively, his consumption when adult and when old[9], and $\beta \leq 1$ denotes the subjective discount factor. The utility from uncertain lifetime consumption is based on Yaari (1965), as in Abel (1985) and in Zhang et al. (2001, 2003). σ_x denotes the utility cost of schooling effort, where $\sigma \in [0, \infty)$ represents learning ability. As shown by Huggett et al. (2006), earnings differences are first explained by differences in learning ability across individuals. In our setting, a talented child characterized by $\sigma = 0$ endures no cost in making the effort. By contrast, a lazy or untalented child characterized by $\sigma \to \infty$ will endure an infinite cost and will then always choose not to make the effort, i.e. $e_{t-1} = 0$. Note that σ can be considered as an inherited (perfectly here) trait which represents both family background and genetic transmission[10]. We denote $G(\sigma)$ the cumulative dis-

[9] As in Boldrin and Montes (2005) and Docquier et al. (2007), we assume that the only decision of children concerns education as their consumption is part of their parents' consumption. As a consequence and without loss of generality, consumption when young does not appear in the utility function.

[10] Stating that earnings are very significantly tied to the earnings of the parents (Bowles and Gintis, 2002, d'Addio, 2007), this suggests that the intergenerational earnings per-

tribution function of learning ability through the population, and we assume it is of class C^2.

During the second life period, individuals consume a part of their disposable income, and save via a perfect annuity market such as:

$$c_t + s_t = W_t\left(1 - \tau\right) \tag{13}$$

where s_t denotes private savings.

In the third life period, old-age survivors are retired. They get back their savings with interest, receive their pension from the public retirement system and consume their wealth. The budget constraint is then:

$$d_{t+1} = \frac{R_{t+1}}{\rho} s_t + p_{t+1} \tag{14}$$

where R_{t+1} denotes the real interest factor. Note that, with a perfect annuity market, old-age survivors share the savings of deceased individuals. The expected return to savings is then equal to the actuarially fair factor $\frac{R_{t+1}}{\rho}$ as in Zhang et al. (2001). The alternative would be the existence of unvolontary bequests as in Abel (1985) and Zhang et al. (2003).

Let $\Omega_t^i = W_t^i\left(1 - \tau\right) + \frac{\rho p_{t+1}^i}{R_{t+1}}$ be the expected lifetime income of a worker of type i, $i = u, s$. Considering the calculation of pension benefits (11), an individual who has chosen to make the effort at school will maximize his

sistence is based on the inheritability of learning ability within families. Supporting such a view, education is a major contributor to intergenerational earnings mobility and educational differences tend to persist across generations (d'Addio, 2007). Nevertheless, as shown by Bowles and Gintis (2002), it does not imply that the intergenerational earnings determination is only based on genetic transmission. Learning ability also reflects noncognitive personality traits such as, for example, a taste for learning at school which can be influenced by the family background as much as by the genes.

lifetime income by spending the following time in training during his second life period:

$$h_t = \inf\left\{h^0\left[1 + \frac{1}{1-\tau}\frac{\rho\xi\theta_{t+1}}{R_{t+1}}\right];1\right\} \tag{15}$$

where $h^0 = \frac{\delta}{1+\delta}$ is the training length with no retirement system.

Proposition 1 *Linking pension benefits to partial earnings history generates an incentive to be trained longer.*

Conventional retirement systems whose pension benefits are based, even partially, on the best or last years generate an incentive for longer training. Initially, the lengthening of training has a negative effect on income. During this period individuals have indeed no earnings capacity. However, they earn more afterwards. In addition, as pensions are linked to the best or last years they also benefit, all things being equal, from an increase in their benefits. Following equation (15), individuals who undertake training may find it profitable to be trained longer as an investment in their pension benefits. Note that this incentive disappears completely if pension benefits are based on full lifetime average earnings ($\xi = 0$), or if the system is totally flat-rate ($\theta_{t+1} = 0$). Moreover, this incentive is weaker as the interest rate increases. Indeed, the higher the interest rate, the lower the present actuarial value of pension benefits.

To summarize, the incentive to be trained longer, generated by conventional retirement systems, is due to the interaction of two factors:

- pension benefits are linked to the best or last years
- training results in steeper age-earnings profiles

The utility maximization of an individual subject to budgetary constraints (13) and (14) leads to the following saving function:

$$s_t = \frac{\beta\rho}{1+\beta\rho} W_t \left(1-\tau\right) - \frac{\rho}{1+\beta\rho} \frac{p_{t+1}}{R_{t+1}} \tag{16}$$

By reducing simultaneously the disposable income and the need for future income, a retirement system reduces private savings. This result holds irrespective of the calculation of pension benefits and their financing.

Last, an individual will choose to make the effort at school if the opportunity of complementary training entails a monetary benefit higher than the utility cost associated with the effort, i.e. if $(1+\beta\rho)\ln\Omega_t^s - \sigma_x e_{t-1} \geq (1+\beta\rho)\ln\Omega_t^u$. Considering an interior solution, the proportion of individuals q_t who choose to be trained in t (and then to make the school effort in $t-1$) and become skilled workers is defined by:

$$q_t = \bar{e}_{t-1} = G\left[(1+\beta\rho)\ln I_t\right] \tag{17}$$

where $I_t = \frac{\Omega_t^s}{\Omega_t^u}$ represents the lifetime income inequality between skilled and unskilled workers in t. Following (17), the higher this inequality, the larger the proportion of individuals incited to be trained: $\frac{dq_t}{dI_t} > 0$.

3.2 Firms

We consider a competitive sector characterized by a representative firm producing a good, which can be either consumed or invested, according to a Cobb-Douglas technology with constant return to scale:

$$Y_t = F\left(K_t, L_t^u, L_t^s\right) = AK_t^{\alpha}\left(Z_t^u L_t^u + \left(1-h_t\right) Z_t^s L_t^s\right)^{1-\alpha},\ 0<\alpha<1, \tag{18}$$

where Y_t denotes the output, K_t the physical capital stock, L_t^i the number of worker of type i in t, $i = u, s$, and A the total factor productivity. Assuming for simplicity as in Docquier and Paddison (2003) and Le Garrec (2012) that skilled and unskilled labors are perfect substitutes[11], $H_t = Z_t^u L_t^u + (1 - h_t) Z_t^s L_t^s$ represents the labor supply in efficiency units.

Denoting per capita efficient capital by $k_t = \frac{K_t}{H_t}$ and assuming a total capital depreciation, the optimal conditions resulting from the maximization of the profit are:

$$R_t = A\alpha k_t^{\alpha-1} \tag{19}$$

$$w_t = A(1-\alpha) k_t^{\alpha} \tag{20}$$

Before studying the impact of retirement systems and the calculation of pension benefits on economic growth, we have to characterize the equilibrium and its properties.

4 Equilibrium

The economy is composed of four markets corresponding to the unskilled labor, the skilled labor, the physical capital and the good. In a closed-economy setting, the general equilibrium can be obtained by considering only the clearing of three markets, as according to the Walras law, the fourth is necessarily cleared. In our case, we consider the clearing of the following

[11] Assuming alternatively they are imperfect substitutes would affect the skill choice by introducing a wage premium for human capital. However, it would not change the training length as defined in eq. (15) at all. The incentive to be trained longer as specified in Proposition 1 would then not be affected.

markets:

unskilled labor:

$$L_t^u = (1 - q_t) \tag{21}$$

skilled labor:

$$L_t^s = q_t \tag{22}$$

physical capital:

$$K_{t+1} = q_t s_t^s + (1 - q_t)\, s_t^u \tag{23}$$

4.1 PAYG social security and the capital accumulation

Retirement systems have PAYG features, i.e. within a period, pension benefits are financed by contributions of workers of the same period. In other words, retirement systems transfer workers' income towards pensioners. Since workers are either skilled or unskilled, the social security balanced budget is defined as follows:

$$\rho L_{t-1}^u p_t^u + \rho L_{t-1}^s p_t^s = \tau \left[L_t^u W_t^u + L_t^s W_t^s \right] \tag{24}$$

Since at date t there is a proportion q_t and $1 - q_t$ of respectively skilled and unskilled workers as specified in eqs. (21) and (22), the balanced budget of the retirement system (24), with eqs. (1)-(11), is rewritten as:

$$\theta_t = \left[\frac{\tau}{\rho} \left[q_t \left(1 - h_t\right) B h_t^\delta + 1 - q_t \right] - \nu_t \right] \frac{w_t}{w_{t-1}} \quad \text{if } \xi = 1 \tag{25}$$

or:

$$\psi_t = \frac{q_t\left(1-h_t\right)Bh_t^{\delta}+1-q_t}{q_{t-1}\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}+1-q_{t-1}}\left(q_{t-1}Bh_{t-1}^{\delta}+1-q_{t-1}\right)\frac{w_t}{w_{t-1}} \quad \text{if } \xi=0 \tag{26}$$

Considering the social security balanced budget, either eq. (25) for a conventional system or eq. (26) for a NDC system, and the physical capital market clearing (23), with eqs. (1)-(11), (16), (19) and (20), the dynamics of capital accumulation in the model can be expressed independently of the calculation of pension benefits as:

$$\begin{aligned} & k_{t+1}\left[q_{t+1}\left(1-h_{t+1}\right)Bh_{t+1}^{\delta}+1-q_{t+1}\right] \\ = \; & \frac{A\alpha\beta\rho\left(1-\alpha\right)\left(1-\tau\right)}{\alpha\left(1+\beta\rho\right)+\tau\left(1-\alpha\right)}\frac{q_t\left(1-h_t\right)Bh_t^{\delta}+1-q_t}{q_tBh_t^{\delta}+1-q_t}k_t^{\alpha} \end{aligned} \tag{27}$$

As retirement systems reduce private savings (eq. 16), all things being equal, PAYG retirement systems are harmful for the accumulation of physical capital: $\frac{\partial \frac{1-\tau}{\alpha(1+\beta)+\tau(1-\alpha)}}{\partial \tau} < 0$ (eq. 27). In addition, as q and h are both forward-looking variables, their specification is crucial to determine the dynamic properties of the model and the convergence towards its steady-state (balanced growth) path.

4.2 Human capital and actuarial fairness

As is obvious, a NDC system ($\xi = 0$) is actuarially fair. As noted in the introduction and characterized by eq. (9), most conventional retirement systems in the industrial world are also close to actuarial fairness.

Proposition 2 *Conventional retirement systems whose pensions are linked to the best or last years are actuarially fair if including a flat-rate component indexed on the unskilled earnings,* $\overline{p}_t = \nu_t W_t^u$, *such as* $\nu_t = \widetilde{\nu}_t = \frac{Bh_{t-1}^{1+\delta}}{Bh_{t-1}^{\delta}-1}\frac{\tau}{\rho}\frac{q_t(1-h_t)Bh_t^{\delta}+1-q_t}{q_{t-1}(1-h_{t-1})Bh_{t-1}^{\delta}+1-q_{t-1}}$.

If $\nu_t > \widetilde{\nu}_t$, the retirement system is fiscally favorable to low-income earners, $\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} > \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}}$, and is then progressive. In the opposite case, $\nu_t < \widetilde{\nu}_t$, it is regressive. This feature is easily understandable. On one hand, the flat-rate part of the pension benefits is clearly favorable to low-income earners: they receive as much as high-income earners whereas they have contributed less. A flat-rate system is obviously progressive. On the other hand, the pension part which is linked to the best or last years, characterized by θ_t, is favorable to high-income earners as they have a steeper lifetime income profile, as explained by Lindbeck and Persson (2003) and Bozio and Piketty (2008). If there is no flat-rate part then the system is regressive. Therefore, there is a unique combination of the flat-rate and earning-related parts, the one defined in Proposition 2, that characterizes actuarial fairness in a conventional system.

Consider an actuarially fair retirement system, i.e. either $\xi = 0$ or $\xi = 1$ and $\nu_{t+1} = \widetilde{\nu}_{t+1}$. In such a case, the lifetime income inequality $I_t = \frac{\Omega^s_t}{\Omega^u_t}$ becomes $I_t = \frac{W^s_t}{W^u_t}$. Using eqs. (1)-(4), the proportion of skilled workers in t defined by eq. (17) becomes:

$$q_t = G\left([1+\beta\rho]\ln\left[(1-h_t)\,Bh^{\delta}_t\right]\right) \tag{28}$$

In this configuration, the choice for a young individual to make the effort at school in $t-1$ to become a skilled worker in t depends only on his personal talent, his life expectancy and the length of the training he anticipates to complete. As h^0 corresponds to $\max\left\{(1-h)\,Bh^{\delta}\right\}$, we can deduce from eq. (28) that any increase in the training length compared to the basic level h^0 will lead to a decrease in the skilled workers proportion: $\left.\frac{\partial q_t}{\partial h_t}\right|_{h_t \geq h^0} \leq 0$. Following Proposition 1, we can then expect that conventional actuarially fair retirement systems whose pension benefits are based on partial earnings history reduce the proportion of skilled workers.

When the retirement system is purely contributory as a NDC system ($\xi = 0$), it has no impact on the training length in the second period of life (eq. 17). By contrast, as characterized by eq. (15), linking pension benefits to partial earnings history as in a conventional system ($\xi = 1$) generates an incentive to be trained longer which depends crucially on the actualized Bismarckian component $\frac{\theta_{t+1}}{R_{t+1}}$. Using eqs. (19)-(20) and (25)-(27) yields $\frac{\rho\theta_{t+1}}{R_{t+1}} = \left[\frac{(1-h_t)Bh_t^{\delta}-1}{Bh_t^{\delta}-1}\right]\frac{\beta\rho(1-\alpha)(1-\tau)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}$. Thus, the training length according to the social security features can be summarized as:

$$h_t = \begin{cases} h^0 & \text{if } \xi = 0 \\ h^0\left[1 + \frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{(1-h_t)Bh_t^{\delta}-1}{Bh_t^{\delta}-1}\right] & \text{if } \xi = 1 \text{ and } \nu_{t+1} = \widetilde{\nu}_{t+1} \end{cases} \tag{29}$$

If $\xi = 1$ and $\nu_{t+1} = \widetilde{\nu}_{t+1}$, we derive from (29) that $\lim\limits_{h\to h^0} RHS > h^0$ and $\lim\limits_{h\to 1} RHS < h^0$. In this case, the training is expressed as a function $h_t = h(\tau,\rho)$ such as $h^0 \leq h(\tau,\rho) \leq 1$. In the case of a NDC system, as the latter has no impact on the training length, we will note conveniently $h_t = h(\xi\tau,\rho)$, where $\xi = 0$, i.e. $h(0,\rho) = h^0 = \frac{\delta}{1+\delta}\ \forall\rho$. Thereafter, as the skill choice depends only on the training length and on the longevity (eq. 28), it can also be expressed as $q_t = q(\xi\tau,\rho) = Q(h(\xi\tau,\rho),\rho)$, where $\xi = 1$ or $\xi = 0$. In the latter case, $q_t = Q(h(0,\rho),\rho) = Q(h^0,\rho)$ corresponds to an unchanged proportion of skilled workers in t compared to a situation with no retirement system: $q(0,\rho) = G\left([1+\beta\rho]\ln\left[(1-h^0)Bh^{0\delta}\right]\right)$.

4.3 Dynamic properties

As underlined by eqs. (28) and (29), human capital variables are in their steady-state values independently of the calculation of pension benefits. Accordingly, considering an actuarially fair retirement system, the physical capital accumulation dynamics (27) can be rewritten as:

$$k_{t+1} = \frac{A\alpha\beta(1-\alpha)(1-\tau)}{\alpha(1+\beta)+\tau(1-\alpha)} \frac{1}{Q(h(\xi\tau,\rho),\rho)\, Bh(\tau,\rho)^{\delta} + 1 - Q(h(\xi\tau,\rho),\rho)} k_t^{\alpha} \tag{30}$$

Since $\alpha < 1$, given $k_0 > 0$, the model has good dynamic properties and converges to its steady-state (balanced growth) path characterized by $h = h(\xi\tau,\rho)$, $q = Q(h(\xi\tau,\rho),\rho)$ and $k = \left[\frac{A\alpha\beta\rho(1-\alpha)(1-\tau)}{\alpha(1+\beta\rho)+\tau(1-\alpha)} \frac{1}{qBh^{\delta}+1-q}\right]^{\frac{1}{1-\alpha}}$, where $\xi = 0$ or $\xi = 1$.

As the convergence is verified, the impact of retirement systems and the calculation of pension benefits on investment in human capital and on growth can now be discussed.

5 Social security and economic growth

On the balanced growth path, we deduce from the labor market clearing relations (21) and (22) as well as eqs. (1), (3) and (18) the economic growth rate g:

$$1 + g = \frac{Y}{Y_{-1}} = \frac{\bar{Z}}{\bar{Z}_{-1}} = 1 + q\left(Bh^{\delta} - 1\right) \tag{31}$$

In line with the new growth literature initiated by Lucas (1988) and Romer (1990), equation (31) stresses that long-term economic growth positively depends on the rate of knowledge accumulation which is driven both by the proportion of skilled workers in the economy (i.e. those who have made the effort at school) and the length of training. From this perspective, it is worth noting that, after mixed empirical supports (see Benhabib and Spiegel, 1994; Bils and Klenow, 2000), the positive impact of education on economic growth has received more recently a clear support from empirical studies conducted with improved data quality (see de la Fuente and Domenech, 2006;

Cohen and Soto, 2007). We then study how NDC and conventional systems impact differently economic growth through change in the training length and the proportion of skilled workers.

5.1 NDC systems

A pure contributory NDC system ($\xi = 0$) has no impact on the training length (eq. 17). As a consequence, as underlined in equation (28), it has also no impact on the proportion of skilled workers. Indeed, in that case, as pension benefits are proportional to all contributions the retirement system can no longer alter the skill choice. NDC systems are then characterized by an unchanged investment in human capital, i.e. $h = h^0$ and $q = Q\left(h^0, \rho\right)$, and it follows that:

Proposition 3 *NDC systems have no impact on economic growth.*

5.2 Conventional systems

Consider alternativelly conventional systems whose pensions are linked to the best or last years, i.e. $\xi = 1$. Admitting they are actuarially fair, i.e. $\nu = \widetilde{\nu}$, following eq. (29) the training is specified by $h = h^0\left[1 + \frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{(1-h)Bh^\delta-1}{Bh^\delta-1}\right]$, where $\lim\limits_{h\to h^0} RHS > h^0$ and $\lim\limits_{h\to 1} RHS < h^0$. This equation thus defines a relation between the training and the contribution rate of the retirement system such as $h = h\left(\tau, \rho\right) < 1$ and $\frac{\partial h}{\partial \tau} > 0$. In addition, as the skill choice is specified by $q = G\left(\left[1+\beta\rho\right]\ln\left[\left(1-h\right)Bh^\delta\right]\right)$ where $h \geq h^0$, it follows that $\frac{\partial q}{\partial \tau} \leq 0$. The negative impact of conventional systems on the proportion of skilled workers can appear at first glance counter-intuitive. Indeed, as such a system results in the lengthening of the skilled workers' training, it raises the difference between skilled and unskilled workers' earnings. However, from a life cycle perspective, with no retirement system or with a pure contributory system (Proposition 1), individuals who decide to undertake training choose the length h^0 which maximizes their expected lifetime income; h^0 thus maximizes the lifetime income inequality between skilled and unskilled workers. A lengthening of the training thus raises lifetime income inequality when $h < h^0$. Conversely, when $h > h^0$, a lengthening of the training reduces lifetime income inequality because we move away from the individually optimal training length. Therefore, even if the retirement system does not carry out transfers from high-income to low-income earners, we know from Proposition 1 that such a earnings-related pension benefit formula generates an incentive for longer training. Skilled workers are then encouraged to train themselves more than their individually optimal level. Consequently, actuarially fair retirement systems whose pensions are linked to the best or last years reduce lifetime income inequality compared to a situation with no retirement system (or purely contributory as NDC systems) and then reduce the proportion of skilled workers (eq. 17). Denoting ε_Q^h the elasticity of Q with respect to h, the following Proposition then holds:

Proposition 4 *As long as* $0 < Q\left(h^0, \rho\right) < 1$*, assuming* $d^2G\left(.\right) \leq 0 \; \forall \tau \geq 0$*,* $-\varepsilon_Q^h > \frac{Bh^\delta \delta}{Bh^\delta - 1}$ *for* $\tau = 1$ *is a necessary and sufficient condition such as economic growth exhibits an inverse U-shaped pattern with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years.*

By reducing the proportion of skilled workers in the economy, actuarially fair conventional systems negatively impact economic growth. On the other hand, they incite skilled workers to train longer. For sufficiently low size of the system, the latter effect always dominates the former and we can stress a positive impact of PAYG retirement systems on economic growth as empirically reported by Sala-i-Martin (1996) and Zhang and Zhang (2004). Initiated by the lengthening of training, the underlined mechanism is directly related to Kemnitz and Wigger (2000) and Le Garrec (2001)[12].

However, when the size of the system increases, everything else being equal, the leading effect can reverse if lifetime income inequality strongly matters in the skill choice, or more formally if $-\varepsilon_Q^h > \frac{Bh^\delta \delta}{Bh^\delta - 1}$, where $-\varepsilon_Q^h = \varepsilon_Q^I \left(\frac{h}{1-h} - \delta\right)$, ε_Q^I being the elasticity of Q with respect to I. In that case, similarly to Docquier and Paddison (2003), conventional PAYG retirement systems based on partial earnings history are harmful for economic growth because they first reduce the proportion of skilled workers. We then shed light on the existence of an inverse U-shaped pattern of economic growth with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years. Moreover, it sustains the existence of an optimally designed retirement system regarding economic growth which is not an NDC system but a conventional system based on partial earnings history. At least if its size is not too high compared to the optimal size. Indeed, in that case economic growth with a conventional system could be potentially lower than with no system or with a NDC system. To illustrate this point, consider for example any distribution characterized by $(1+\beta\rho)\ln\left[(1-h(1,\rho))\,Bh(1,\rho)^\delta\right] < \sigma_{\min} < (1+\beta\rho)\ln\left[(1-h(0,\rho))\,Bh(0,\rho)^\delta\right]$. In this case, it exists $\tilde{\tau} < 1$ such as $\tau \geq \tilde{\tau}$ entails $g = 0$ with a conventional system whereas $g = \left(Bh^{0\delta} - 1\right) Q\left(h^0, \rho\right) > 0$ with a NDC system (or with

[12] In Zhang (1995) and Zhang and Zhang (2003), PAYG retirement systems result in more growth by reducing fertility of altruistic parents who consequently invest more in the education of their children. In Sala-i-Martin (1996), old workers are associated with negative externalities in the average stock of human capital. By inducing earlier retirement, PAYG retirement systems then stimulate growth.

no retirement system). It stresses the importance of evaluating the impact of aging on economic growth and the size of the conventional system which maximizes it to determine the desirable adjustment.

6 Aging and optimal growth

6.1 Longevity, education and growth

As noted in the introduction, the coming century will be characterized by increased longevity. Nowadays equal to 80 years, life expectancy at birth should reach 85 years in 2050 in Western Europe (United Nations, 2009). It will have important consequences on public finance. It will have also important consequences for individuals, involving significant changes in their choices. First, individuals will need to finance a longer period in retirement. With low pension benefits, they will inevitably need to save more before retirement. As increased longevity raises the value of investments that pay over time, it will also encourage investment in education. For an economy with high life expectancy, Kalemli-Ozcan et al. (2000) have hence estimated the elasticity of schooling years with respect to life expectancy to 0.7. Allowing economic growth to be driven by investment in education, we can also expect from an increased longevity a positive impact on economic growth.

Proposition 5 *With no retirement system (or with a NDC system), increased longevity stimulates economic growth by increasing the proportion of skilled workers while letting unchanged the time they have spent to be trained.*

This result is directly related to Proposition 1. Indeed, if there is no retirement system or a NDC system, the training length is $h = h^0 = \frac{\delta}{1+\delta}$. In that case, there is then no impact of aging on the length of training. The impact only comes from a change in the proportion of skilled workers such as, following eq. (28), $\frac{\partial Q}{\partial \rho} = dG\left(.\right)\beta \ln\left[\left(1-h^{0}\right)Bh^{0\delta}\right] \geq 0$.

By contrast, in the case of an actuarially fair conventional system ($\xi = 1$ and $\nu = \widetilde{\nu}$) it follows from eq. (29) that the training length increases with longevity: $\frac{\partial h}{\partial \rho} = \frac{\delta \frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1} \frac{\beta(1-\alpha)[\alpha+\tau(1-\alpha)]}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^{2}}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^{2}}\right]} > 0$[13] $\forall \tau > 0$. Indeed, such a retirement system provides incentives to invest in pension benefits through longer training. As increased longevity favors investments that pay out over time, it then increases the training length. The impact on the proportion of skilled workers is therefore no more trivial. On one hand, everything else being equal, an increased longevity encourages individuals to become skilled worker: $\frac{\partial Q}{\partial \rho} \geq 0$. On the other hand, as $h > h^{0}$, the induced lengthening of the training reduces lifetime income inequality and then $\frac{\partial Q}{\partial h} \leq 0$.

Proposition 6 *With an actuarially fair retirement system whose pensions are linked to the best or last years, increased longevity enhances economic growth both by increasing the proportion of skilled workers and the time they have spent to be trained if the latter is moderate enough.*

Formally, the condition in Proposition 6 applies as long as $\frac{\partial h}{\partial \rho} \leq \frac{\beta}{1+\beta\rho}\frac{\ln\left[(1-h)Bh^{\delta}\right]}{\frac{1}{1-h}-\frac{\delta}{h}}$. As $\lim\limits_{\tau=0}\frac{\partial h}{\partial \rho} = 0$ and $\lim\limits_{\tau=0}\left(\frac{1}{1-h}-\frac{\delta}{h}\right) = 0$, it is always the case at least if the size of the retirement system is sufficiently low.

6.2 Optimal growth

An important distinction must be made between the two systems when considering aging. As a NDC system has no impact on economic growth, no adjustment is required when the population is aging. By contrast, if a conventional system can enhance economic growth, it can also be potentially

[13] Note that $Bh^{\delta} - 1 - \delta = \left(1 - \frac{1-h}{h}\delta\right)\left(Bh^{\delta} - 1\right) + \frac{\delta}{h}\left[\left(1-h\right)Bh^{\delta} - 1\right] \geq 0\ \forall h \geq h^{0}$.

harmful if its size is too high compared to the optimal size. We must then study the evolution of the latter to verify, at a minimum, that an unchanged size does not become harmful for economic growth, i.e. that the optimal size is not decreasing with longevity. Assuming $\tau^* = \arg\max_{\tau} \{1+g\} < 1$, it follows from Proposition 4 that:

$$sign \frac{d\tau^*}{d\rho} = sign \left\{ \begin{array}{c} \frac{\partial^2 Q}{\partial\rho\partial h}\frac{h}{q} - \frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial\rho} \\ +\frac{\partial h}{\partial\rho}\left[\frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2} + \frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right] \end{array} \right\} \tag{32}$$

On one hand, assuming $d^2G(.) \leq 0\ \forall\tau \geq 0$ yields $\frac{\partial^2 Q}{\partial h^2} \leq 0$ and then $\frac{\partial h}{\partial\rho}\left[\frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2} + \frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right] \leq 0$. However, at least if considering low levels of τ^*, the length of training is weakly related to longevity. As underlined in Proposition 3, if $\tau = 0$, $\frac{\partial h}{\partial\rho} = 0$. Considering then a sufficiently low impact of longevity on the length of training entails that $sign\frac{d\tau^*}{d\rho} = sign\left\{\frac{\partial^2 Q}{\partial\rho\partial h}\frac{h}{q} - \frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial\rho}\right\}$, where $\frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial\rho} \leq 0$. It follows that if the negative impact on the proportion of skilled workers initiated by the training lenghthening is reduced by the increased longevity, $\frac{\partial^2 Q}{\partial\rho\partial h} \geq 0$, then the optimal size τ^* increases. Such a condition is verified if the elasticity of the density function is, in absolute value, higher than unity: $-\varepsilon_{dG} \geq 1$.

Proposition 7 *Assuming* $\tau^* = \arg\max_{\tau}\{1+g\} < 1$, *if the length of training is weakly related to longevity,* $-\varepsilon_{dG} \geq 1$ *is a sufficient condition such as a marginal increase in longevity raises the size of a conventional retirement system rate which maximizes economic growth.*

Note that the condition $-\varepsilon_{dG} \geq 1$ in Proposition 7 is not restrictive. Let us consider for example a Pareto distribution. In that case, the density function $dG(\sigma) = \frac{\mu\sigma_{\min}^{\mu}}{\sigma^{\mu+1}}\ \forall\sigma \geq \sigma_{\min}$, 0 otherwise, where $\sigma_{\min} > 0$ and $\mu > 0$. It follows that $-\varepsilon_{dG} = -d^2G\frac{\sigma}{dG} = 1+\mu > 1$, i.e. the condition always holds.

For policy-making, as underlined by Le Garrec (2005), maximizing economic growth is equivalent to maximizing an intertemporal social welfare if the weight assigned to future generations is high enough. From this perspective, as there is no guarantee that the effective size τ of conventional systems corresponds to the optimal size τ^*, different configurations can arise with increased longevity. Starting from $\tau_0 \leq \tau^*(\rho_0)$, increased longevity must be associated with increased contributions to obtain the optimal economic growth and then increased social welfare. Note nevertheless that increased longevity with unchanged pension benefits can define a new contribution rate higher than the optimal rate. In such a case, both increased contributions and lower pension benefits are required to satisfy $\tau_1 = \tau^*(\rho_1)$.

Starting from $\tau_0 > \tau^*(\rho_0)$, the desirable adjustment resulting from increased longevity can also corresponds to increased contributions if $\tau_0 \leq \tau^*(\rho_1)$, but most likely to decreased benefits. If $\tau_0 > \tau^*(\rho_1)$, as contributions can not optimally increase, a reduction of pension benefits is certainly beneficial. Moreover, a reduction of contributions such as $\tau_1 = \tau^*(\rho_1)$ can also be desirable for all generations. Indeed, as suggested by Belan et al. (1998), if such a reduction results in a significant increase in economic growth, a Pareto-improving transition may exist. If such a transition does not exist and if the conventional system is harmful for economic growth, $g(\rho_1, \tau_0) < g(\rho_1, \tau = 0)$, a switch to NDC system can then be considered to increase economic growth and then social welfare. Note however that, as PAYG retirement systems seem to enhance economic growth as empirically reported by Sala-i-Martin (1996) and Zhang and Zhang (2004), i.e. $g(\rho_0, \tau_0) > g(\rho_0, \tau = 0)$, such a switch towards NDC systems associated with increased longevity can not be optimal. The adjustment resulting from increased longevity which appears most likely optimal is then an increase in contribution rates.

7 Conclusion

Equal to 80 years, life expectancy at birth in Western Europe should reach 85 years in 2050 (United Nations, 2009). Financed on a PAYG basis, public retirement systems will have then to cope with the increasingly larger number of pensioners compared to the number of contributors. Changes are unavoidable and of major importance in OECDs' countries. In 2005, the payment of pension benefits represented 38% of all their public social expenditures. As a matter of fact, retirement systems are the major program of industrial countries' redistributive policies and their importance should still grow with the aging of their population.

Claiming the broad inefficiency of PAYG retirement systems (being accused of low return and of distorting individual behaviors), some economists as Feldstein (1995a, 1995b, 1996) stress that these financial difficulties give opportunities to move to fully funded systems. However, replacing conventional PAYG systems by financial -or funded- defined contribution (FDC) systems would involve such a large cost of transition that it appears socially and politically difficult to implement such a reform in Western democracies. For that reason, in recent years a large focus has been put on non-financial -or notional- defined contribution (NDC) systems as legislated in Sweden in 1994. By basing benefits on individual accounts, NDC systems have undoubtedly desirable features in terms of transparency. However, as existing retirement systems (except in Anglo-Saxon countries) appear close to actuarial fairness, we can not expect from NDC systems a significant decrease in negative incentive effects. In many respects, introducing a NDC system largely involves moving from a defined benefit to a defined contribution system aiming at the contribution rates' stabilization. Objective which can be achieved similarly within the scope of more conventional defined benefit systems. As stressed by Börsch-Supan (2006), cleverly designed conventional retirement systems can often do the same job as NDC systems. As shown in this article, they can even do better.

In particular, as most conventional retirement systems have pension benefits linked only to partial earnings history, they can stimulate economic growth by promoting the accumulation of human capital, at least if their size is not too high. When considering the aging process, regarding economic growth, the optimal adjustment is then likely an increase in the size of existing retirement systems rather than a switch to notional systems. Such recommandation appears strengthened when observing in addition that actuarially fair retirement systems whose pensions are linked to the best or last years lower lifetime income inequality while NDC systems do not. More generally, moving to a NDC system, by nature purely contributory, definitively closes the debate about the progressivity of the retirement system which is an important one in democracy. Following Le Garrec (2012), compared to any actuarially fair system, greater progressivity would result in negative incentive effects that would lead to less economic growth, but also to less lifetime inequality. To decide whether greater progressivity involving less lifetime inequality would be worth the cost in terms of economic growth, an intertemporal social welfare, as used in Boadway et al. (1991), Marchand et al. (1996), Le Garrec (2005) and Docquier et al. (2007), would then be interesting.

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Appendix A: proof of propositions

proposition 1

Assume an interior solution. Following (15), $\Delta h = h(\xi = 1) - h(\xi = 0) = \frac{\delta}{1+\delta}\frac{\rho\theta_{t+1}}{(1-\tau)R_{t+1}}\Delta\xi$, where $\Delta\xi = 1$. It follows that $\frac{\Delta h}{\Delta\xi} > 0$ as long as $\theta_{t+1} > 0$.

proposition 2

Assuming $\xi = 1$, actuarial fairness defined by (9) entails that:

$$\frac{\rho\theta_t Bh_{t-1}^{\delta}\overline{Z}_{t-2}w_{t-1} + \rho\nu_t\overline{Z}_{t-1}w_t}{\tau\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}\overline{Z}_{t-2}w_{t-1}} = \frac{\rho\theta_t\overline{Z}_{t-2}w_{t-1} + \rho\nu_t\overline{Z}_{t-1}w_t}{\tau\overline{Z}_{t-2}w_{t-1}}$$

and then:

$$\frac{\theta_t Bh_{t-1}^{\delta}w_{t-1} + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)w_t}{\tau\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}w_{t-1}} = \frac{\theta_t w_{t-1} + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)w_t}{\tau w_{t-1}}$$

Introducing (25) entails then that:

$$\begin{aligned}&\frac{\left[\frac{\tau}{\rho}\left[q_t\left(1-h_t\right)Bh_t^{\delta} + 1 - q_t\right] - \nu_t\right]Bh_{t-1}^{\delta} + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)}{\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}}\\ = &\left[\frac{\tau}{\rho}\left[q_t\left(1-h_t\right)Bh_t^{\delta} + 1 - q_t\right] - \nu_t\right] + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)\end{aligned}$$

Actuarial fairness with $\xi = 1$ is then obtained if:

$$\nu_t = \frac{Bh_{t-1}^{1+\delta}}{Bh_{t-1}^{\delta} - 1}\frac{\tau}{\rho}\frac{q_t\left(1-h_t\right)Bh_t^{\delta} + 1 - q_t}{q_{t-1}\left(1-h_{t-1}\right)Bh_{t-1}^{\delta} + 1 - q_{t-1}}$$

proposition 3

Following equation (28), if $\xi = 0$, $h = h^0 = \frac{\delta}{1+\delta}$. Accordingly, following equation (28), $q = Q\left(h^0, \rho\right)$. Therefore, as $g = q\left(Bh^{\delta} - 1\right)$, $\frac{dg}{d\tau} = 0$ if $\xi = 0$.

proposition 4

From eq. (31) we derive $d(1+g) = \left(Bh^{\delta}-1\right)dq + qB\delta h^{\delta-1}dh$. Knowing that $dq = \frac{\partial Q}{\partial h}dh$ everything else being equal, it follows that $\frac{d(1+g)}{d\tau} = 0$ is equivalent to $\left[\left(Bh^{\delta}-1\right)\frac{\partial Q}{\partial h} + qB\delta h^{\delta-1}\right]\frac{dh}{d\tau} = 0$ or to $\left[\left(Bh^{\delta}-1\right)\varepsilon_Q^h + B\delta h^{\delta}\right]\frac{dh}{d\tau} = 0$.

From eq. (29), if $\xi = 1$ and $\nu = \widetilde{\nu}$, we have $\frac{\partial h}{\partial \tau} = \frac{\delta\frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1}\frac{\beta\rho(1+\beta\rho)\alpha(1-\alpha)}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]} > 0\ \forall\rho > 0$. It follows that $\frac{d(1+g)}{d\tau} = 0$ is equivalent to $-\varepsilon_Q^h = \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$.

Thereafter, $\frac{\partial Q}{\partial h} = dG(.)(1+\beta\rho)\frac{-Bh^{\delta}+(1-h)B\delta h^{\delta-1}}{(1-h)Bh^{\delta}} = dG(.)(1+\beta\rho)\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)$. In $\tau = 0$, it follows that $\varepsilon_Q^h = \frac{\partial Q}{\partial h}\frac{h}{q} = 0$ and then that $-\varepsilon_Q^h < \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$.

On one hand we have: $\frac{d\left(\frac{B\delta h^{\delta}}{Bh^{\delta}-1}\right)}{dh} = \frac{-B\delta^2h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2} < 0$.

On the other hand: $d\varepsilon_Q^h = \left(\frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right)dh\ \forall d\rho = 0$, where $\frac{\partial^2 Q}{\partial h^2} = d^2G(.)(1+\beta\rho)^2\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)^2 + dG(.)(1+\beta\rho)\left(-\frac{1}{(1-h)^2}-\frac{\delta}{h^2}\right)$. As $\frac{-1}{1-h}+\frac{\delta}{h} \leq 0\ \forall\ h \geq h^0 = \frac{\delta}{1+\delta}$, assuming $d^2G(.) \leq 0\ \forall\tau \geq 0$ results in $-\frac{d\varepsilon_Q^h}{dh} \geq 0$.

Assuming $d^2G(.) \leq 0\ \forall\tau \geq 0$, $-\varepsilon_Q^h > \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$ in $\tau = 1$ is then a necessary and sufficient condition such as economic growth exhibits an inverse U-shaped pattern with respect to the size of an actuarially fair retirement system whose pensions are tied to the best or last years.

proposition 5

Following equation (29), if $\xi = 0$, $h = h^0 = \frac{\delta}{1+\delta}$. Accordingly, following equation (28), $q = Q\left(h^0, \rho\right)$. Therefore, as $g = q\left(Bh^{\delta}-1\right)$, $\frac{dg}{d\rho} = \frac{\partial Q}{\partial \rho}\left(Bh^{0\delta}-1\right)$ if $\xi = 0$, where $\frac{\partial Q}{\partial \rho} = dG(.)\beta\ln\left[\left(1-h^0\right)Bh^{0\delta}\right] \geq 0$ and $\frac{\partial h}{\partial \rho} = 0$.

proposition 6

Following equation (29), if $\xi = 1$ and $\nu = \tilde{\nu}$, $h = h^0\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{(1-h_t)Bh_t^{\delta}-1}{Bh_t^{\delta}-1}\right]$.

It follows that $\frac{\partial h}{\partial \rho} = \frac{\delta\frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1}\frac{\beta(1-\alpha)(\alpha+\tau(1-\alpha))}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]} \geq 0$. Following equation (28), $dq = dG(.)\left\{\beta\ln\left[(1-h)Bh^{\delta}\right]d\rho + (1+\beta\rho)\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)dh\right\}$. It follows that $\frac{dq}{d\rho}\geq 0$ if $\frac{dh}{d\rho}\leq\frac{\beta}{1+\beta\rho}\frac{\ln\left[(1-h)Bh^{\delta}\right]}{\frac{1}{1-h}-\frac{\delta}{h}}$. If the latter condition holds, as $dg = dq\left(Bh^{\delta}-1\right)+q\delta Bh^{\delta-1}dh$, $\frac{dg}{d\rho}\geq 0$ everything else being equal.

proposition 7

Assuming that $-\varepsilon_Q^h > \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$ for $\tau = 1$, $-\frac{\partial Q}{\partial h}\left(h\left(\tau^*,\rho\right),\rho\right)\frac{h(\tau^*,\rho)}{Q(h(\tau^*,\rho),\rho)} = \frac{Bh(\tau^*,\rho)^{\delta}\delta}{Bh(\tau^*,\rho)^{\delta}-1}$ defines the contribution rate which maximizes economic growth. It follows:

$$-\left\{\frac{\partial^2 Q}{\partial h^2}\left[\frac{\partial h}{\partial\tau}d\tau^*+\frac{\partial h}{\partial\rho}d\rho\right]+\frac{\partial^2 Q}{\partial\rho\partial h}d\rho\right\}\frac{h}{q}-\frac{\partial Q}{\partial h}\frac{1}{q}\left[\frac{\partial h}{\partial\tau}d\tau^*+\frac{\partial h}{\partial\rho}d\rho\right]+\frac{\partial Q}{\partial h}h\frac{\frac{\partial Q}{\partial h}\left[\frac{\partial h}{\partial\tau}d\tau^*+\frac{\partial h}{\partial\rho}d\rho\right]+\frac{\partial Q}{\partial\rho}d\rho}{q^2}$$
$$=\frac{-B\delta^2h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\left[\frac{\partial h}{\partial\tau}d\tau^*+\frac{\partial h}{\partial\rho}d\rho\right]$$
$$\Longleftrightarrow -\left\{\frac{\partial^2 Q}{\partial h^2}\frac{h}{q}+\frac{\partial Q}{\partial h}\frac{1}{q}-\left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}+\frac{-B\delta^2h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\right\}\frac{\partial h}{\partial\tau}d\tau^*$$
$$=\left\{\left[\frac{\partial^2 Q}{\partial h^2}\frac{\partial h}{\partial\rho}+\frac{\partial^2 Q}{\partial\rho\partial h}\right]\frac{h}{q}+\frac{\partial Q}{\partial h}\frac{1}{q}\frac{\partial h}{\partial\rho}-\frac{\partial Q}{\partial h}h\frac{\frac{\partial Q}{\partial h}\frac{\partial h}{\partial\rho}+\frac{\partial Q}{\partial\rho}}{q^2}+\frac{-B\delta^2h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\frac{\partial h}{\partial\rho}\right\}d\rho$$

As $-\left\{\frac{\partial^2 Q}{\partial h^2}\frac{h}{q}+\frac{\partial Q}{\partial h}\frac{1}{q}-\left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}+\frac{-B\delta^2h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\right\}>0$ (see proof of proposition 4), it follows that:

$$sign\frac{d\tau^*}{d\rho}=sign\left\{\begin{array}{c}\frac{\partial^2 Q}{\partial\rho\partial h}\frac{h}{q}-\frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial\rho}\\+\frac{\partial h}{\partial\rho}\left[\frac{-B\delta^2h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}+\frac{\partial^2 Q}{\partial h^2}\frac{h}{q}+\frac{\partial Q}{\partial h}\frac{1}{q}-\left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right]\end{array}\right\}$$

We have then $\frac{\partial^2 Q}{\partial\rho\partial h}=d^2G(.)\beta\ln\left[(1-h)Bh^{\delta}\right](1+\beta\rho)\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)+dG(.)\beta\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)$
$=\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)\left(d^2G(.)\beta\ln\left[(1-h)Bh^{\delta}\right](1+\beta\rho)+dG(.)\beta\right)$
$=\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)dG(.)\beta\left(\frac{d^2G(.)}{dG(.)}\ln\left[(1-h)Bh^{\delta}\right](1+\beta\rho)+1\right)$
$=\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)dG(.)\beta\left(\varepsilon_{dG}+1\right)$

It follows that if $\varepsilon_{dG}\leq -1$, $\frac{\partial^2 Q}{\partial\rho\partial h}\geq 0$ $\forall h\geq h^0=\frac{\delta}{1+\delta}$. As $\frac{\partial Q}{\partial h}\leq 0$ and $\frac{\partial Q}{\partial\rho}\geq 0$, it follows from (32) that if $\frac{\partial h}{\partial\rho}$ is sufficiently low, $\frac{d\tau^*}{d\rho}\geq 0$. As $\frac{\partial h}{\partial\rho}=\frac{\delta\frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1}\frac{\beta(1-\alpha)(\alpha+\tau(1-\alpha))}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]}$, it is in particular the case if τ^* is sufficiently low.