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Increased longevity and social security reform: questioning the optimality of individual accounts when education matters

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Abstract

In many European countries, population aging had led to debate about a switch from conventional unfunded public pension systems to notional systems characterized by individual accounts. In this article, we develop an overlapping generations model in which endogenous growth is based on an accumulation of knowledge driven by the proportion of skilled workers and by the time they have spent in training. In such a framework, we show that conventional pension systems, contrary to notional systems, can enhance economic growth by linking benefits only to the partial earnings history. Thus, to ensure economic growth, the optimal adjustment to increased longevity could consist in increasing the size of existing retirement systems rather than switching to notional systems.

Keywords: social security, intertemporal choice, human capital

JEL classification: H55, D91, E24

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1 Introduction

In 1950, life expectancy at birth in Western Europe was 68 years. It is now 80 years and should reach 85 by 2050 (United Nations, 2009). The downside of this trend is the serious threat that is hanging over the financing of our public retirement systems. Financed on a PAYG basis, i.e. pension benefits are paid through contributions of contemporary workers, the systems must cope with an increasingly large number of pensioners compared to the number of contributors. With an unchanged average age of retirement, in France, for example, the ratio of pensioners to workers (the dependency ratio) should reach 70.1% in 2040, whereas it was 35.8% in 1990. Changes are unavoidable. If we want to guarantee in the near future the current level of benefits within the same system, it will be necessary to increase either the contribution rate or the length of contribution (by delaying the age of retirement).

This financing problem calls into question the role of PAYG retirement systems in our societies. For instance, by evaluating the real pre-tax return on non-financial corporate capital at 9.3%[1] and the growth rate over the same period (1960 to 1995) at 2.6%, Feldstein (1995a, 1995b, 1996) unequivocally advocates the privatization of retirement systems and a switch to fully funded systems. He assesses the potential present-value gain at nearly $20 trillion for the United States. However, replacing conventional PAYG systems by financial - or funded - defined contribution (FDC) systems would certainly involve prohibitive social and political costs. One generation will have to pay twice. Implementing such a reform in Western democracies thus appears difficult. For that reason, in recent years a large focus has been

[1] This return combines profits before all federal, state, and local taxes with the net interest paid. The method of calculation is described in Feldstein, Poterba and Dicks-Mireaux (1983).

put on non-financial - or notional - defined contribution (NDC) systems as legislated in Sweden in 1994. As described by Palmer (2006), NDC systems are PAYG systems that mimic FDC systems. Individual contributions are noted on individual accounts. Accounts are credited with a rate of return that reflects demographic and productivity changes. Obviously, replacing conventional PAYG systems by NDC systems does not address the main concern of Feldstein (1995a, 1995b, 1996), that is, the low return associated with the PAYG financing method. However, supporters of NDC systems claim that conventional systems, by linking pension benefits only partially to contributions, distort individual behaviors, inducing reduced work efforts or earlier retirements. In addition, they claim that only an explicit defined contribution system will be able to stabilize contributions in spite of aging populations.

However, looking at the empirical facts, the supposed inefficiency of conventional retirement systems must be reconsidered. Firstly, even if their pension benefits are linked to partial earnings history, conventional systems are close to actuarial fairness[2] as NDC systems because high-income earners live longer (Deaton and Paxton, 1998, 1999; Breyer and Hupfeld, 2009) and have steeper age-earnings profiles (Lindbeck and Persson, 2003; Bozio and Piketty, 2008). Secondly, stabilizing contributions can be achieved similarly within the scope of more conventional defined benefit systems, as seen in

[2] See Burkhauser and Walick (1981), Stahlberg (1990), Garrett (1995), Gustman and Steinmeier (2001), Coronado et al. (1999, 2000) and Brown et al. (2006). Strictly speaking, a retirement system is said actuarially fair if its return is equal to the interest rate (Lindbeck and Persson, 2003; Cigno, 2008). Considering that the economic growth rate, which is the retirement system return, is lower than the interest rate, retirement systems could be described more properly as quasi-actuarial fair as noted by Lindbeck and Persson (2003).

the "point system" in France or in Germany. In that case, the unit of pension rights is earnings points (not euros) and can be adjusted according to demographic and productivity changes, as in an NDC system. As stressed by Börsch-Supan (2006), cleverly designed conventional retirement systems can often do the same job as NDC systems. Finally, empirical findings from Sala-i-Martin (1996) and Zhang and Zhang (2004) tend to support a positive impact of retirement systems on economic growth through the human capital channel. From these perspectives, it is then not straightforward to determine whether the introduction of individual accounts or the stabilization of contributions are desirable objectives. In this article, we investigate whether the introduction of individual accounts into social security may be a desirable reform, starting from the fact that conventional retirement systems yield more economic growth.

To explain the positive link between PAYG retirement systems and economic growth that is suggested by the empirical findings, the theoretical literature has focused on the human capital channel, and particularly on parental altruism[3]. According to this strand of the literature, PAYG retirement systems result in higher economic growth because they provide an incentive for altruistic parents to invest more in their children's education. However, as highlighted by Cigno (2010), they also provide an incentive for parents to have fewer children, while the investment per child stays insufficient to be socially optimal. In that context, when private behaviour is not observable, Cigno et al. (2003) and Cigno and Luporini (2011) show that a second-best policy would be to provide parents with subsidies linked to the number of their children and their future capacity to pay taxes. To that end,

[3]See Zhang (1995), Kaganovich and Zilcha (1999), Zhang et al. (2001), Zhang and Zhang (2003), Lambrecht et al. (2005) and Glomm and Kaganovich (2008).

Cigno (2010) suggests that unconventional children-related pension systems be added to conventional retirement systems so as to allow individuals to earn a pension by raising children and by investing in their human capital. While introducing such an unconventional system could stimulate both fertility and economic growth, Fanti and Gori (2013) nevertheless stress the associated risk of increasing cyclical instability.

Besides the impact of PAYG systems on parents' behavior, Kemnitz and Wigger (2000) and Le Garrec (2001) have shown that they may also provide an incentive for people to invest in their own education. Interestingly, in this second strand of the literature, results have been obtained in models where the learning ability of individuals is identical. By contrast, when considering heterogeneous learning ability, Docquier and Paddison (2003) show that conventional retirement systems dissuade people from investing in their education. To explain these conflicting results, one can observe that in Kemnitz and Wigger (2000) and Le Garrec (2001) the positive impact of conventional PAYG retirement systems on economic growth occurs through the lengthening of training, while the negative impact in Docquier and Paddison (2003) corresponds to a decrease of the proportion of individuals who decide to train themselves with a fixed training length. By embedding both effects, Le Garrec (2012) then shows that the positive effect always dominates the negative one, at least for low contribution rates. In the spirit of Cigno (2010), the findings of Kemnitz and Wigger (2000) and Le Garrec (2001, 2012) suggest that an optimal feature of retirement systems would also consist in subsidizing people who invest in their own education by linking benefits to the best or last years, and not to full lifetime average earnings, as in NDC systems.

In this article, to analyze the relevancy of the switch from conventional unfunded public pension systems to notional systems we extend the social security-growth literature in two directions. First, following Le Garrec (2012), we consider investment in human capital through both the proportion of individuals who decide to invest and the time they invest. However, by not specifying a particular distribution of learning abilities, we can provide explicit and general conditions so that the positive effect associated with the lengthening of training may be dominated by the negative effect, i.e. the decrease of the proportion of educated individuals. We then show that economic growth may exhibit an inverse U-shaped pattern with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years, while an NDC system has no impact on economic growth. Second, we consider the aging process, not through decreased fertility as is usual, but through increased longevity. This has important consequences. Indeed, as shown by Cipriani (2013), for given contribution rates, in contrast to decreased fertility, increased longevity unambiguously yields lower pension benefits. In addition, as increased longevity raises the value of investments that pay over time, it generates stronger incentives for people to invest in their education, as is well documented in the literature[4]. Therefore, social security interacts with longevity in determining the individual investment in education. We then show that increased longevity may raise the size of the conventional retirement system rate that maximizes economic growth. This result suggests that the optimal adjustment to aging could consist in increasing the size of existing retirement systems rather than switching to notional systems.

The rest of this paper is organized as follows. In section 2, we present the

[4]See de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Boucekkine et al. (2002), Cervellati and Sunde (2005, 2011), Soares (2005), Jayachandran and Lleras-Muney (2009). Challenging the conventional wisdom, Acemoglu and Johnson (2007) find no effect of life expectancy on schooling.

basic assumptions related to the age-earnings profiles and the calculation of pension benefits. In section 3, we analyze optimal behaviors of individuals and firms in light of the basic assumptions. We assume in particular that individuals differ in their learning abilities, as in Docquier and Paddison (2003) and Le Garrec (2012). In section 4, we specify the equilibrium features with actuarially fair retirement systems. In section 5, we then show that actuarially fair retirement systems, depending on their size and on the calculation of pension benefits, can enhance economic growth. In section 6, we then specify optimal adjustments for economic growth when longevity increases. The last section provides a brief conclusion.

2 Earnings profile and pension benefits: basic assumptions

The model used here is an extended version of the Ben-Porath model (1967) with uncertain lifetimes. Individuals live for either two or three periods: they are respectively young, adult, and old. Survival is complete through adulthood. Each adult has a probability $\rho \in (0, 1)$ to survive to old age. The size of the young generation is normalized to one at each date. Due to complete survivance, the size of the adult generation is also equal to one at each date, whereas the size of the retired generation is equal to ρ. Aging then occurs in the model through increased longevity.

2.1 Human capital and age-earnings profiles

When young, individuals go to school. During this period, which corresponds to primary and secondary education (compulsory schooling), individuals born in $t-1$ learn basic knowledge represented by the average knowledge $\overline{Z}_{t-1}$ of the contemporary working generation. In addition, they can choose to make an effort e_{t-1} in learning (where $e_{t-1} = 0$ or 1) to pass the final secondary school examination, qualifying for university entrance. In the second period, those who have made the effort can then complement their basic knowledge by pursuing training during a period h_t instead of directly entering the labor market[5]. At the end of their complementary training, their human capital is characterized by:

$$Z_t^s = Bh_t^\delta \overline{Z}_{t-1},\ B > 0,\ \ \delta > 0 \tag{1}$$

where δ denotes the return to complementary training in terms of human capital.

Skilled workers, those who have completed their training before entering the labor market, are thus characterized by a first period h_t with no earnings. Afterwards, they earn $Z_t^s w_t$, where w_t is the wage rate per unit of effective labor. Earnings of skilled workers W_t^s over their whole active period are thus:

$$W_t^s = (1 - h_t)\, Z_t^s w_t \tag{2}$$

and are then characterized by a steep profile. By contrast, unskilled workers are characterized by the basic human capital during all their working period:

$$Z_t^u = \overline{Z}_{t-1} \tag{3}$$

and are then characterized by flat age-earnings profiles:

[5] In that case training is a full-time activity that can be assimilated to higher education. We could have assumed alternatively that training is a part-time activity without changing the qualitative results (see Le Garrec, 2005).

$$W_t^u = Z_t^u w_t \tag{4}$$

From eqs. (1)-(4), making sure that skilled workers earn more than unskilled workers during their whole active period requires that:

$$(1 - h_t) B h_t^{\delta} > 1 \tag{5}$$

In a simple way, the economy is then characterized in line with Lilliard (1977) and Andolfatto et al. (2000) by age-earnings profiles of workers increased with the time spent in training and by high-school dropouts with flat age-earnings profiles.

2.2 Pension benefits

In conventional systems, the calculation of pension benefits is specific to each country, and sometimes can be very complex. In the theoretical literature on social security[6], two different pension parts are generally distinguished: a redistributive part (the Beveridgean part) characterized by a basic flat-rate benefit, and an insurance part (the Bismarckian part) characterized by earnings-related benefits. The latter is not generally proportional to all contributions, in which case it is not based on full lifetime average earnings (see OECD, 2007). This is particularly the case in Greece and Spain where benefits are linked only to final salary. This also used to be the case in Sweden before the 1994 legislation introducing NDC systems. In France, before the Balladur reform of 1993, earnings-related benefits were linked to the ten best years, and then after the reform were gradually switched to

[6] See Casamatta et al. (2000), Docquier and Paddison (2003), Sommacal (2006), Cremer et al. (2007), Hachon (2010), Le Garrec (2012).

the 25 best years. In the United States, the 35 best years are considered to calculate the benefits, and in Norway the best 20.

Let us define $\tilde{W}_t^i$, $i = s, u$, as the representative earnings on which benefits are based in a conventional system. It does not matter which period is used to calculate the unskilled representative earnings because the age-earnings profile is consistently flat. It follows that:

$$\tilde{W}_t^u = W_t^u \tag{6}$$

For the skilled workers, as the reference earnings $\tilde{W}_t^s$ corresponds to the best or last years, this is specified as:

$$\tilde{W}_t^s = Z_t^s w_t \tag{7}$$

Assuming that the basic flat-rate benefit $\overline{p}_{t+1}$ is linked to the contemporary wage of unskilled workers[7], the calculation of pension benefits for any worker in t in a conventional system is then given by:

$$p_{CONV,t+1} = \theta_{t+1}\tilde{W}_t + \nu_{t+1}W_{t+1}^u \tag{8}$$

where ν_{t+1} represents the size of the flat-rate component of the pension benefits and θ_{t+1} the size of the earnings-related component.

As noted in the introduction, most conventional retirement systems in the industrialized economies are close to actuarial fairness. In terms of the retirement system's implicit return, i.e. the ratio between the expected pension benefits of an individual and the amount of his contributions, this means that:

[7]It is designed to ensure that pensioners achieve some minimum standard of living.

$$\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} \approx \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}} \tag{9}$$

where τ denotes the public pension system contribution rate, and p^i_t denotes the pension benefits in t of a worker of type i in $t-1$, $i = u, s$[8]. If $\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} > \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}}$, then the retirement system is fiscally favorable to low-income earners. In this case the system is progressive. In the opposite case, $\frac{\rho p^u_{CONV,t}}{\tau W^u_{t-1}} < \frac{\rho p^s_{CONV,t}}{\tau W^s_{t-1}}$, it is regressive.

Consider alternatively an NDC system. In that case, individual contributions are noted on individual accounts which are credited with a factor of return ψ. By definition, such a pure contributory system whose pensions are calculated proportionally to all contributions is actuarially fair. Explicitly adjusted to life expectancy, pension benefits are then as follows:

$$p_{NDC,t+1} = \frac{\psi_{t+1}}{\rho} \tau W_t \tag{10}$$

For convenience, we will further note the calculation of pension benefits as:

$$p_{t+1} = \xi \left(p_{CONV,t+1} \right) + (1-\xi)\, p_{NDC,t+1} \tag{11}$$

where $\xi = 1$ for a conventional system, $\xi = 0$ for an NDC system.

[8] If we had considered socioeconomic inequalities in mortality, actuarial fairness would have been defined as $\frac{\rho^u p^u_{CONV,t}}{\tau W^u_{t-1}} \approx \frac{\rho^s p^s_{CONV,t}}{\tau W^s_{t-1}}$, where $\rho^s \geq \rho^u$.

3 Optimal behaviors

3.1 Individuals

As specified in the previous section, individuals live for three periods. They invest in education in the first and possibly in the second period, work in the second one and retire in the third one with probability ρ. The preferences of an individual of type x born in $t-1$ are described by the following utility function:

$$U_x = \ln c_t + \beta\rho \ln d_{t+1} - \sigma_x e_{t-1} \tag{12}$$

where c_t and d_{t+1} denote, respectively, his consumption when an adult and when old[9], and $\beta \leq 1$ denotes the subjective discount factor. The utility from uncertain lifetime consumption is based on Yaari (1965), as in Abel (1985) and in Zhang et al. (2001, 2003). σ_x denotes the utility cost of schooling effort, where $\sigma \in [0, \infty)$ represents learning ability. As shown by Huggett et al. (2006), earnings differences are first explained by differences in learning ability across individuals. In our setting, a talented child characterized by $\sigma = 0$ incurs no cost in making the effort. By contrast, a lazy or untalented child characterized by $\sigma \to \infty$ will incur an infinite cost and will then always choose not to make the effort, i.e. $e_{t-1} = 0$. Note that σ can be considered as an inherited (perfectly here) trait that represents both family background and genetic transmission[10]. We denote $G(\sigma)$ the cumulative distribution function

[9] As in Boldrin and Montes (2005) and Docquier et al. (2007), we assume that the only decision of children concerns education, as their consumption is part of their parents' consumption. As a consequence and without loss of generality, consumption when young does not appear in the utility function.

[10] Stating that earnings are very significantly tied to the earnings of the parents (Bowles and Gintis, 2002, d'Addio, 2007), this suggests that the intergenerational earnings per-

of learning ability through the population, and we assume it is of class C^2.

During the second life period, individuals consume a part of their disposable income, and save via a perfect annuity market such as:

$$c_t + s_t = W_t (1 - \tau) \tag{13}$$

where s_t denotes private savings.

In the third life period, old-age survivors are retired. They get back their savings with interest, receive their pension from the public retirement system and consume their wealth. The budget constraint is then:

$$d_{t+1} = \frac{R_{t+1}}{\rho} s_t + p_{t+1} \tag{14}$$

where R_{t+1} denotes the real interest factor. Note that, with a perfect annuity market, old-age survivors share the savings of deceased individuals. The expected return to savings is then equal to the actuarially fair factor $\frac{R_{t+1}}{\rho}$, as in Zhang et al. (2001). The alternative would be the existence of involontary bequests, as in Abel (1985) and Zhang et al. (2003).

Let $\Omega_t^i = W_t^i (1 - \tau) + \frac{\rho p_{t+1}^i}{R_{t+1}}$ be the expected lifetime income of a worker of type i, $i = u, s$. Considering the calculation of pension benefits (11), an individual who has chosen to make the effort at school will maximize his lifetime income by spending the following time in training during his second life period:

sistence is based on the inheritability of learning ability within families. Supporting such a view, education is a major contributor to intergenerational earnings mobility and educational differences tend to persist across generations (d'Addio, 2007). Nevertheless, as shown by Bowles and Gintis (2002), this does not imply that the intergenerational earnings determination is based only on genetic transmission. Learning ability also reflects non-cognitive personality traits such as, for example, a taste for learning at school which can be influenced by the family background as much as by the genes.

$$h_t = \inf \left\{ h^0 \left[1 + \frac{1}{1-\tau} \frac{\rho \xi \theta_{t+1}}{R_{t+1}} \right] ; 1 \right\} \tag{15}$$

where $h^0 = \frac{\delta}{1+\delta}$ is the training length with no retirement system.

Proposition 1 *Linking pension benefits to the partial earnings history generates an incentive to be trained longer.*

Conventional retirement systems whose pension benefits are based, even partially, on the best or last years generate an incentive for longer training. Initially, the lengthening of training has a negative effect on income, as during this period individuals have no earnings capacity. However, they earn more afterwards. In addition, as pensions are linked to the best or last years, individuals also benefit, all things being equal, from an increase in their benefits. Following equation (15), individuals who undertake training may find it profitable to be trained longer, since this represents an investment in their pension benefits. Note that this incentive disappears completely if pension benefits are based on full lifetime average earnings ($\xi = 0$), or if the system is totally flat-rate ($\theta_{t+1} = 0$). Moreover, this incentive is weaker as the interest rate increases. Indeed, the higher the interest rate, the lower the present actuarial value of pension benefits.

To summarize, the incentive to be trained longer generated by conventional retirement systems is due to the interaction of two factors:

- pension benefits are linked to the best or last years
- training results in steeper age-earnings profiles

The utility maximization of an individual subject to budgetary constraints (13) and (14) leads to the following savings function:

$$s_t = \frac{\beta\rho}{1+\beta\rho} W_t (1-\tau) - \frac{\rho}{1+\beta\rho} \frac{p_{t+1}}{R_{t+1}} \quad (16)$$

By reducing simultaneously the disposable income and the need for future income, a retirement system reduces private savings. This result holds irrespective of the calculation of pension benefits and their financing.

Last, an individual will choose to make the effort at school if the opportunity of complementary training yields a monetary benefit higher than the utility cost associated with the effort, i.e. if $(1+\beta\rho)\ln\Omega_t^s - \sigma_x e_{t-1} \geq (1+\beta\rho)\ln\Omega_t^u$. Considering an interior solution, the proportion of individuals q_t who choose to be trained in t (and then to make the school effort in $t-1$) and become skilled workers is defined by:

$$q_t = \bar{e}_{t-1} = G\left[(1+\beta\rho)\ln I_t\right] \quad (17)$$

where $I_t = \frac{\Omega_t^s}{\Omega_t^u}$ represents the lifetime income inequality between skilled and unskilled workers in t. Following (17), the higher this inequality, the larger the proportion of individuals induced to be trained: $\frac{dq_t}{dI_t} > 0$.

3.2 Firms

We consider a competitive sector characterized by a representative firm producing a good, which can be either consumed or invested, according to a Cobb-Douglas technology with constant return to scale:

$$Y_t = F\left(K_t, L_t^u, L_t^s\right) = AK_t^{\alpha}\left(Z_t^u L_t^u + (1-h_t) Z_t^s L_t^s\right)^{1-\alpha},\ 0<\alpha<1 \quad (18)$$

where Y_t denotes the output, K_t the physical capital stock, L_t^i the number of workers of type i in t, $i = u, s$, and A the total factor productivity. Assuming for simplicity as in Docquier and Paddison (2003) and Le Garrec (2012) that skilled and unskilled labors are perfect substitutes[11], $H_t = Z_t^u L_t^u + (1-h_t) Z_t^s L_t^s$ represents the labor supply in efficiency units.

Denoting per capita efficient capital by $k_t = \frac{K_t}{H_t}$ and assuming a total capital depreciation, the optimal conditions resulting from the maximization of the profit are:

$$R_t = A\alpha k_t^{\alpha-1} \tag{19}$$

$$w_t = A(1-\alpha) k_t^{\alpha} \tag{20}$$

Before studying the impact of retirement systems and the calculation of pension benefits on economic growth, we need to characterize the equilibrium and its properties.

4 Equilibrium

The economy is composed of four markets corresponding to the unskilled labor, the skilled labor, the physical capital and the good. In a closed-economy setting, the general equilibrium can be obtained by considering only the clearing of three markets, as according to the Walras law, the fourth is necessarily cleared. In our case, we consider the clearing of the following markets:

unskilled labor:

[11] Assuming alternatively that they are imperfect substitutes would affect the skill choice by introducing a wage premium for human capital. However, it would not change the training length as defined in eq. (15) at all. The incentive to be trained longer as specified in Proposition 1 would then not be affected.

$$L_t^u = (1 - q_t) \tag{21}$$

skilled labor:

$$L_t^s = q_t \tag{22}$$

physical capital:

$$K_{t+1} = q_t s_t^s + (1 - q_t)\, s_t^u \tag{23}$$

4.1 PAYG social security and capital accumulation

Retirement systems have PAYG features, i.e. within a period, pension benefits are financed by contributions of workers of the same period. In other words, retirement systems transfer workers' income towards pensioners. Since workers are either skilled or unskilled, the social security balanced budget is defined as follows:

$$\rho L_{t-1}^u p_t^u + \rho L_{t-1}^s p_t^s = \tau \left[L_t^u W_t^u + L_t^s W_t^s\right] \tag{24}$$

Since at date t there is a proportion q_t and $1 - q_t$ of respectively skilled and unskilled workers as specified in eqs. (21) and (22), the balanced budget of the retirement system (24), with eqs. (1)-(11), is rewritten as:

$$\theta_t = \left[\frac{\tau}{\rho}\left[q_t\,(1 - h_t)\,Bh_t^\delta + 1 - q_t\right] - \nu_t\right]\frac{w_t}{w_{t-1}} \quad \text{if } \xi = 1 \tag{25}$$

or:

$$\psi_t = \frac{q_t\,(1 - h_t)\,Bh_t^\delta + 1 - q_t}{q_{t-1}\,(1 - h_{t-1})\,Bh_{t-1}^\delta + 1 - q_{t-1}}\left(q_{t-1}Bh_{t-1}^\delta + 1 - q_{t-1}\right)\frac{w_t}{w_{t-1}} \quad \text{if } \xi = 0 \tag{26}$$

Considering the social security balanced budget, either eq. (25) for a conventional system or eq. (26) for an NDC system, and the clearing of the physical capital market (23), with eqs. (1)-(11), (16), (19) and (20), the dynamics of capital accumulation in the model can be expressed independently of the calculation of pension benefits as:

$$\begin{aligned} & k_{t+1}\left[q_{t+1}\left(1-h_{t+1}\right)Bh_{t+1}^{\delta}+1-q_{t+1}\right] \\ = \; & \frac{A\alpha\beta\rho\left(1-\alpha\right)\left(1-\tau\right)}{\alpha\left(1+\beta\rho\right)+\tau\left(1-\alpha\right)}\frac{q_{t}\left(1-h_{t}\right)Bh_{t}^{\delta}+1-q_{t}}{q_{t}Bh_{t}^{\delta}+1-q_{t}}k_{t}^{\alpha} \end{aligned} \tag{27}$$

As retirement systems reduce private savings (eq. 16), all things being equal, PAYG retirement systems are harmful for the accumulation of physical capital: $\frac{\partial \frac{1-\tau}{\alpha(1+\beta)+\tau(1-\alpha)}}{\partial\tau} < 0$ (eq. 27). In addition, as q and h are both forward-looking variables, their specification is crucial to determine the dynamic properties of the model and the convergence towards its steady-state (balanced growth) path.

4.2 Human capital and actuarial fairness

As is obvious, an NDC system ($\xi = 0$) is actuarially fair. As noted in the introduction and characterized by eq. (9), most conventional retirement systems in the industrial world are also close to actuarial fairness.

Proposition 2 *Conventional retirement systems whose pensions are linked to the best or last years are actuarially fair if they include a flat-rate component indexed on the unskilled earnings,* $\overline{p}_t = \nu_t W_t^u$, *such as* $\nu_t = \widetilde{\nu}_t = \frac{Bh_{t-1}^{1+\delta}}{Bh_{t-1}^{\delta}-1}\frac{\tau}{\rho}\frac{q_t(1-h_t)Bh_t^{\delta}+1-q_t}{q_{t-1}(1-h_{t-1})Bh_{t-1}^{\delta}+1-q_{t-1}}$.

If $\nu_t > \widetilde{\nu}_t$, the retirement system is fiscally favorable to low-income earners, $\frac{\rho p_{CONV,t}^u}{\tau W_{t-1}^u} > \frac{\rho p_{CONV,t}^s}{\tau W_{t-1}^s}$, and is then progressive. In the opposite case, $\nu_t < \widetilde{\nu}_t$, it is regressive. This feature is easily understandable. On the one hand, the flat-rate part of the pension benefits is clearly favorable to low-income earners: they receive as much as high-income earners whereas they have contributed less. A flat-rate system is obviously progressive. On the other hand, the pension part that is linked to the best or last years, characterized by θ_t, is favorable to high-income earners as they have a steeper lifetime income profile, as explained by Lindbeck and Persson (2003) and Bozio and Piketty (2008). If there is no flat-rate part then the system is regressive. Therefore, there is a unique combination of the flat-rate and earning-related parts, the one defined in Proposition 2, that characterizes actuarial fairness in a conventional system.

Consider an actuarially fair retirement system, i.e. either $\xi = 0$ or $\xi = 1$ and $\nu_{t+1} = \widetilde{\nu}_{t+1}$. In such a case, the lifetime income inequality $I_t = \frac{\Omega_t^s}{\Omega_t^u}$ becomes $I_t = \frac{W_t^s}{W_t^u}$. Using eqs. (1)-(4), the proportion of skilled workers in t defined by eq. (17) becomes:

$$q_t = G\left([1 + \beta\rho] \ln \left[(1 - h_t)\, Bh_t^{\delta}\right]\right) \tag{28}$$

In this configuration, the choice for a young individual to make the effort at school in $t - 1$ to become a skilled worker in t depends only on their personal talent, life expectancy and the length of the training anticipated. As h^0 corresponds to $\max\left\{(1 - h)\, Bh^{\delta}\right\}$, we can deduce from eq. (28) that any increase in the training length compared to the basic level h^0 will lead to a decrease in the proportion of skilled workers: $\left.\frac{\partial q_t}{\partial h_t}\right|_{h_t \geq h^0} \leq 0$. Following Proposition 1, we can then expect that conventional actuarially fair retirement systems whose pension benefits are based on the partial earnings history reduce the proportion of skilled workers.

A retirement system that is purely contributory, such as an NDC system ($\xi = 0$), has no impact on the training length in the second period of life (eq. 17). By contrast, as characterized by eq. (15), linking pension benefits to the partial earnings history as in a conventional system ($\xi = 1$) generates an incentive to be trained longer that depends crucially on the actualized Bismarckian component $\frac{\theta_{t+1}}{R_{t+1}}$. Using eqs. (19)-(20) and (25)-(27) yields $\frac{\rho\theta_{t+1}}{R_{t+1}} = \left[\frac{(1-h_t)Bh_t^{\delta}-1}{Bh_t^{\delta}-1}\right] \frac{\beta\rho(1-\alpha)(1-\tau)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}$. Thus, the training length according to the social security features can be summarized as:

$$h_t = \begin{cases} h^0 & \text{if } \xi = 0 \\ h^0 \left[1 + \frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)} \frac{(1-h_t)Bh_t^{\delta}-1}{Bh_t^{\delta}-1}\right] & \text{if } \xi = 1 \text{ and } \nu_{t+1} = \widetilde{\nu}_{t+1} \end{cases} \tag{29}$$

If $\xi = 1$ and $\nu_{t+1} = \widetilde{\nu}_{t+1}$, we derive from (29) that $\lim_{h \to h^0} RHS > h^0$ and $\lim_{h \to 1} RHS < h^0$. In this case, the training is expressed as a function $h_t = h(\tau, \rho)$ such that $h^0 \leq h(\tau, \rho) \leq 1$. In the case of an NDC system, as the latter has no impact on the training length, we will note conveniently $h_t = h(\xi\tau, \rho)$, where $\xi = 0$, i.e. $h(0, \rho) = h^0 = \frac{\delta}{1+\delta}\ \forall \rho$. Thereafter, as the skill choice depends only on the training length and on the longevity (eq. 28), it can also be expressed as $q_t = q(\xi\tau, \rho) = Q(h(\xi\tau, \rho), \rho)$, where $\xi = 1$ or $\xi = 0$. In the latter case, $q_t = Q(h(0, \rho), \rho) = Q(h^0, \rho)$ corresponds to an unchanged proportion of skilled workers in t compared to a situation with no retirement system: $q(0, \rho) = G\left([1 + \beta\rho] \ln\left[(1 - h^0) Bh^{0\delta}\right]\right)$.

4.3 Dynamic properties

As underlined by eqs. (28) and (29), human capital variables are in their steady-state values independently of the calculation of pension benefits. Accordingly, considering an actuarially fair retirement system, the physical capital accumulation dynamics (27) can be rewritten as:

$$k_{t+1} = \frac{A\alpha\beta\left(1-\alpha\right)\left(1-\tau\right)}{\alpha\left(1+\beta\right)+\tau\left(1-\alpha\right)}\frac{1}{Q\left(h\left(\xi\tau,\rho\right),\rho\right)Bh\left(\tau,\rho\right)^{\delta}+1-Q\left(h\left(\xi\tau,\rho\right),\rho\right)}k_t^{\alpha} \tag{30}$$

Since $\alpha < 1$, given $k_0 > 0$, the model has good dynamic properties and converges to its steady-state (balanced growth) path characterized by $h = h\left(\xi\tau,\rho\right)$, $q = Q\left(h\left(\xi\tau,\rho\right),\rho\right)$ and $k = \left[\frac{A\alpha\beta\rho(1-\alpha)(1-\tau)}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{1}{qBh^{\delta}+1-q}\right]^{\frac{1}{1-\alpha}}$, where $\xi = 0$ or $\xi = 1$.

As the convergence is verified, the impact of retirement systems and the calculation of pension benefits on investment in human capital and on growth can now be discussed.

5 Social security and economic growth

On the balanced growth path, we deduce from the labor market clearing relations (21) and (22) as well as eqs. (1), (3) and (18) the economic growth rate g:

$$1+g = \frac{Y}{Y_{-1}} = \frac{\bar{Z}}{\bar{Z}_{-1}} = 1+q\left(Bh^{\delta}-1\right) \tag{31}$$

In line with the new growth literature initiated by Lucas (1988) and Romer (1990), equation (31) stresses that long-term economic growth positively depends on the rate of knowledge accumulation, which is driven both by the proportion of skilled workers in the economy (i.e. those who have made the effort at school) and the length of training. From this perspective, it is worth noting that, following mixed empirical support (see Benhabib and Spiegel, 1994; Bils and Klenow, 2000), more recently the positive impact of education on economic growth has received clear backing from empirical studies conducted with improved data quality (see de la Fuente and Domenech, 2006; Cohen and Soto, 2007). We then study how NDC and conventional systems have differing impacts on economic growth through changes in the training length and the proportion of skilled workers.

5.1 NDC systems

A pure contributory NDC system ($\xi = 0$) has no impact on the training length (eq. 17). As a consequence, as underlined in equation (28), it also has no impact on the proportion of skilled workers. Indeed, in that case, as pension benefits are proportional to all contributions the retirement system can no longer alter the skill choice. NDC systems are then characterized by an unchanged investment in human capital, i.e. $h = h^0$ and $q = Q(h^0, \rho)$, and it follows that:

Proposition 3 *NDC systems have no impact on economic growth.*

5.2 Conventional systems

Consider alternatively conventional systems whose pensions are linked to the best or last years, i.e. $\xi = 1$. Admitting that they are actuarially fair, i.e. $\nu = \widetilde{\nu}$, following eq. (29) the training is specified by $h = h^0 \left[1 + \frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)} \frac{(1-h)Bh^\delta - 1}{Bh^\delta - 1}\right]$, where $\lim_{h \to h^0} RHS > h^0$ and $\lim_{h \to 1} RHS < h^0$. This equation thus defines a relation between the training and the contribution rate of the retirement system such that $h = h(\tau, \rho) < 1$ and $\frac{\partial h}{\partial \tau} > 0$. In addition, as the skill choice is specified by $q = G\left([1 + \beta\rho] \ln\left[(1-h) Bh^\delta\right]\right)$ where $h \geq h^0$, it follows that $\frac{\partial q}{\partial \tau} \leq 0$. The negative impact of conventional systems on the proportion of skilled workers can at first glance appear counter-intuitive. Indeed, as such a system results in lengthening the training of skilled workers, it widens the gap between skilled and unskilled workers' earnings. However, from a life cycle perspective, with no retirement system or with a pure contributory system (Proposition 1), individuals who decide to undertake training choose the length h^0 that maximizes their expected lifetime income; h^0 thus maximizes the lifetime income inequality between skilled and unskilled workers. Lengthening the training thus increases lifetime income inequality when $h < h^0$. Conversely, when $h > h^0$, lengthening the training reduces lifetime income inequality, because we move away from the individually optimal training length. Therefore, even if the retirement system does not carry out transfers from high-income to low-income earners, we know from Proposition 1 that such an earnings-related pension benefit formula generates an incentive for longer training. Skilled workers are then encouraged to train themselves beyond their individually optimal level. Consequently, actuarially fair retirement systems whose pensions are linked to the best or last years reduce lifetime income inequality compared to a situation with no retirement system (or a purely contributory one, such as NDC systems) and then reduce the proportion of skilled workers (eq. 17). Denoting ε_Q^h as the elasticity of Q with respect to h, the following Proposition then holds:

Proposition 4 *As long as* $0 < Q(h^0, \rho) < 1$, *assuming* $d^2G(.) \leq 0 \ \forall \tau \geq 0$, $-\varepsilon_Q^h > \frac{Bh^\delta \delta}{Bh^\delta - 1}$ *for* $\tau = 1$ *is a necessary and sufficient condition such that economic growth exhibits an inverse U-shaped pattern with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years.*

By reducing the proportion of skilled workers in the economy, actuarially fair conventional systems negatively impact economic growth. On the other hand, they induce skilled workers to train longer. For a sufficiently low size of the system, the latter effect always dominates the former, and we can highlight the positive impact of PAYG retirement systems on economic growth as empirically reported by Sala-i-Martin (1996) and Zhang and Zhang (2004). The underlying mechanism, which is initiated by the lengthening of training, is directly related to Kemnitz and Wigger (2000) and Le Garrec (2001)[12].

However, when the size of the system increases, everything else being equal, the leading effect can be reversed if lifetime income inequality strongly matters in the skill choice, or more formally if $-\varepsilon_Q^h > \frac{Bh^\delta \delta}{Bh^\delta - 1}$, where $-\varepsilon_Q^h = \varepsilon_Q^I \left(\frac{h}{1-h} - \delta\right)$, ε_Q^I being the elasticity of Q with respect to I. In that case, similarly to Docquier and Paddison (2003), conventional PAYG retirement systems based on partial earnings history are harmful for economic growth because they first reduce the proportion of skilled workers. This sheds light on the existence of an inverse U-shaped pattern of economic growth with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years. Moreover, this inverse U-shaped pattern sustains the existence of an optimally designed retirement system regarding economic growth which is not an NDC system but a conventional system based on partial earnings history, at least if its size is not too high compared to the optimal size. Indeed, in the latter case economic growth with a conventional system could be potentially lower than with no sys-

[12] In Zhang (1995), Zhang et al. (2001) and Zhang and Zhang (2003), PAYG retirement systems reduce fertility, while they relax parents' liquidity constraints in Kaganovich and Zilcha (1999), Lambrecht et al. (2005) and Glomm and Kaganovich (2008). In both cases, PAYG retirement systems result in more growth because they provide an incentive for altruistic parents to invest more in their childrens' education. In Sala-i-Martin (1996), older workers are associated with negative externalities in the average stock of human capital. By inducing earlier retirement, PAYG retirement systems then stimulate growth. In Kaganovich and Meier (2012), PAYG retirement systems stimulate growth because they help bolster the majority of voters who support the funding of public education.

tem or with an NDC system. To illustrate this point, consider for example any distribution characterized by $(1+\beta\rho)\ln\left[(1-h(1,\rho))Bh(1,\rho)^{\delta}\right] < \sigma_{\min} < (1+\beta\rho)\ln\left[(1-h(0,\rho))Bh(0,\rho)^{\delta}\right]$. In this case, there exists $\tilde{\tau} < 1$ such that $\tau \geq \tilde{\tau}$ entails $g = 0$ with a conventional system whereas $g = \left(Bh^{0\delta} - 1\right)Q\left(h^{0},\rho\right) > 0$ with an NDC system (or with no retirement system). This stresses the importance of evaluating the impact of aging on economic growth and the size of the conventional system that maximizes growth so as to determine the desirable adjustment.

6 Aging and optimal growth

6.1 Longevity, education and growth

As noted in the introduction, the coming century will be characterized by increased longevity. Life expectancy at birth, nowadays equal to 80 years, should reach 85 by 2050 in Western Europe (United Nations, 2009). This will have important consequences on public finance. It will have also important consequences for individuals, involving significant changes in their choices. First, individuals will need to finance a longer period in retirement. With low pension benefits, they will inevitably need to save more before retirement. As increased longevity raises the value of investments that pay over time, it will also encourage investment in education. For an economy with high life expectancy, Kalemli-Ozcan et al. (2000) have hence estimated the elasticity of schooling years with respect to life expectancy at 0.7. Assuming that economic growth is driven by investment in education, we can also expect that increased longevity will have a positive impact on economic growth.

Proposition 5 *With no retirement system (or with an NDC system), in-*

creased longevity stimulates economic growth by increasing the proportion of skilled workers while leaving unchanged the time they have spent in training.

This result is directly related to Proposition 1. Indeed, if there is no retirement system or an NDC system, the training length is $h = h^0 = \frac{\delta}{1+\delta}$. In that case, there is then no impact of aging on the length of training. The impact comes only from a change in the proportion of skilled workers such that, following eq. (28), $\frac{\partial Q}{\partial \rho} = dG\left(.\right) \beta \ln \left[\left(1-h^0\right) Bh^{0\delta}\right] \geq 0$.

By contrast, in the case of an actuarially fair conventional system ($\xi = 1$ and $\nu = \widetilde{\nu}$) it follows from eq. (29) that the training length increases with longevity: $\frac{\partial h}{\partial \rho} = \frac{\delta \frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1} \frac{\beta(1-\alpha)[\alpha+\tau(1-\alpha)]}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)} \frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]} > 0$[13] $\forall \tau > 0$. Indeed, such a retirement system provides incentives to invest in pension benefits through longer training. As increased longevity favors investments that pay out over time, it then increases the training length. The impact on the proportion of skilled workers is therefore no longer trivial. On the one hand, everything else being equal, increased longevity encourages individuals to become skilled workers: $\frac{\partial Q}{\partial \rho} \geq 0$. On the other hand, as $h > h^0$, the induced lengthening of the training reduces lifetime income inequality and then $\frac{\partial Q}{\partial h} \leq 0$.

Proposition 6 *With an actuarially fair retirement system whose pensions are linked to the best or last years, increased longevity enhances economic growth by increasing both the proportion of skilled workers and the time they have spent to be trained if the latter is moderate enough.*

Formally, the condition in Proposition 6 applies as long as $\frac{\partial h}{\partial \rho} \leq \frac{\beta}{1+\beta\rho} \frac{\ln\left[(1-h)Bh^{\delta}\right]}{\frac{1}{1-h}-\frac{\delta}{h}}$. As $\lim_{\tau=0} \frac{\partial h}{\partial \rho} = 0$ and $\lim_{\tau=0} \left(\frac{1}{1-h} - \frac{\delta}{h}\right) = 0$, this is always the case so long as the size of the retirement system is sufficiently low.

[13] Note that $Bh^{\delta} - 1 - \delta = \left(1 - \frac{1-h}{h}\delta\right)\left(Bh^{\delta} - 1\right) + \frac{\delta}{h}\left[\left(1-h\right)Bh^{\delta} - 1\right] \geq 0 \ \forall h \geq h^0$.

6.2 Optimal growth

An important distinction must be made between the two systems when considering aging. As an NDC system has no impact on economic growth, no adjustment is required when the population is aging. By contrast, if a conventional system can enhance economic growth, it can also be potentially harmful if its size is too high compared to the optimal size. We must then study the evolution of the latter to verify, at a minimum, that an unchanged size does not become harmful for economic growth, i.e. that the optimal size is not decreasing with longevity. Assuming $\tau^* = \arg\max_{\tau} \{1+g\} < 1$, it follows from Proposition 4 that:

$$sign\frac{d\tau^*}{d\rho} = sign\left\{\begin{array}{c} \frac{\partial^2 Q}{\partial\rho\partial h}\frac{h}{q} - \frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial \rho} \\ +\frac{\partial h}{\partial \rho}\left[\frac{-B\delta^2 h^{\delta-1}}{\left(Bh^\delta-1\right)^2} + \frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right]\end{array}\right\} \tag{32}$$

On the one hand, assuming $d^2G(.) \leq 0\ \forall \tau \geq 0$ yields $\frac{\partial^2 Q}{\partial h^2} \leq 0$ and then $\frac{\partial h}{\partial \rho}\left[\frac{-B\delta^2 h^{\delta-1}}{\left(Bh^\delta-1\right)^2} + \frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right] \leq 0$. However, at least if considering low levels of τ^*, the length of training is weakly related to longevity. As underlined in Proposition 3, if $\tau = 0$, $\frac{\partial h}{\partial \rho} = 0$. Considering a sufficiently low impact of longevity on the length of training then yields $sign\frac{d\tau^*}{d\rho} = sign\left\{\frac{\partial^2 Q}{\partial\rho\partial h}\frac{h}{q} - \frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial \rho}\right\}$, where $\frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial \rho} \leq 0$. It follows that if the negative impact on the proportion of skilled workers initiated by the lengthening of training is reduced by the increased longevity, $\frac{\partial^2 Q}{\partial\rho\partial h} \geq 0$, then the optimal size τ^* increases. Such a condition is verified if the elasticity of the density function is, in absolute value, higher than unity: $-\varepsilon_{dG} \geq 1$.

Proposition 7 *Assuming* $\tau^* = \arg\max_{\tau} \{1+g\} < 1$*, if the length of training is weakly related to longevity,* $-\varepsilon_{dG} \geq 1$ *is a sufficient condition so that* *a marginal increase in longevity raises the growth-maximizing size of a conventional retirement system.*

Note that the condition $-\varepsilon_{dG} \geq 1$ in Proposition 7 is not restrictive. Let us consider for example a Pareto distribution. In that case, the density function $dG(\sigma) = \frac{\mu\sigma_{\min}^{\mu}}{\sigma^{\mu+1}}$ $\forall\sigma \geq \sigma_{\min}$, 0 otherwise, where $\sigma_{\min} > 0$ and $\mu > 0$. It follows that $-\varepsilon_{dG} = -d^2G\frac{\sigma}{dG} = 1 + \mu > 1$, i.e. the condition always holds.

For policy-making, as underlined by Le Garrec (2005), maximizing economic growth is equivalent to maximizing intertemporal social welfare if the weight assigned to future generations is high enough. From this perspective, in the case of a benevolent planner able to implement the optimal policy, the message is clear: increased longevity should be associated with an increase in the size of the existing conventional retirement systems, not with a switch towards NDC systems. However, there is no guarantee that the political process leads to this optimal size. According to Browning (1975), there are even good reasons to think that the political process leads to a PAYG size exceeding the growth-maximizing level. Indeed, he showed that workers tend to increase their support for the PAYG retirement system as they approach retirement. Considering then that the pivotal voter is middle-aged worker, by definition closer to retirement than a young worker, this could strengthen support for a PAYG size that exceeds the growth-maximizing (or the welfare-maximizing) level. Does this mean that in practice an NDC system is preferable to a conventional system? Not necessarily. Indeed, an assessment that the conventional PAYG size $\hat{\tau}(\rho_0)$ when considering longevity ρ_0 exceeds the growth-maximizing level $\tau^*(\rho_0)$ does not necessarily mean that an NDC system would allow greater economic growth. By contrast, if we give credit to the empirical results reported by Sala-i-Martin (1996) and Zhang and Zhang (2004), economic growth is such that: $g(\rho_0, \hat{\tau}(\rho_0)) > g(\rho_0, \tau = 0)$, where

$g(\rho_0, \tau = 0)$ denotes the level of economic growth that would be obtained with an NDC system of any size (Proposition 5).

Starting then from a situation characterized by $g(\rho_0, \hat{\tau}(\rho_0)) > g(\rho_0, \tau = 0)$, what may happen with increased longevity $\rho_1 > \rho_0$? Firstly, as the pivotal voter approaches retirement, it is likely that the PAYG size supported by a majority will grow, $\frac{\partial \hat{\tau}}{\partial \rho} \geq 0$. Two configurations may then occur according to the variation of $\hat{\tau}$ compared with τ^*. If $\frac{\partial[\hat{\tau} - \tau^*]}{\partial \rho}$ is low enough, the superiority of a conventional system over an NDC system may be preserved, i.e. $g(\rho_1, \hat{\tau}(\rho_1)) > g(\rho_1, \tau = 0)$. In that case, a switch towards NDC systems will not be optimal. By contrast, if $\frac{\partial[\hat{\tau} - \tau^*]}{\partial \rho}$ is high enough, we may observe the reverse, i.e. $g(\rho_1, \hat{\tau}(\rho_1)) < g(\rho_1, \tau = 0)$. In that case, as suggested by Belan et al. (1998), a Pareto-improving transition towards a fully funded system may exist if it results in a significant increase in economic growth. If such a transition does not exist, a switch to NDC system can then be considered as a desirable policy for increasing economic growth and social welfare.

Note that all the solutions above have been considered while keeping the calculation of pension benefits actuarially fair. However, if the main problem of existing retirement systems is that they are too large, another solution would be to increase the progressivity of the system. Indeed, as highlighted by Koethenbuerger (2007), the size of the retirement system chosen by the median voter tends to decrease as the link between contributions and benefits is loosened. It is a fact that progressive systems appear smaller than actuarially fair systems. However, as argued by Le Garrec (2005), more progressivity also leads to fewer incentives for people to invest in their education. At this stage, the impact on economic growth of introducing more progressivity would then appear uncertain, unless it also strengthens majority support for public education funding, as argued by Kaganovich and

Meier (2012). It would be very important for future research to investigate whether these incentives on public education could compensate the loss in private education incentives so that introducing progressivity may be a credible alternative to switching to NDC systems in certain circumstances. To do that, it is necessary to introduce public education explicitly into the model.

7 Conclusion

Life expectancy at birth in Western Europe, which is currently 80 years, should reach 85 by 2050 (United Nations, 2009). PAYG-financed public retirement systems will have then to cope with an increasingly large number of pensioners compared to the number of contributors. Important changes are thus unavoidable in the OECD countries . In 2005, the payment of pension benefits represented 38% of all OECD public social expenditure. As a matter of fact, retirement systems are the main plank in the industrialized countries' redistributive policies, and their importance will increase even further with the aging of their populations.

Based on the claim that the PAYG retirement systems are generally inefficient (accused of low returns and of distorting individual behavior), some economists such as Feldstein (1995a, 1995b, 1996) stress that these financial difficulties are providing an opportunity to move to fully funded systems. However, replacing conventional PAYG systems by financial - or funded - defined contribution (FDC) systems would involve such large transitional costs that it would be socially and politically difficult to implement such a reform in the Western democracies. For that reason, in recent years there has been a large focus on non-financial - or notional - defined contribution (NDC) systems of the kind enacted in Sweden in 1994. By basing benefits on individual accounts, NDC systems undoubtedly do have desirable features in terms of transparency. However, as existing retirement systems (except in the Anglo-Saxon countries) appear close to actuarial fairness, we can not expect a significant decrease in negative incentive effects from NDC systems. In many respects, introducing an NDC system largely involves moving from a defined benefit to a defined contribution system while aiming to stabilize the contribution rate, an objective that can be achieved similarly within the scope of more conventional defined benefit systems. As emphasized by Börsch-Supan (2006), cleverly designed conventional retirement systems can often do the same job as NDC systems. As shown in this article, they can even do better.

In particular, as most conventional retirement systems link pension benefits only to the partial earnings history, they can stimulate economic growth by promoting the accumulation of human capital, at least if their size is not too high. When population aging is taken into account, the optimal adjustment in terms of economic growth is then likely to be an increase in the size of the existing retirement systems rather than a switch to notional systems. This recommendation appears strengthened by the further observation that actuarially fair retirement systems whose pensions are linked to the best or last years lower lifetime income inequality, whereas NDC systems do not. More generally, moving to an NDC system, by nature purely contributory, definitively closes the debate about the progressivity of the retirement system, which is an important one in a democracy. According to Le Garrec (2012), compared to any actuarially fair system, greater progressivity would result in negative incentive effects that would lead to less economic growth, but also to less lifetime inequality. In that case, it would then be interesting to evaluate with an intertemporal social welfare, as used in Boadway et al.

(1991), Marchand et al. (1996), Le Garrec (2005) and Docquier et al. (2007), whether greater progressivity with its concomitant lower lifetime inequality would be worth the cost in terms of economic growth – unless there is no trade-off between economic growth and greater progressivity in the calculation of pension benefits, as is argued by Kaganovich and Meier (2012). From that perspective, incorporating public education in the analysis appears to be a promising avenue for further research.

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References

[1] Abel A. (1985), Precautionary saving and accidental bequests, *American Economic Review*, 75(4), pp. 777-791.

[2] Acemoglu D. and Johnson S. (2007), Disease and development: the effect of life expectancy on economic growth, *Journal of Political Economy*, 115(6), pp. 925-985.

[3] Andolfatto D., Ferrall C., Gomme P. (2000), Life-cycle learning, earning, income and wealth, mimeo, November.

[4] Belan P., Michel P. and Pestieau P. (1998), Pareto-improving social security reform, *Geneva Risk and Insurance Review*, 23(2), pp. 119-125.

[5] Benhabib J. and Spiegel M.M. (1994), The role of human capital in economic development: evidence from aggregate cross-country data, *Journal of Monetary Economics*, 34(2), pp.143-173.

[6] Ben-Porath, Y. (1967), The production of human capital and the life cycle of earnings, *Journal of Political Economy*, 75, pp. 352-365.

[7] Bils M. and Klenow P. (2000), Does schooling cause growth?, *American Economic Review*, 90(5), pp. 1160-83.

[8] Boadway R., Marchand M. and Pestieau P. (1991), Pay-as-you-go social security in a changing environment, *Journal of Population Economics*, 4, pp. 257-280.

[9] Boldrin M. and Montes A. (2005), The intergenerational State education and pensions, *Review of Economic Studies*, 72(3), pp. 651-664.

[10] Börsch-Supan A. (2006), What are NDC systems? What do they bring to reform strategies?, in *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer eds., chap. 3, The World Bank.

[11] Boucekkine R., de la Croix D. and Licandro O. (2002), Vintage human capital, demographic trends and endogenous growth, *Journal of Economic Theory*, 104, pp. 340-375.

[12] Bowles S. and Gintis H. (2002), The inheritance of inequality, *Journal of Economic Perspective*, 16(3), pp. 3-30.

[13] Bozio A. and Piketty T. (2008), *Pour un nouveau système de retraite: Des comptes individuels de cotisations financés par répartition*, Collection du CEPREMAP, éditions Rue d'Ulm.

[14] Breyer F. and Hupfeld S. (2009), Fairness of public pensions and old-age poverty, *FinanzArchiv: Public Finance Analysis*, 65(3), pp. 358-380.

[15] Brown J., Coronado, J. L. and Fullerton, D. (2006), The progressivity of social security, *NBER Papers on Retirement Research Center Projects,* NB06-10, December.

[16] Burkhauser R. and Walick J. (1981), Disentangling the annuity from the redistributive aspects of social security in United States, *Review of Income and Wealth*, vol 28, pp 401-422.

[17] Casamatta G., Cremer H. and Pestieau P. (2000), The political economy of social security, *Scandinavian Journal of Economics*, 102(3), pp. 503-522.

[18] Cervellati M. and Sunde U. (2005), Human capital, life expectancy, and the process of development, *American Economic Review*, 95(5), pp. 1653-1672.

[19] Cervellati M. and Sunde U. (2011), Life expectancy and economic growth: The role of the demographic transition, *Journal of Economic Growth*, 16(2), pp. 99-133.

[20] Cigno A. (2008), Is there a social security tax wedge?, *Labour Economics*, 15(1), pp.68–77.

[21] Cigno A. (2010), How to avoid a pension crisis: a question of intelligent system design, *CESifo Economic Studies*, 56, pp. 21-37.

[22] Cigno A. and A. Luporini (2011), Optimal family policy in the presence of moral hazard when the quality and quantity of children are stochastic, *CESifo Economic Studies*, 57, pp. 349-364.

[23] Cigno A., A. Luporini and A. Pettini (2003), Transfers to families with children as a principal-agent problem, *Journal of Public Economics*, 87, pp. 1165-1172.

[24] Cipriani G. P. (2013), Population ageing and PAYG pensions in the OLG model, *Journal of Population Economics*, 27, pp. 251-256.

[25] Cohen D. and Soto M. (2007), Growth and education—good data, good results, *Journal of Economic growth*, 12(1), pp. 51-76.

[26] Coronado, J. L., Fullerton, D. and Glass, T. (1999), Distributional impacts of proposed changes to the social security system, *NBER WP 6989*, March.

[27] Coronado, J. L., Fullerton, D. and Glass, T. (2000), The progressivity of social security, *NBER WP 7520*, February.

[28] Cremer H., De Donder P., Maldonado D., Pestieau P. (2007), Voting over type and generosity of a pension system when some individuals are myopic, *Journal of Public Economics*, 91, pp. 2041-2061.

[29] d'Addio, A. C. (2007), Intergenerational transmission of disadvantage: mobility or immobility across generations?, *OECD Social, Employment and Migration Working Papers 52*, OECD Publishing.

[30] Deaton A. and C. Paxton (1998), Aging and inequality in income and health, *American Economic Review*, 88(2), pp. 248-253.

[31] Deaton A. and C. Paxton (1999), Mortality, education, income and inequality among american cohort, *NBER WP 7140*, May.

[32] de la Croix D. and Licandro O. (1999), Life expectancy and endogenous growth, *Economics Letters*, 65, pp. 255-263.

[33] de la Fuente A. and Domenech A. (2006), Human capital in growth regressions: how much difference does data quality make?, *Journal of the European Economic Association*, 4(1), pp. 1-36.

[34] Docquier F. and Paddison O. (2003), Social security benefit rules, growth and inequality, *Journal of Macroeconomics*, 25, pp. 47-71.

[35] Docquier F., Paddison O. and Pestieau P. (2007), Optimal accumulation in an endogenous growth setting with human capital, *Journal of Economic Theory*, 134, pp. 361-378.

[36] Fanti L. and L. Gori (2013), Fertility-related pensions and cyclical instability, *Journal of Population Economics*, 26, 1209-1232.

[37] Feldstein M. (1995a), Fiscal policies, capital formation, and capitalism, *European Economic Review*, 39, pp 399-420.

[38] Feldstein M. (1995b), Would privatizing Social Security raise economic welfare?, *NBER WP 5281*.

[39] Feldstein M. (1996), The missing piece in policy analysis: Social Security reform, *American Economic Review*, 86(2), pp. 1-14.

[40] Feldstein M., Dicks-Mireaux L. and Poterba J. (1983), The effective taxe rate and the pretax rate of return, *Journal of Public Economics*, 21(2), pp129-158.

[41] Garrett D. (1995), The effects of differential mortality rates on the progressivity of social security, *Economic Inquiry*, vol 33(3), pp 457-475.

[42] Glomm G. and Kaganovich M. (2008), Social security, public education and the growth-inequality relationship, *European Economic Review*, 52, pp. 1009-1034.

[43] Gustman, A. and Steinmeier, T. (2001), How effective is redistribution under the social security benefit formula?, *Journal of Public Economics*, 82, pp. 1-28.

[44] Hachon C. (2010), Do Beveridgian pension systems increase growth?, *Journal of Population Economics*, 23, pp. 825-831.

[45] Huggett M., Ventura G. and Yaron A. (2006), Human capital and earnings distribution dynamics, *Journal of Monetary Economics,* 53(2), pp. 265-290.

[46] Jayachandran S. and Lleras-Muney A. (2009), Life expectancy and human capital investments: evidence from maternal mortality declines, *Quarterly Journal of Economics*, 124(1), pp. 349-398.

[47] Kaganovich M. and Zilcha I. (1999), Education, social security and growth, *Journal of Public Economics*, 71, pp. 289-309.

[48] Kaganovich M. and Meier V. (2012), Social security systems, human capital, and growth in a small open economy, *Journal of Public Economic Theory*, 14(4), pp. 573-600.

[49] Kalemli-Ozcan S., Ryder H. and Weil D. (2000), Mortality decline, human capital investment, and economic growth, *Journal of Development Economics*, 62, pp. 1-23.

[50] Kemnitz, A. and Wigger, B. (2000), Growth and social security: the role of human capital, *European Journal of Political Economy*, 16, pp. 673-683.

[51] Koethenbuerger M., Poutvaara P. and Profeta P. (2007), Why are more redistributive social security systems smaller? A median voter approach, *Oxford Economic Papers*, 60, pp. 275-292.

[52] Lambrecht S., Michel P. and Vidal J.-P. (2005), Public pensions and growth, *European Economic Review*, 49, pp. 1261-1281.

[53] Le Garrec G. (2001), Systèmes de retraite par répartition, mode de calcul des droits à pension et croissance, *Louvain Economic Review*, 67(4), pp. 357-380.

[54] Le Garrec G. (2005), Social security, inequality and growth, *WP n°2005-22*, OFCE/Sciences Po, December.

[55] Le Garrec G. (2012), Social security, income inequality and growth, *Journal of Pension Economics and Finance*, 11(1), pp. 53-70.

[56] Lillard, L. (1977), Inequality: earnings vs. human capital, *American Economic Review*, vol 67(2), pp. 42-53.

[57] Lindbeck A. and Persson M. (2003), The gains from pension reform, *Journal of Economic Literature*, 41(1), pp. 74-112.

[58] Lucas R.E. (1988), On the mechanics of economic development, *Journal of Monetary Economics*, 21, pp. 3-42.

[59] Marchand M., Michel P. and Pestieau P. (1996), Optimal intergenerational transfers in an endogenous growth model with fertility changes, *European Journal of Political Economy*, 12, pp. 33-48.

[60] OECD (2007), *Pensions at a glance: public policies across OECD countries.*

[61] Palmer E. (2006), What is NDC?, in *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer eds., chap. 2, The World Bank.

[62] Romer P. (1990), Endogenous technical change, *Journal of Political Economy*, 98(5), pp. S71–S102.

[63] Sala-i-Martin X. (1996), A Positive Theory of Social Security, *Journal of Economic Growth*, 1(2), pp 277-304.

[64] Saint-Paul G. (1992), Fiscal policy in an endogeneous growth model, *Quarterly Journal of Economics*, 107(4), pp 1243-1259.

[65] Soares R. (2005), Mortality reductions, educational attainment, and fertility choice, *American Economic Review*, 95(3), pp. 580-601.

[66] Sommacal A. (2006), Pension systems and intragenerational redistribution when labor supply is endogenous, *Oxford Economic Papers*, 58(3), pp. 379-406.

[67] Stahlberg A.-C. (1990), Life cycle income redistribution of the public sector: inter- and intra-generational effects, in *Generating Equality in the Welfare State*, Persson I. ed., Oslo:Norvegian University Press.

[68] United Nations (2009), *World Population Prospects: The 2008 Revision*.

[69] Yaari M. (1965), Uncertain lifetime, life insurance, and the theory of the consumer, *Review of Economic Studies*, 32(2), pp. 137-150.

[70] Zhang J. (1995), Social Security and Endogenous growth, *Journal of Public Economics*, 58, pp. 185-213.

[71] Zhang J., Zhang J. and Lee R. (2001), Mortality decline and long-run economic growth, *Journal of Public Economics*, 80, pp. 485-507.

[72] Zhang J., Zhang J. and Lee R. (2003), Rising longevity, education, savings, and growth, *Journal of Development Economics*, 70, pp. 83-101.

[73] Zhang J. and Zhang J. (2003), Long-run effects of unfunded social security with earnings-dependent benefits, *Journal of Economic Dynamics and Control*, 28, pp. 617-641.

[74] Zhang J. and Zhang J. (2004), How does social security affect economic growth? Evidence from cross-country data, *Journal of Population Economics*, 17, pp. 473-500.

Appendix A: proof of propositions

proposition 1

Assume an interior solution. Following (15), $\Delta h = h(\xi = 1) - h(\xi = 0) = \frac{\delta}{1+\delta}\frac{\rho\theta_{t+1}}{(1-\tau)R_{t+1}}\Delta\xi$, where $\Delta\xi = 1$. It follows that $\frac{\Delta h}{\Delta \xi} > 0$ as long as $\theta_{t+1} > 0$.

proposition 2

Assuming $\xi = 1$, actuarial fairness defined by (9) entails that:

$$\frac{\rho\theta_t B h_{t-1}^{\delta}\overline{Z}_{t-2}w_{t-1} + \rho\nu_t\overline{Z}_{t-1}w_t}{\tau\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}\overline{Z}_{t-2}w_{t-1}} = \frac{\rho\theta_t\overline{Z}_{t-2}w_{t-1} + \rho\nu_t\overline{Z}_{t-1}w_t}{\tau\overline{Z}_{t-2}w_{t-1}}$$

and then:

$$\frac{\theta_t B h_{t-1}^{\delta}w_{t-1} + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)w_t}{\tau\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}w_{t-1}} = \frac{\theta_t w_{t-1} + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)w_t}{\tau w_{t-1}}$$

Introducing (25) entails then that:

$$\frac{\left[\frac{\tau}{\rho}\left[q_t\left(1-h_t\right)Bh_t^{\delta} + 1 - q_t\right] - \nu_t\right]Bh_{t-1}^{\delta} + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)}{\left(1-h_{t-1}\right)Bh_{t-1}^{\delta}}$$
$$= \left[\frac{\tau}{\rho}\left[q_t\left(1-h_t\right)Bh_t^{\delta} + 1 - q_t\right] - \nu_t\right] + \nu_t\left(q_{t-1}Bh_{t-1}^{\delta} + 1 - q_{t-1}\right)$$

Actuarial fairness with $\xi = 1$ is then obtained if:

$$\nu_t = \frac{Bh_{t-1}^{1+\delta}}{Bh_{t-1}^{\delta} - 1}\frac{\tau}{\rho}\frac{q_t\left(1-h_t\right)Bh_t^{\delta} + 1 - q_t}{q_{t-1}\left(1-h_{t-1}\right)Bh_{t-1}^{\delta} + 1 - q_{t-1}}$$

proposition 3

Following equation (28), if $\xi = 0$, $h = h^0 = \frac{\delta}{1+\delta}$. Accordingly, following equation (28), $q = Q\left(h^0, \rho\right)$. Therefore, as $g = q\left(Bh^{\delta} - 1\right)$, $\frac{dg}{d\tau} = 0$ if $\xi = 0$.

proposition 4

From eq. (31) we derive $d(1+g) = \left(Bh^{\delta}-1\right)dq + qB\delta h^{\delta-1}dh$. Knowing that $dq = \frac{\partial Q}{\partial h}dh$ everything else being equal, it follows that $\frac{d(1+g)}{d\tau} = 0$ is equivalent to $\left[\left(Bh^{\delta}-1\right)\frac{\partial Q}{\partial h} + qB\delta h^{\delta-1}\right]\frac{dh}{d\tau} = 0$ or to $\left[\left(Bh^{\delta}-1\right)\varepsilon_Q^h + B\delta h^{\delta}\right]\frac{dh}{d\tau} = 0$.

From eq. (29), if $\xi = 1$ and $\nu = \widetilde{\nu}$, we have $\frac{\partial h}{\partial \tau} = \frac{\delta\frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1}\frac{\beta\rho(1+\beta\rho)\alpha(1-\alpha)}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]} > 0 \ \forall\rho > 0$. It follows that $\frac{d(1+g)}{d\tau} = 0$ is equivalent to $-\varepsilon_Q^h = \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$.

Thereafter, $\frac{\partial Q}{\partial h} = dG(.)(1+\beta\rho)\frac{-Bh^{\delta}+(1-h)B\delta h^{\delta-1}}{(1-h)Bh^{\delta}} = dG(.)(1+\beta\rho)\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)$. In $\tau = 0$, it follows that $\varepsilon_Q^h = \frac{\partial Q}{\partial h}\frac{h}{q} = 0$ and then that $-\varepsilon_Q^h < \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$.

On the one hand we have: $\frac{d\left(\frac{B\delta h^{\delta}}{Bh^{\delta}-1}\right)}{dh} = \frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2} < 0$.

On the other hand: $d\varepsilon_Q^h = \left(\frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right)dh \ \forall d\rho = 0$, where $\frac{\partial^2 Q}{\partial h^2} = d^2G(.)(1+\beta\rho)^2\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)^2 + dG(.)(1+\beta\rho)\left(-\frac{1}{(1-h)^2}-\frac{\delta}{h^2}\right)$. As $\frac{-1}{1-h}+\frac{\delta}{h} \leq 0 \ \forall\ h \geq h^0 = \frac{\delta}{1+\delta}$, assuming $d^2G(.) \leq 0 \ \forall\tau \geq 0$ results in $-\frac{d\varepsilon_Q^h}{dh} \geq 0$.

Assuming $d^2G(.) \leq 0 \ \forall\tau \geq 0$, $-\varepsilon_Q^h > \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$ in $\tau = 1$ is then a necessary and sufficient condition such that economic growth exhibits an inverse U-shaped pattern with respect to the size of an actuarially fair retirement system whose pensions are linked to the best or last years.

proposition 5

Following equation (29), if $\xi = 0$, $h = h^0 = \frac{\delta}{1+\delta}$. Accordingly, following equation (28), $q = Q\left(h^0, \rho\right)$. Therefore, as $g = q\left(Bh^{\delta}-1\right)$, $\frac{dg}{d\rho} = \frac{\partial Q}{\partial \rho}\left(Bh^{0\delta}-1\right)$ if $\xi = 0$, where $\frac{\partial Q}{\partial \rho} = dG(.)\beta\ln\left[\left(1-h^0\right)Bh^{0\delta}\right] \geq 0$ and $\frac{\partial h}{\partial \rho} = 0$.

proposition 6

Following equation (29), if $\xi = 1$ and $\nu = \tilde{\nu}$, $h = h^0\left[1 + \frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{(1-h_t)Bh_t^{\delta}-1}{Bh_t^{\delta}-1}\right]$.

It follows that $\frac{\partial h}{\partial \rho} = \frac{\delta\frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1}\frac{\beta(1-\alpha)(\alpha+\tau(1-\alpha))}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]} \geq 0$. Following equation (28), $dq = dG(.)\left\{\beta \ln\left[(1-h)Bh^{\delta}\right]d\rho + (1+\beta\rho)\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)dh\right\}$. It follows that $\frac{dq}{d\rho} \geq 0$ if $\frac{dh}{d\rho} \leq \frac{\beta}{1+\beta\rho}\frac{\ln\left[(1-h)Bh^{\delta}\right]}{\frac{1}{1-h}-\frac{\delta}{h}}$. If the latter condition holds, as $dg = dq\left(Bh^{\delta}-1\right)+q\delta Bh^{\delta-1}dh$, $\frac{dg}{d\rho} \geq 0$ everything else being equal.

proposition 7

Assuming that $-\varepsilon_Q^h > \frac{B\delta h^{\delta}}{Bh^{\delta}-1}$ for $\tau = 1$, $-\frac{\partial Q}{\partial h}\left(h\left(\tau^*,\rho\right),\rho\right)\frac{h(\tau^*,\rho)}{Q(h(\tau^*,\rho),\rho)} = \frac{Bh(\tau^*,\rho)^{\delta}\delta}{Bh(\tau^*,\rho)^{\delta}-1}$ defines the contribution rate that maximizes economic growth. It follows:

$$-\left\{\frac{\partial^2 Q}{\partial h^2}\left[\frac{\partial h}{\partial \tau}d\tau^* + \frac{\partial h}{\partial \rho}d\rho\right] + \frac{\partial^2 Q}{\partial \rho \partial h}d\rho\right\}\frac{h}{q} - \frac{\partial Q}{\partial h}\frac{1}{q}\left[\frac{\partial h}{\partial \tau}d\tau^* + \frac{\partial h}{\partial \rho}d\rho\right] + \frac{\partial Q}{\partial h}h\frac{\frac{\partial Q}{\partial h}\left[\frac{\partial h}{\partial \tau}d\tau^* + \frac{\partial h}{\partial \rho}d\rho\right]+\frac{\partial Q}{\partial \rho}d\rho}{q^2}$$
$$= \frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\left[\frac{\partial h}{\partial \tau}d\tau^* + \frac{\partial h}{\partial \rho}d\rho\right]$$
$$\iff -\left\{\frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2} + \frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\right\}\frac{\partial h}{\partial \tau}d\tau^*$$
$$= \left\{\left[\frac{\partial^2 Q}{\partial h^2}\frac{\partial h}{\partial \rho} + \frac{\partial^2 Q}{\partial \rho \partial h}\right]\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q}\frac{\partial h}{\partial \rho} - \frac{\partial Q}{\partial h}h\frac{\frac{\partial Q}{\partial h}\frac{\partial h}{\partial \rho}+\frac{\partial Q}{\partial \rho}}{q^2} + \frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\frac{\partial h}{\partial \rho}\right\}d\rho$$

As $-\left\{\frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2} + \frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2}\right\} > 0$ (see proof of proposition 4), it follows that:

$$sign\frac{d\tau^*}{d\rho} = sign\left\{\begin{array}{c}\frac{\partial^2 Q}{\partial \rho \partial h}\frac{h}{q} - \frac{\partial Q}{\partial h}\frac{h}{q^2}\frac{\partial Q}{\partial \rho} \\ +\frac{\partial h}{\partial \rho}\left[\frac{-B\delta^2 h^{\delta-1}}{\left(Bh^{\delta}-1\right)^2} + \frac{\partial^2 Q}{\partial h^2}\frac{h}{q} + \frac{\partial Q}{\partial h}\frac{1}{q} - \left(\frac{\partial Q}{\partial h}\right)^2\frac{h}{q^2}\right]\end{array}\right\}$$

We have then $\frac{\partial^2 Q}{\partial \rho \partial h} = d^2G(.)\beta\ln\left[(1-h)Bh^{\delta}\right](1+\beta\rho)\left(\frac{-1}{1-h}+\frac{\delta}{h}\right) + dG(.)\beta\left(\frac{-1}{1-h}+\frac{\delta}{h}\right)$

$= \left(\frac{-1}{1-h}+\frac{\delta}{h}\right)\left(d^2G(.)\beta\ln\left[(1-h)Bh^{\delta}\right](1+\beta\rho) + dG(.)\beta\right)$

$= \left(\frac{-1}{1-h}+\frac{\delta}{h}\right)dG(.)\beta\left(\frac{d^2G(.)}{dG(.)}\ln\left[(1-h)Bh^{\delta}\right](1+\beta\rho)+1\right)$

$= \left(\frac{-1}{1-h}+\frac{\delta}{h}\right)dG(.)\beta\left(\varepsilon_{dG}+1\right)$

It follows that if $\varepsilon_{dG} \leq -1$, $\frac{\partial^2 Q}{\partial \rho \partial h} \geq 0$ $\forall h \geq h^0 = \frac{\delta}{1+\delta}$. As $\frac{\partial Q}{\partial h} \leq 0$ and $\frac{\partial Q}{\partial \rho} \geq 0$, it follows from (32) that if $\frac{\partial h}{\partial \rho}$ is sufficiently low, $\frac{d\tau^*}{d\rho} \geq 0$. As $\frac{\partial h}{\partial \rho} = \frac{\delta\frac{(1-h)Bh^{\delta}-1}{Bh^{\delta}-1}\frac{\beta(1-\alpha)(\alpha+\tau(1-\alpha))}{[\alpha(1+\beta\rho)+\tau(1-\alpha)]^2}\tau}{1+\delta\left[1+\frac{\beta\rho(1-\alpha)\tau}{\alpha(1+\beta\rho)+\tau(1-\alpha)}\frac{Bh^{\delta}\left(Bh^{\delta}-1-\delta\right)}{\left(Bh^{\delta}-1\right)^2}\right]}$, this is in particular the case if τ^* is sufficiently low.