

Working paper

2016-08

COMMERCIAL REVOLUTIONS, SEARCH, AND DEVELOPMENT

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March 2016

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Abstract

A society can reap more benefits from an improvement in communication and transportation when the gap between privileged and less-privileged individuals is narrow. This hypothesis is investigated in a Kiyotaki-Wright search environment where people enjoy different rental positions. Historical evidence concerning the rise of Italian seaport cities during the Middle Ages, of the Netherlands in the 17th century, and the nobility's resistance to trade in pre-revolutionary France is used to support the hypothesis.

Keywords: Privileges, Resistance to Trade, Speculative Strategies.

JEL codes: O11, N13, C61, C63, D63.

1 Introduction

It has been argued that when an economy stagnates, it is often because the most capable people choose to be occupied in rent-seeking activities over alternatives that would improve the society's welfare (Krueger, 1974, Baumol, 1990, 1993, 2002, and Murphy, Shleifer and Vishny, 1991, 1993). Murphy, Shleifer and Vishny (1991) (henceforth MSV) suggest that good communications and transportation that facilitate trade can direct people toward more socially useful occupations (see their Table 1, p. 519).

Though this suggestion is sound, their contribution lacks a formal analysis of how and under what conditions improved contacts among individuals can induce some to devote greater effort to socially useful activities. This paper provides such an analysis using a variant of the Kiyotaki and Wright (1989) (henceforth KW) search model. There are two aspects of the KW environment that are useful for evaluating MSV's conjecture. First, because exchanges occur in decentralized markets, it is possible to be explicit about shocks that

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affect the intensity of communication among individuals, as this is measured by a matching rate. Second, because the population is heterogenous with respect to rental income, it is possible to evaluate how the society's stratification affects its ability to reap the benefits of a communication shock.

The notion that a large gap between privileged and unprivileged individuals can slow economic progress has been widely discussed in the literature. Landes (1998), for instance, contends that people in Europe did not exhibit the same dynamism as the North American settlers because the presence of a wealthy landed aristocracy in Europe abetted inaction. By contrast, in America, "Equality bred self-esteem, ambition, a readiness to enter and compete in the marketplace, a spirit of individualism and contentiousness" (1998, p. 297). Lopez, in a seminal summary of the medieval European commercial revolution, recounts the circumstances of the rise of Italian cities as follows:

The business fever, when it came, left almost no one untouched. Perhaps the most telling change occurred among those noble families that grew too large to live comfortably off their inherited land. Outside Italy the supernumerary children tried to escape poverty and boredom through an ecclesiastic career, marriage to a noble heiress, or military service at somebody's hire. In the Italian towns they more often found the same opportunities and thrills by pooling their capital in business ventures, which involved a chance of pick up along the way a fight with pirates, brigands, unfriendly lords, and possibly Infidels. ...Indeed it is usually difficult, in the scanty documents of the tenth and eleventh centuries, to tell apart merchants who had bought real estates with the profits of trade, and were called "honorable" or "noble", from the noblemen who had sold their estate, invested the proceed in trade, and married merchants' daughters. (1976, p. 66)

Conversely, France, where the stratification of society was particularly pronounced until the Revolution (Chaussinand-Nogaret, 1985), missed the opportunity to rival the Genoese and the Venetians during the medieval trade revolution and in 17th century could hardly match the maritime commercial power of the Dutch Republic.

To formalize these arguments, I consider a KW economy consisting of three groups of people with different privileges, measured by their flows of rents. An individual's only choice is whether to engage in (indirect) trade. For people at the bottom of the privilege distribution, namely commoners, trading is always the best option. Conversely, for those at the top, identified as the higher nobility, it never is. Those individuals in the middle, however, can go either way. They abandon the existing rent-seeking practices if the lure of commercial activities is strong enough to compensate them for foregoing the comfort of receiving a generous regular rent. When the middle group chooses to trade, a new trade link is created and all groups trade at higher frequencies. The consequences of this network externality effect are illustrated by a quantitative experiment that captures the main stylized

facts of a commercial revolution.

Despite the policy debate spurred by Baumol (1990) and Murphy et al. (1991) on how to reduce unproductive occupations, formal analysis of why trade could be an effective instrument in pursuing such an objective is lacking. A notable exception is Holmes et al. (2001). In a growth model with vertical innovation, they evaluate the effects of regulatory entry restrictions imposed on out-of-state firms. Their analysis and the type of shock they consider are, nevertheless, different than mine. I focus on a pre-industrial economy, in which sudden booms in income are typically driven by trade (Lopez, 1976, p. 85-91), and use the matching rate to capture the communication shock.

Another related work is Acemoglu (1995), who employs evidence on the medieval commercial revolution to argue that the reward system is not exogenous as, for instance, Baumol (1990) does but adjusts over time as people move from unproductive to productive occupations. A similar result emerges in my analysis: When the middle group switches to trade, the rental over production income ratio of privileged individuals declines.

This paper also contributes to the literature on growth spurts as the result of qualitative changes in the structure of the economy. Peretto (2015) highlights that an important condition for the onset of modern growth is market expansion. Similarly, Kelly (1997) notes that market expansion was the main driver of medieval Chinese economic growth. In this paper, a larger market, generated by an acceleration of the matching rate, leads, as in Kelly, to a temporary growth spurt – in the long run, the economy converges to a higher but constant level of production.

The remainder of the paper is organized as follows. The next section briefly describes the economic environment, illustrates the evolution of the distribution of inventories under a given profile of strategies, and defines the best response functions of three types of representative agents. The section that follows studies the dynamics of the system. Section (4) proposes two quantitative experiments to illustrate the propagation mechanism of a shock to the matching rate. Section (5) discusses key stylized facts of the medieval and Atlantic trade revolutions and comments on the resistance of the French aristocracy to trade. Section (6) addresses multiple steady states. Section (7) concludes.

2 The Model Economy

There are only minor differences with respect to the decentralized economy described in KW: Time is continuous; the ranking of returns across assets is allowed to change; and agents are not necessarily equally distributed across types.¹ The overall size of the population is N . There are three types of infinitely lived agents, measured by N_i , for $i = 1, 2, 3$. The fraction

¹Lagos et al. (2016), section two, also presents a similar summary of the model.

of each type is $\mu_i = N_i/N$. The type i agent ² consumes only good i and can produce only good $i + 1 \pmod{3}$. Production occurs immediately after consumption. Agent i 's instantaneous utility from consumption and the disutility of producing good $i + 1$ (the terms good, commodity, inventory, and asset will be used interchangeably) are denoted by U_i and D_i , respectively, and their difference is $u_i = U_i - D_i$. Without loss of generality, I assume that $D_i = 0$ and $U_i = 1$, for all i . There is a capacity constraint: At each instant in time, an individual can hold only one unit of asset i that offers a return r_i , measured in units of utility. The discount rate is denoted by $\rho > 0$. A pair of agents is randomly and uniformly chosen from the population to meet for a possible trade. After a pair is formed, the waiting time for the next pair to be called is governed by a Poisson process with intensity α . A bilateral trade occurs if, and only if, it is mutually agreeable. Agent i always accepts good i but never holds it because there is immediate consumption. Therefore, agent i always enters the market with either one unit of good $i + 1$ or one unit of good $i + 2$.

The proportion of type- i agents that hold good j at time t is denoted by $p_{i,j}(t)$. Then, the vector $\tilde{\mathbf{p}}(t) = \{p_{i,j}(t)\}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$ describes the state of the economy at time t (henceforth, i and j go from 1 to 3). However, because $p_{i,i}(t) = 0$,

$$p_{i,i+1}(t) + p_{i,i+2}(t) = \mu_i, \quad (1)$$

for any $t > 0$, the state of the economy can be represented in a more parsimonious way by $\mathbf{p}(t) = \{p_{1,2}(t), p_{2,3}(t), p_{3,1}(t)\}$. To simplify the notation, sometimes $p_{i,i+1}$ becomes p_i . An individual i has only to decide whether to exchange his production good for the other type of good. Agent i 's choice in favor of indirect trade is denoted $\tau_i(t) = 1$ and that against it $\tau_i(t) = 0$. Agent i has to select a time path, $\tau_i(t)$, that maximizes her expected stream of present and future net utility, given other agents' strategy paths, $\boldsymbol{\theta}(t) = [\theta_1(t), \theta_2(t), \theta_3(t)]$, and $\mathbf{p}(t)$, for any $t > 0$.

2.1 Distribution of Assets and Value Functions

For a given profile of strategies $\boldsymbol{\theta}(t)$, the evolution in the stock of good $i + 1$ held by agents of type i is given by³

$$\dot{p}_{i,i+1} = \alpha \{ p_{i,i+2} [p_{i+1,i} (1 - \theta_{i+1}) + p_{i+2,i} + p_{i+2,i+1} (1 - \theta_i)] - p_{i,i+1} [p_{i+1,i+2} \theta_i] \}. \quad (2)$$

The terms inside the brackets before the minus sign calculate the probability that a type- i agent is called for a match while holding good $i + 2$ and ultimately holds good $i + 1$. Such

²When no confusion arises, I will use the loose language of calling an agent of type i simply agent or individual i .

³Duffie and Sun (2012) show that in a similar matching environment frequency coincides with probability. See, in particular, their Theorem 1.

an event materializes either because of barter or because the agent leaves the meeting with good i , consumes it, and then immediately produces good $i + 1$. The following expression accounts for the probability that an agent of type i who holds good $i + 1$ ultimately has good $i + 2$. The behavior of $p_{i,i+2}$ is derived through (1). The ensemble of the system that describes the evolution of the distribution of inventories is denoted by $F(\mathbf{p}(t))$.

Consider now a representative agent of type i that has to compute her best profile of strategies, given a pattern of inventories, $\mathbf{p}(t)$, and a pattern of strategies for other agents, $\boldsymbol{\theta}(t)$ – including those of her own type. Let $V_{i,j}(t)$ be the value function when holding good j at time t . When $j = i + 1$, we have that

$$\begin{aligned} V_{i,i+1}(t) = & \max_{\{\tau_i(s)\}_{s \geq t}} \int_t^\infty \alpha e^{-\alpha(s-t)} \{ e^{-\rho(s-t)} ([p_{i,i+2}\tau_i(1-\theta_i) + p_{i+1,i+2}\tau_i]V_{i,i+2} + \\ & + [1 - p_{i,i+2}\tau_i(1-\theta_i) - p_{i+1,i+2}\tau_i]V_{i,i+1} + [p_{i+1,i} + p_{i+2,i}\theta_{i+2}]u_i) + \\ & + \frac{1 - e^{-(s-t)\rho}}{\rho} r_{i+1} \} ds, \end{aligned} \quad (3)$$

where the term $\alpha e^{-\alpha(s-t)} ds$ measures the probability that an agent of type i is called to form a match for the first time after time t in the time interval $(s, s + ds)$. The term after the discount factor is the probability that this agent engages in indirect trading – in which case, she is left with $V_{i,i+2}$ as her continuation value. Otherwise, she ultimately has good $i + 1$ either because no trade occurs or because she acquired her consumption good – an event that occurs with probability $p_{i+1,i} + p_{i+2,i}\theta_{i+2}$ – and then produces good $i + 1$. The last term is the return on holding asset $i + 1$ from time t to time s . An additional equation for $V_{i,i+2}(t)$, reported in the Appendix, completes the description of the optimization problem.

Let $\Delta_i(s) \equiv V_{i,i+1}(s) - V_{i,i+2}(s)$, and let $\tilde{\tau}_i(s; \boldsymbol{\theta}(s), \mathbf{p}(s))$ denote the optimal (or best) response profile of strategies of representative agent i to other players' strategies $\boldsymbol{\theta}(s)$ along the pattern of inventories $\mathbf{p}(s)$ for $s > t$. Then, it must be that

$$\tilde{\tau}_i(s; \boldsymbol{\theta}(s), \mathbf{p}(s)) = \begin{cases} 1 & \text{if } \Delta_i(s) < 0 \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

for any $s \geq t$. Hence, the formulation of the problem corresponds to a Markov decision process in which the representative agent optimizes over a sequence of functions, $\tilde{\tau}_i(t)$, that allows the ex post decision on indirect trading to vary with the current state of the inventory distribution and the pattern of the strategies of other agents.

2.2 Definition of Dynamic Nash Equilibrium

Given an initial distribution of inventories, $\mathbf{p}(0) = \mathbf{p}_0$, a *Dynamic Nash Equilibrium (DNE)* is a path of strategies $\boldsymbol{\theta}^*(t)$ together with a distribution of inventories $\mathbf{p}^*(t)$ such that for all

$t > 0$:

- i. $\mathbf{p}^*(t)$ and $\boldsymbol{\theta}^*(t)$ satisfy the dynamics equations (2) with the initial condition $\mathbf{p}^*(0) = \mathbf{p}_0$ and subject to the constraint (1);
- ii. For all $t > 0$, every agent maximizes his or her expected utility given the strategy profiles of the rest of the population; and
- iii $\tilde{r}_i(t; \boldsymbol{\theta}^*(t), \mathbf{p}^*(t)) = \theta_i^*(t)$ for all $t > 0$.

2.3 Rent Seeking and Production

Following Lagos et al. (2016), I interpret a good as a Lucas tree producing a fruit. Let Q_i and R_i be the level of production and the rental income of the ensemble of type- i agents. It is easy to show that

$$Q_i = \alpha \{p_{i,i+1}(p_{i+1,i} + p_{i+2,i}\theta_{i+2}) + p_{i,i+2}[p_{i+1,i}(1 - \theta_{i+1}) + p_{i+2,i}]\}. \quad (5)$$

and that

$$R_i = (r_{i+1} - r_{i+2})p_{i,i+1} + r_{i+2}\mu_i, \quad (6)$$

Therefore, the overall income of the ensemble of type- i agents is

$$Y_i = Q_i + R_i$$

Summing up over the three types of agents, we obtain the aggregate level of rent, production, and income, that is: $R = \sum_1^3 R_i$, $Q = \sum_1^3 Q_i$, and $Y = \sum_1^3 Y_i$, respectively. The rental ratio, r_i , is defined as

$$\tilde{r}_i \equiv \frac{R_i}{Y_i} = \frac{1}{1 + Q_i/R_i}$$

Hence, the rental ratio for the overall economy is

$$\tilde{r} \equiv \frac{R}{Y} = \frac{1}{1 + Q/R}. \quad (7)$$

A first insight from equation (5) is that the flow of production of group i , Q_i , depends on the trade strategies of the other two groups of agents, θ_{i+1} and θ_{i+2} . A second insight is that the set of strategies of the three types of agents affects Q_i through the distribution of assets, \mathbf{p} . Section (2.5) analytically characterizes the dependence of production from the strategies for some interesting equilibria.

Finally, it is also useful to define the frequency of trade, $t_i(s)$. This measures the number of times that a good is traded within a unit of time. It can be shown that

$$t_i(s) = \alpha \{ p_{i+2}(\mu_i - p_i(1 - \theta_{i+2}) + \theta_{i+1}p_{i+1}) + (\mu_{i+1} - p_{i+1})[p_i + (\mu_i - p_i)(1 - \theta_{i+1}) + (\mu_{i+2} - p_{i+2})(1 - \theta_{i+2})] \}.$$

Then, the average trade frequency is

$$\bar{t}(s) = x_1(s)t_1(s) + x_2(s)t_2(s) + x_3(s)t_3(s)$$

where $x_i(s) = \sum_{j=1}^3 p_{j,i}$, is the outstanding stock of the type- i good.

2.4 Privileges

I turn now to the description of economic privileges in an environment similar to Model A of KW in which $r_3 < r_2 < r_1$. Because $u_i = 1$ for $i = 1, 2, 3$, the order of privileges can be defined through the returns, r_i . An agent i benefits from two types of advantages when his production good fetches a higher return, r_{i+1} : he receives a greater flow of dividends while waiting for a successful trade; he can market his production good more easily, as this becomes more attractive for the buyer. Clearly, the most privileged individuals are type-3 agents, as they produce the good that yields the highest rent. Next come type-1 agents. Type-2 agents, who produce the good with the lowest return, are at the bottom. A variety of circumstances lead to the creation of rents (Baumol, 1990 and MSV). Here, however, I am concerned with their consequences for production. Baumol remarks, "If the rules are such as to impede the earning of much wealth via activity A, or are such as to impose social disgrace on those who engage in it, then, other things being equal, entrepreneurs' efforts will tend to be channeled to other activities, call them B. But if B contributes less to production or welfare than A, the consequences for society may be considerable" (1990, p. 898). Next, I characterize this statement in the KW environment.

2.5 Steady States

There are six possible return rankings. When $r_3 < r_2 < r_1$ (Model A in KW), the steady-state (pure-strategies) equilibrium is given by

$$\boldsymbol{\theta} = [0, 1, 0] \text{ and } \mathbf{p} = \frac{1}{3}[1, 0.5, 1], \text{ if } \frac{r_2 - r_3}{\alpha} > \frac{1}{6} \quad (\text{F})$$

and by

$$\boldsymbol{\theta} = [1, 1, 0] \text{ and } \mathbf{p} = \frac{1}{3}[a, b, 1], \text{ if } \frac{r_2 - r_3}{\alpha} < \frac{1}{3}(\sqrt{2} - 1) \quad (\text{S})$$

where $a = \frac{1}{2}\sqrt{2}$ and $b = \sqrt{2} - 1$. Following KW, I will refer to these two equilibria as fundamental and speculative equilibrium, respectively, or as (0,1,0) and (1,1,0) equilibrium.

Two other rankings, $r_2 < r_1 < r_3$ and $r_1 < r_3 < r_2$, are simply relabeled depictions of the same Model A-type economy. The three remaining rankings are more than relabeling exercises, as they can give rise to multiple equilibria. They correspond to Model B of KW and will be discussed in Section (6). Until then, I will focus on the case in which $r_3 < r_2 < r_1$. (Table 8 summarizes the main features of the equilibria under the six possible configurations of the asset returns).

Returning now to the $r_3 < r_2 < r_1$ case, I compare the volume of production and the level of rents that prevail in the speculative and the fundamental equilibrium. The objective is to obtain the main insight into the long-run effects of communication shocks when this induces type-1 individuals to switch strategies.

Proposition 1 *Assume that $r_3 < r_2 < r_1$, α satisfies the condition in (F) and that α' satisfies the condition in (S) – namely, $\alpha' > \frac{r_2 - r_3}{\frac{1}{3}(\sqrt{2} - 1)}$. The equilibrium steady-state volumes of production and the levels of rents when the matching rate is α and α' are those reported in Table 1.*

Proof. To obtain the results in Table 1, it suffices to evaluate the expressions (5) and (6) on the fundamental (F) and speculative (S) equilibria. ■

An inspection of Panel A of Table 1 reveals the following:

Corollary 1 *The volume of production, Q_i and Q , is strictly higher in the speculative than in the fundamental equilibrium.*

A comparison of the level of rents (Panel B) obtained by groups over the two equilibria yields the following result:

Corollary 2 *The rents gained by type-3 individuals, R_3 , are the same in the two equilibria. The rent earned by type-1 individuals, R_1 , is higher in the fundamental equilibrium. Conversely, type-2 individuals receive higher rents, R_2 , in the speculative equilibrium.*

This result implies that there would be a convergence of rents between the bottom and the middle groups (i.e., commoners and lower nobility) when going from the fundamental to the speculative equilibrium. From the two corollaries, the following also holds:

Corollary 3 *The rental ratios of the two most privileged groups, \tilde{r}_1 and \tilde{r}_3 , are always lower in the speculative than in the fundamental equilibrium.*

Table 1: Production and Rents

	Lower Nobility	Commoners	Higher Nobility	Aggregate
Panel A: Production				
	Q_1	Q_2	Q_3	Q
F	$\frac{1}{18}\alpha$	$\frac{1}{18}\alpha$	$\frac{1}{18}\alpha$	$\frac{1}{6}\alpha$
S	$\frac{2\sqrt{2}}{18}(\sqrt{2}-1)\alpha'$	$\frac{\sqrt{2}}{18}\alpha'$	$\frac{\sqrt{2}}{18}\alpha'$	$\frac{2}{9}\alpha'$
Panel B: Rents				
	R_1	R_2	R_3	R
F	$\frac{1}{3}r_2$	$\frac{1}{3}(\frac{1}{2}r_1 + \frac{1}{2}r_3)$	$\frac{1}{3}r_1$	$\frac{1}{2}r_1 + \frac{1}{3}(r_2 + \frac{1}{2}r_3)$
S	$\frac{1}{3}[ar_2 + (1-a)r_3]$	$\frac{1}{3}[(1-b)r_1 + br_3]$	$\frac{1}{3}r_1$	$(1 - \frac{\sqrt{2}}{3})r_1 + \frac{1}{3\sqrt{2}}(r_2 + r_3)$

- Note: F and S denote the fundamental and speculative equilibrium, respectively; $a = \frac{1}{2}\sqrt{2}$ and $b = \sqrt{2} - 1$.

A final observation concerns the relationship among the average levels of rents, R_i , in the two equilibria. Section (2.4) defined the privileges of an individual on the basis of the rent generated by his production good relative to that of the other two goods. Panel B of Table 1 shows that in the fundamental equilibrium, the average rent of the higher nobility is greater than that of the lower nobility and that the lower nobility earns a larger rent than do the commoners. In the speculative equilibrium, it remains true that the higher nobility receives a larger rent than the other two groups. Nevertheless, for the lower nobility's average rent to be larger than that of the commoners, $r_1 - r_2$ relative to $r_2 - r_3$ has to be below a certain threshold. These observations are summarized as follows:

Corollary 4 *In the fundamental steady-state equilibrium, $R_3 > R_1 > R_2$. In the speculative equilibrium, $R_3 > R_1$ and $R_3 > R_2$. Furthermore, if $\frac{r_1 - r_2}{r_2 - r_3} < \frac{1}{(\sqrt{2}-1)2} - 1 \simeq 0.2071$, $R_1 > R_2$.*

Next, I describe the adjustment process of the economy when going from a fundamental to a speculative equilibrium. First, I study the dynamics and establish some convergence properties and then propose two experiments. In one, a shock to the matching parameter triggers a switch from a fundamental to a speculative equilibrium, whereas it does not in the other.

3 Dynamics

KW provide a characterization of the steady-state equilibrium properties of their model but do not attempt to describe the process by which the economy converges to a steady-state equilibrium, nor do they offer an account of how a new trading strategy may emerge during the transition. Several researchers have contributed to filling this gap via different methodological approaches. Marimon et al. (1990) and Başçı (1999) ask whether artificially intelligent agents can learn to play equilibrium strategies. A similar question is tested in a number of controlled laboratory experiments with real people (Brown, 1996, and Duffy and Ochs, 1999). Matsuyama et al. (1993), Wright (1995), Luo (1999) and Sethi (1999) approach the issue through evolutionary dynamics.

The dynamics of this class of models pose difficulties not encountered in economies in which similar individuals trade their goods and services in centralized markets because the evolution of the distribution of goods across individuals depends on the history of exchanges, that is, on agents' trading strategies. These, in turn, are affected by the expected evolution of the distribution of goods.

I obtain the dynamics by building directly on the concept of an open-loop Nash equilibrium with many players as in Fudenberg and Levine (1988). The idea is to identify an equilibrium such that, given the actions of all other players, no player can make any gain by changing her action. The analysis in this section does not require any specific ordering of the rents.

The law of motion of $\mathbf{p}(t)$ is a function of the profile of strategies, $\boldsymbol{\theta}(t)$, that can change over time in discrete steps. Specifically, Eq. (2) states that the system of the distribution of assets is given by the following (the time index is dropped):

$$\dot{p}_{1,2} = \alpha\{p_{1,3}[p_{2,1}(1 - \theta_2) + p_{3,1} + p_{3,2}(1 - \theta_1)] - p_{1,2}p_{2,3}\theta_1\}, \quad (8)$$

$$\dot{p}_{2,3} = \alpha\{p_{2,1}[p_{3,2}(1 - \theta_3) + p_{1,2} + p_{1,3}(1 - \theta_2)] - p_{2,3}p_{3,1}\theta_2\}, \quad (9)$$

$$\dot{p}_{3,1} = \alpha\{p_{3,2}[p_{1,3}(1 - \theta_1) + p_{2,3} + p_{2,1}(1 - \theta_3)] - p_{3,1}p_{1,2}\theta_3\}. \quad (10)$$

The evolution of the system can be followed for any arbitrary pattern of strategies, $\boldsymbol{\theta}(t)$, and for any set of parameters. Nevertheless, to maintain analytical tractability, I restrict attention to a time-constant set of strategies.

Proposition 2 *Except for the set of strategies (1,1,1), under any other time-constant pure strategies (τ_1, τ_2, τ_3) , with $\tau_i = \{0, 1\}$, for $i = 1, 2, 3$, $\mathbf{p}(t)$ converges to a stationary distribution, from any initial position.*

Proof. See Appendix

This proposition guarantees global convergence of the system for seven of the eight possible sets of strategies. These, however, are not necessarily Nash strategies. The following proposition specifies the paths that lead to a Nash steady-state equilibrium.

Proposition 3 *When the population is equally divided among the three types, the following six steady-state Nash Equilibria (NE) exist:*

Strategies	Assets Distribution	Strategies	Assets Distribution
(0,1,0)	$\frac{1}{3}[1, \frac{1}{2}, 1]$	(1,1,0)	$\frac{1}{3}[a, b, 1]$
(1,0,0)	$\frac{1}{3}[\frac{1}{2}, 1, 1]$	(1,0,1)	$\frac{1}{3}[b, 1, a]$
(0,0,1)	$\frac{1}{3}[1, 1, \frac{1}{2}]$	(0,1,1)	$\frac{1}{3}[1, a, b]$

where $a = \frac{1}{2}\sqrt{2}$ and $b = \sqrt{2} - 1$. In some cases a pair of NE coexist.

Proof. To prove that a given steady-state distribution is a NE, one needs to verify that the sign of Δ_i is consistent with the profile of strategies assumed for that particular steady-state distribution. See Technical Appendix for details.

Note that the (1,1,1) set of strategies, for which it is more difficult to analytically establish the convergence of the distribution, $\mathbf{p}(t)$, does not support a steady-state Nash equilibrium. When the economy is outside of the steady state, the same principle of verifying the consistency of Δ_i with respect to τ_i applies. Nevertheless, obtaining an analytical solution is rather challenging, as doing so depends on the overall forecasted evolution of the asset distribution. Instead, I solve it through a numerical algorithm. Next, I briefly illustrate the working of the algorithm. (Further details are available in a technical appendix.) The algorithm is quite general, as it also permits changing strategies along the dynamics.

The value functions, $V_{i,j}$, are the criteria that agents follow to determine their optimal patterns of strategies. In a steady state, $V_{i,j}$ can be determined analytically. To obtain their values also outside of the steady state, I exploit two properties of the system. First, for any interesting profile of strategies, the state variable, $\mathbf{p}(t)$, converges toward a fixed point (which is not necessarily a Nash equilibrium). Second, along a given pattern of $\mathbf{p}(t)$, the numerical value functions converge to their theoretical values when integrated backward in time. It is then possible to verify whether the value function of a representative agent along a specific trajectory, $\mathbf{p}(t)$, is consistent with the profile of strategies that are used to obtain such a trajectory, $\mathbf{p}(t)$, that is, to verify that no agent has an incentive to deviate at any point in time from the designated profile of strategies. The algorithm generates a sequence of rounds that seeks convergence toward an open-loop Nash equilibrium for $\mathbf{p}(t)$ and $\boldsymbol{\theta}(t)$. To

prove that a given steady-state distribution is a NE, one needs to verify that the sign of Δ_i is consistent with the profile of strategies assumed for that particular steady-state distribution.

4 Quantitative Experiments

I now turn to a quantitative illustration of the central result of this paper: a communication shock fuels an economic boom by inducing type-1 individuals, who are in the middle of the privilege distribution, to switch from fundamental to speculative strategies, as long as the distance between the privileged and unprivileged individuals is not too large.

4.1 The Commercial Revolution

Consider an economy characterized by a set of parameters that satisfies the condition for a fundamental steady-state equilibrium $(0,1,0)$, that is, $\frac{r_2-r_3}{\alpha} > \frac{1}{6}$, as, for example, that reported in Table 2. The economy is in such an equilibrium when it is struck by a communication shock. The magnitude of the shock is large enough that with the new level of the matching rate, α' , the condition for a speculative steady state is verified, that is, $\frac{r_2-r_3}{\alpha'} < \frac{1}{3}(\sqrt{2}-1)$. This shock captures, for instance, the initiatives of the Italian seaport cities that broke the trade freeze in the Mediterranean (see section 5.1) or the Colombian voyages.

After the shock, the three types of agents adopt the $(1,1,0)$ constant profile of Nash strategies, and the economy converges to the speculative steady state, $\mathbf{p} = \frac{1}{3}[a, b, 1]$, with $a = \frac{1}{2}\sqrt{2}$ and $b = \sqrt{2}-1$. The convergence is guaranteed by proposition (3). Fig. 1 depicts the evolution of the distribution of assets and that of the trade frequencies for the three goods. Type-1 individuals, by switching from $\tau_1 = 0$ to $\tau_1 = 1$, that is, by abandoning rent-seeking behavior in favor of trade, fuel a boom in trade and production for all goods. Specifically, immediately after the shock, the trade frequency of good 2 and of good 3 more than doubles, and that of good 1 increases by approximately 40%. Subsequently, the trade frequency of good 1 continues increasing, that of good 2 declines, and that of good 3 exhibits hump-shaped behavior. However, once the adjustment process to the new (speculative) equilibrium is complete, all goods trade at higher frequencies than in the initial (fundamental) equilibrium.

The long-run consequences of the shock on income and rental ratios were described in the corollaries of section (2.5) (notice that the parameters satisfy the condition of corollary 4). The first row (baseline) of Table 3 and of Table 4 summarize the long-run effects of the experiment, and Fig. 2 provides details on the adjustment process. In the short run, type-2 individuals (commoners) benefit the most from the shock, as they are favored by the sudden

Table 2: Parameters

	Population	Discount	Matching	Utility	Returns		
	μ_i	δ	α	u_i	r_1	r_2	r_3
Baseline	$\frac{1}{3}$	0.03	1	1	0.21	0.2	0
Higher Privileges	$\frac{1}{3}$	0.03	1	1	0.31	0.3	0

Table 3: Income Changes – Long Run

	Lower Nobility	Commoners	Higher Nobility	Aggregate
	Y_1	Y_2	Y_3	Y
Baseline	30.01	56.20	40.29	41.37
Higher Privileges	16.43	23.04	16.11	18.03

- Note: The table reports the steady-state effects of a 50% increase in α on Type- i average income and on population-level average income. The underlying parameters of the two economies are specified in Table 2. The figures are in percentages.

increase in the market for their production good 3. Indeed, immediately after the shock, the market effect is so large that type-2 individuals (commoners) overtake type-1 individuals – the lower nobility. Although in the long run they remain at the bottom of the scale, the distance from the lower nobility is considerably reduced. Such a convergence in income is consistent with Lopez’s observation, reported in the Introduction (see also Section 5.2), that the commercial revolution blurred the dividing line between merchants and nobles.

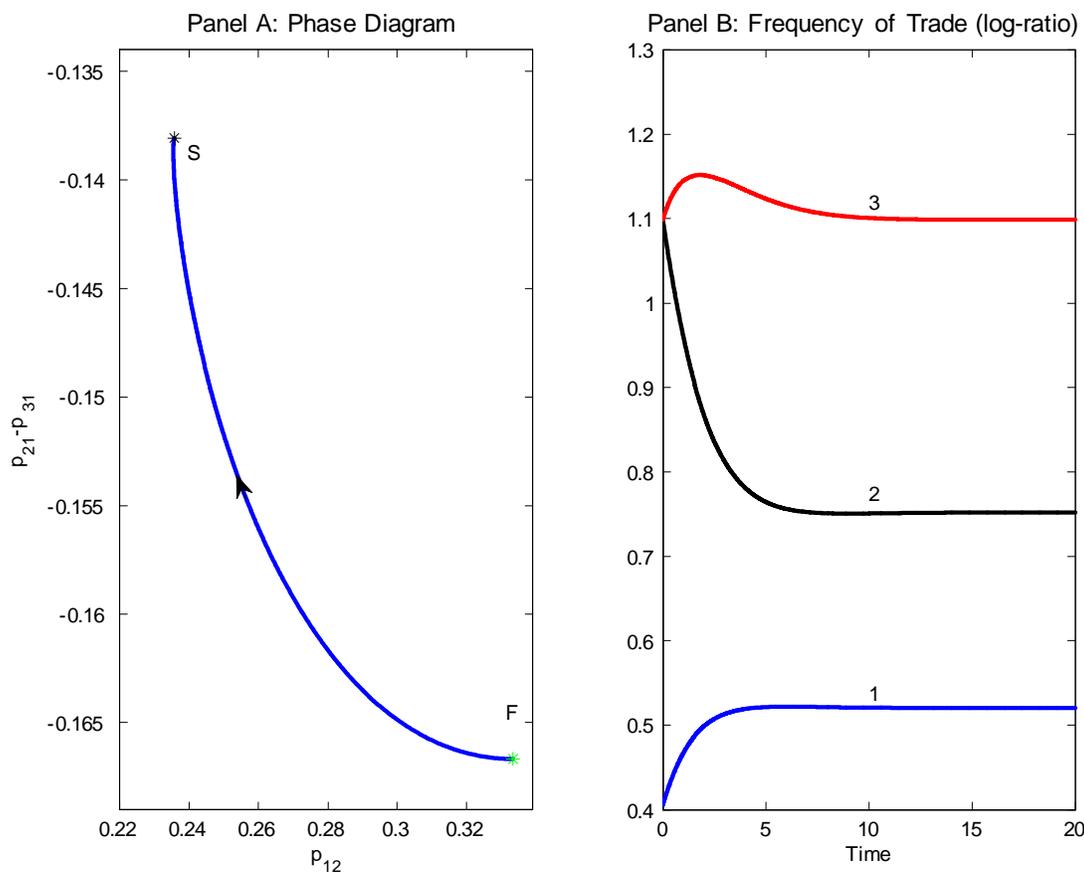
Corollary 3 established that the communication shock always drives down the rental ratios of the two privileged groups. With the current set of parameters, the commoners’ rental ratio, \tilde{r}_3 , also declines. After an initial decline across the three types, the rental ratio, \tilde{r}_i , continues declining for type-1 and type-3 individuals but increases for type-2 individuals (Panel B of Fig. 1), chiefly because the trade frequency of their consumption good declines.

Table 4: Changes in Rent-Income Ratios – Long Run

	Lower Nobility	Commoners	Higher Nobility	Aggregate
	\tilde{r}_1	\tilde{r}_2	\tilde{r}_3	\tilde{r}
Baseline	-64.66	-40.37	-40.29	-49.57
Higher Privileges	-16.43	-23.04	-16.11	-18.03

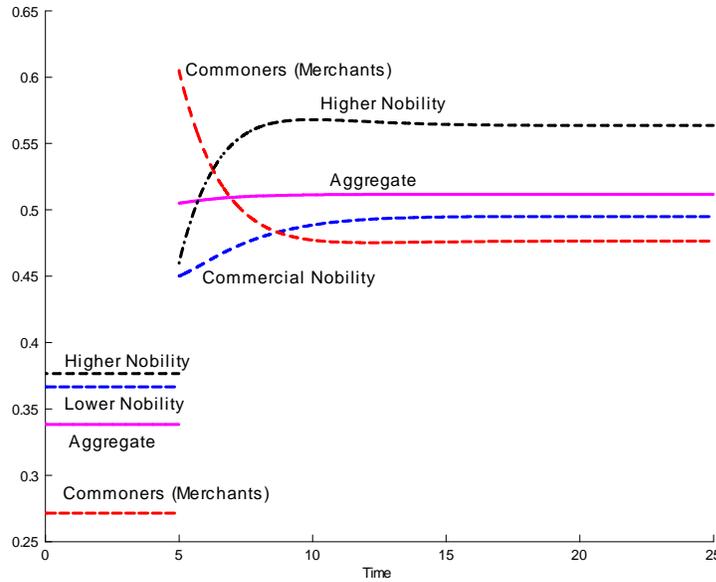
- Note: The table reports the steady-state effects of a 50% increase in α on \tilde{r}_i and \tilde{r} . The underlying parameters of the two economies are specified in Table 2. The figures are in percentages.

Figure 1: The Commercial Revolution, Distribution of Goods, and Trade

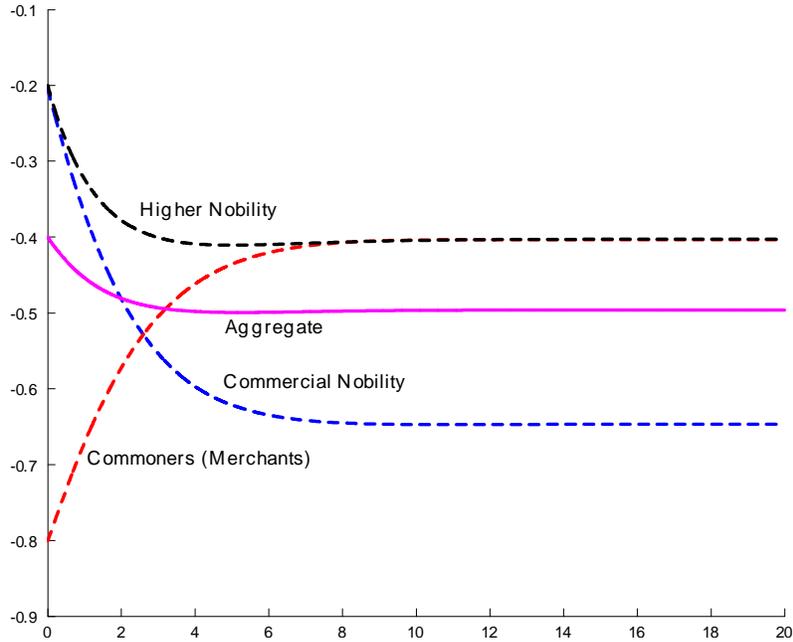


- Note: The baseline economy (see Table 2, first row) is in the fundamental steady-state equilibrium when it is struck by a shock that permanently increases the matching parameter by 50 percent. The ratios and differences are calculated with respect to the pre-shock state. The average frequency of trade increases by approximately 70%.

Figure 2: Communication Shock
 Panel A: Income

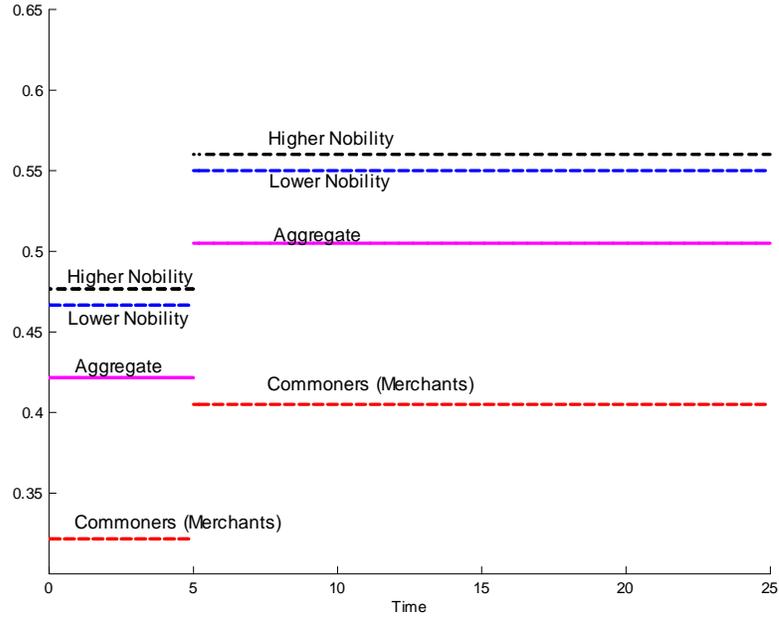


Panel B: Rent-Income Ratio

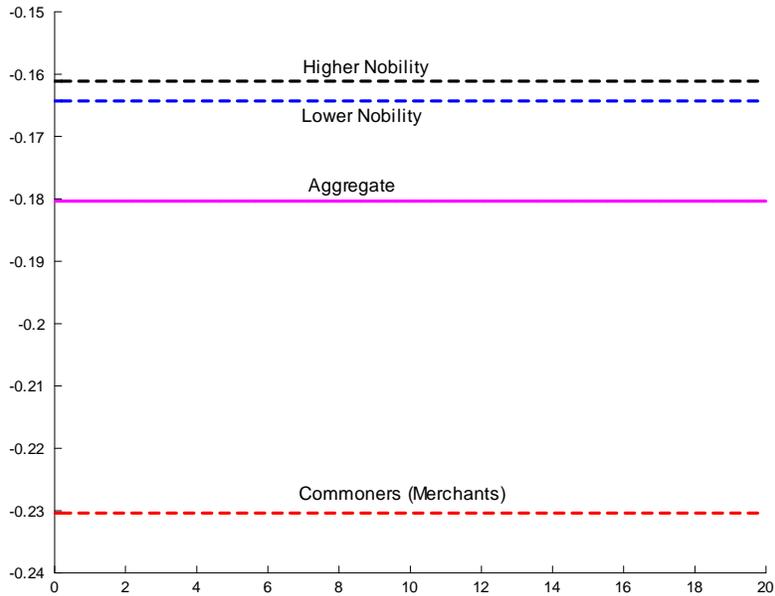


- Note: Panel A plots the average income of type-3 (higher nobility), type-2 (lower nobility), and type-1 (merchants) agents before and after a shock that increases the matching rate by 50 percent. Initially, the economy is in a fundamental-steady state equilibrium. Panel B reports the percentage change in the rent-output ratio \tilde{r}_i and \tilde{r} (see Eqs. (6 and (7))) due to the shock. The parameter values are in Table 3, first row (baseline).

Figure 3: Communication Shock
 Panel A: Income



Panel B: Rent-Income Ratio



- Note: See the note for figure 2. The underlying values of the parameters are in Table 3, second row (higher privileges).

4.2 Missing the Commercial Revolution

The passage from Lopez quoted in the Introduction notes that outside Italy, people were less eager to reap the gains of the 13th century commercial revolution. The analysis suggests that one factor that could have impeded the spread of the benefits of commerce was the relatively high levels of rents enjoyed by the nobility. As will be clarified in Section (5.2), in France and in Saxony, during the Middle Ages, the economic gap between the aristocracy and commoners was substantially larger than in Northern Italy (Lopez, 1976, Wickham, 2009).

In the KW environment, a larger gap between the privileged and the less privileged groups reduces the odds that a communication shock generates an economic boom. Recall that when $\frac{r_2-r_3}{\alpha} > \frac{1}{6}$, the economy's long-run equilibrium is (0,1,0). Hence, if the difference between the rent fetched by goods 2 and 3, which are produced by the lower nobility and the commoners, respectively, is large, a shock to the matching parameter, α , may fail to convince the lower nobility to adopt a speculative strategy. This event is reproduced in Fig. 3, which depicts the effects of a 50% increase in the matching rate in an economy similar to that studied in the previous experiment, except that the goods produced by the two privileged classes offer more generous returns ($r_1 = 0.31$ and $r_2 = 0.3$, see Table 2). The higher matching rate makes everyone better off, but it fails to trigger the economic boom observed in the previous experiment. Indeed, the economy remains in the same pre-shock fundamental steady-state equilibrium, although the trading and production activities are scaled up because of the higher meeting frequency. In the current experiment, in the long-run average income increases by 18 percent, compared to a 41 percent rise in the baseline experiment (Table 3) and the average rent-income ratio declines by approximately 18% (all driven by the rise in production income), while in the baseline experiment, it declines by approximately 50% Table 4.

5 Discussion

In this section, I verify the plausibility of the central results of the model by reviewing historical evidence on the effects of a communication shock. The baseline experiment demonstrated that if the gap between the privileged classes and the commoners is not too large, an improvement in communication favors (1) the emergence of a commercial nobility; (2) the convergence of income between commoners and the commercial nobility; and (3) a substantial increase in per capita income. Conversely, the alternative experiment revealed that if the rental gap, $r_2 - r_3$, is relatively large, the nobility will resist trade, the social and economic gap will persist, and per capita income will increase only moderately, reflecting the more intensive frequency of existing patterns of trade.

To match the phenomena emerging in the two experiments, I will review key facts concerning the onset and dissemination of the 13th commercial revolution and regarding the consequences of Atlantic trade in France and the Netherlands. I will also discuss evidence that leads support for the notion that the landed aristocracy's resistance to trade prevented France from experiencing an economic boom comparable to that observed in Northern and Central Italy during the Middle Ages and in the Low Countries during the 16th century.

5.1 The Middle Age Communication Shock

With the disintegration of the Roman Empire, the regions on the shores of the Mediterranean broke their communication ties. In the period running from approximately the year 700 to 950, trade had reached a low point in Europe, with the exception of those regions under the economic influence of the Arabs and of Byzantium, most notably Spain and Sicily.

"[B]y the time of Charlemagne the sea that had functioned as a central highway for the Greco-Roman community became the border between three different communities, which in the lack of an appropriate economic term may be identified through their predominant religions: Islamic, Orthodox, and Catholic. (Lopez, 1976, p. 22)

A tenuous link between these three separate communities was initially provided by the Jews. Their success in trade, however, was often halted by the reaction of the majority of the population that would drive them out of the community and force them to forfeit their capital (Lopez 1976, p. 62). The positive and lasting shock that significantly improved the communication networks in the Mediterranean was due to the actions of small, poor communities such as the one that took refuge on the Venetian islands. With the rise of Italian seaport cities, the Mediterranean again became a commercial highway. Cipolla (1974) cites the rapid spread of the Black Death as evidence of the intensification of the communication network in Europe and in the Mediterranean, an hypothesis subsequently supported by several studies in economic history, demography, and epidemiology (see Boerner and Severgnini, 2012).

The communication shock was not the result of a policy of a centralized government as in Sung China (Kelly, 1997). Quite the contrary, it occurred in the most politically fragmented part of Europe. Lopez states the following:

Venice was then [829] practically independent, but honored her allegiance to Byzantium by supplying naval assistance, and used her eastern connections to unlock the gates of the Western Empire. She also maintained with Muslim Africa and the Levant as good relations as the sudden transitions from cold to hot war permitted. Thus she gradually built up a thriving triangular trade. (1976, p. 63)

The new fervor for commerce was also not associated with any major developments in

naval engineering. The most common Venetian vessel was an improved version of the Roman and later Byzantine galley. Venetian trade was conducted primarily through short-haul navigation with relatively small galleys.

5.2 Privileges: The Italian Commercial Nobility and the French Aristocracy

In the baseline experiment, the communication shock triggered a major transformation of the model economy. Type-1 individuals (the lower nobility) abandoned rent-seeking activity and turned to trade. As a result, the income of the type-1 and type-2 individuals (the commoners) converged toward a higher level (see Panel A, Figure (2)).

It has been said that the presence of the king with his power to distribute privileges absorbed the attention of the most resourceful in a society (Baumol, 2002). This appears to have been the case in France (Acemoglu et al., 2005). Conversely, as the Venetian nobility was too distant from Constantinople and in conflict with the Frankish and German kings, it did not develop any expectations of earning substantial rents from traditional lordships. It is revealing, for instance, that the Venetian nobility was not determined by land but by wealth accumulated through commercial activities (Madden, 2012). On this subject, Lopez reports the following:

As the progress of Venice and Amalfi was shifting the center of economic and naval power from the Byzantine and Muslim to the Catholic shore of the Mediterranean, two seaports from the barbarian part of Italy joined the race. Pisa and Genoa, too, were driven into a predominately commercial way of life by an original handicap. In the tenth century their territory had been frequently laid waste by Muslim raiders looking for slaves and other booty; not even the towns had been spared. Led by their urban nobility, the Pisans and the Genoese fought back. (1976, p. 66)

Puga and Treffer (2014) also document high social mobility and low income inequality in Venice through the end of the 13th century, that is, until the establishment of entry restrictions into the lucrative long-distance trade (the *Serrata*). Similarly, in Genoa, the dividing line between nobles and merchants was rather blurred. Lopez offers the following account:

The habit of sharing risk on the decks of the ships and rubbing shoulders in the narrow alleys and shops of the business center created a strong equalitarian tradition. Ever since the origins of the Genoese Commune, the law admitted no privileges of birth and ignored the very word "nobility." Naturally, this does not mean that nobility was unknown, for aristocrats turned businessmen played leading roles in town and at sea; but the first recognition it got was through fourteenth-century decrees that excluded noblemen from public offices. The informal term *bonitas*, good or

solid citizens, described the Genoese upper class more accurately. (1964, p. 447- 48)

The second experiment clarified that the switch to a speculative strategy does not occur (see Figure (3)) when the upper classes' rents are above a certain threshold. Medieval France was a region dominated by a rich landed aristocracy. "Aristocracy varied substantially in their wealth across western societies. In Merovingian Francia, there were some really rich landowners, with dozens of landed estates each, and a highly militarized factional politics. Bavaria was like Francia, although probably on a smaller scale; only a handful of families (apart from the ruling dukes) seem to have been important owners. In Lombard Italy, however, the wealth of the aristocratic strata was much more modest" (Wickham,2009, pp. 204-205).

More specifically on the situation of the aristocracy in Northern Italy, Wickham states, "Most Lombard aristocrats were fairly restricted in their wealth. Almost none of our documents show any of them with more than between five and ten estates, which is close to a minimum for aristocrats in Francia. The king and the ruling dukes of the south [Italy] had immense lands, of course, and a small number of powerful ducal families, particularly the north-east were rich, but the bulk of the elite owned only a handful of properties, usually only in the city territory they lived in, plus perhaps its immediate neighbors, with, quite often, a house in Pavia" (2009, pp.145-146).

The accumulation of conspicuous wealth by the French landed aristocracy continued during the Carolingian and post-Carolingian periods, when more land came under aristocratic control than before and less land was under the control of non-aristocrats. This change was particularly evident in Saxony, where Charlemagne's conquest resulted in a rapid takeover of land that had previously been under peasant ownership (Wickham, 2009)

5.3 The Middle Ages Growth Spurt

In the baseline experiment, there is a more pronounced acceleration of trade (70%) than in the alternative high-rent scenario (40%) because the communication shock triggered the emergence of a new trade link. As a result, in the baseline experiment, income also increased substantially more (41%) than in the high-rent scenario (18%) (see Tables 3 and 4).

Some data, admittedly of a rather rough variety, are available to verify the responses of trade and income to a communication shock in places that differed in the level of privilege enjoyed by the upper classes. Table 5 reports the impressive population growth in Genoa and Venice, the two Italian cities that were at the core of the commercial revolution. Bairoch (1988) estimates that the population in Venice went from approximately 45 thousand in the year 1000 to 75 thousand in the year 1200 and reached a medieval peak of 110 thousand in the year 1300. The population explosion in Genoa is even more staggering. The corresponding figures are 15, 30, and 100 thousand, respectively. Bairoch's data concord with

Lopez's remark that "by the end of the thirteenth century, Genoa proper may have reached the 100,000 mark, with probably another half million people in the territory of the state, which included several smaller urban centers. These figures are no more than very rough approximations, but they would indicate that the capital city was the fifth largest in Europe (larger than Paris and London, but smaller than Constantinople, Venice, Milan, and Florence) and the entire state was more thickly settled than Pennsylvania today" (1964, p. 448).

In contrast, during the 13th century, the population of Marseille increased only moderately and even declined in the following century: In the year 1200, Marseille was approximately the same size of Genoa, but by the year 1400, it had a population of only one-fifth that of Genoa or Venice.

The dynamism of northern Italian cities engaging in maritime trade is associated with an increase in income. Angus Maddison's (2009) data indicate that between the year 1000 and 1500, per capita income more than doubled in Italy. Furthermore, Bold and van Zanden's (2014) data indicate that in the year 1500, income in Northern Italy was 40 percent higher than Italy's average reported by Maddison (2009).

The use of the GDP per capita statistics in contemporary prices for earlier centuries inevitably relies on strong assumptions. Nevertheless, they agree with Lopez's description of the 13th century economic situation in Northern and Central Italy: "[N]ever before had such a large proportion of the population been free from want, or such a variety and abundance of goods been constantly available – not in ancient Rome at its peak, not in Byzantine and Islamic countries at their best" (1976, p. 106).

5.4 Atlantic Trade

As observed in Section (4.2), despite France being in a favorable geographical position, it did not benefit from the trade revolutions as did the nearby Low Countries and Northern Italy.

That pre-revolutionary France did not catch the Atlantic trade wave appears evident from the modest change in population in its Atlantic seaport cities compared to that observed in the Dutch Republic. In the year 1700, the population of Bordeaux, the most important French 18th century port, was approximately equal to that of two centuries earlier and only one-quarter that of Amsterdam. Furthermore, the share of French among the merchant population of Bordeaux in the 18th century was rather small: Sephardi, Germans, Dutch and Irish made up one-third of Bordeaux merchants, and half of the French firms were operated by Huguenots linked to the Protestant merchants of Northern Europe. Only after 1793 was a new pattern of trade established in Bordeaux, most notably with the United

States (Marzagalli, 2005).

By contrast, between 1600 and 1700, Amsterdam experienced explosive growth, growing from 54 to 200 thousand inhabitants. Income also increased considerably in the Netherlands. In the year 1500, per capita income in France and the Netherlands was approximately the same. By the year 1600, however, income in the Netherlands was nearly double that observed a century earlier, whereas in France it had increase by less than 20 percent. In the following century, the divergence between the two regions became more pronounced, with a gap factor of 2.3 in the year 1700 (see Table 7).

5.5 More on the Resistance of the French Aristocracy to Commerce

In the experiment in Section (4.2) the lower nobility did not switch to speculative strategies because $r_2 - r_3 > \frac{1}{6}\alpha'$, where α' is the matching rate after the positive communication shock. Clearly, the larger the gap, $r_3 - r_2$, the higher the expected loss in terms of rent associated with a switch to a speculative strategy.

A number of historians argue that the the landed French aristocracy was resistant to embracing commerce due to concerns that such a move would cost them privileges – especially property tax exemptions. Such a belief was well grounded. In 1295, when Philip the Fair exempted the nobles from paying him a certain aid, he specifically excluded those in trade. In 1435, Charles VII refused to exempt from taxation nobles who sold their wines in taverns. Earlier in the 16th century, Francis I voided the tax exemptions of nobles who meddled in commercial activities.

The position of the French monarchy with respect to having a commercial nobility was ambiguous, as two conflicting interests were at stake. On the one hand, the monarch had a clear interest in developing trade, as this was an important source of revenue to finance warfare. For instance, Lopez (1964) reports that at the end of the 13th century, the taxes Genoa raised from her sea trade amounted to approximately ten times the receipts of the French royal treasury. On the other hand, the king could not afford to lose the support of the nobility that had long opposed the idea of mingling with merchants.

It is revealing that Louis XI, who was very familiar with the state of affairs in the Italian cities, took steps to establish commercial routes to compete with Venice and Genoa. In the second half of the 15th century, the monarch prepared legislation to eliminate the separation between the nobility and merchants, in imitation of what was already in place in Italian cities. However, the law was never implemented. Instead, a few decades later, in 1560, prior legislation and customs that sanctioned the nobility's participation in commerce was codified (the Code of Ordinances of Orleans, see Zeller, 1946). It was not until 1629, under

Table 5: Population in French and Italian Seaport Cities ('000)

Year	1000	1200	1300	1400
Venice	45	70	110	100
Genoa	15	30	100	100
Marseille	7	25	31	21
Bordeaux	n.a.	15	30	30

- Source: Bairoch (1988)

Table 6: Population in French and Dutch Seaport Cities ('000)

Year	1500	1600	1700
Amsterdam	15	54	200
Bordeaux	50	40	45

- Source: Bairoch (1988).

Richelieu, that nobles were allowed to engage in maritime commerce, either directly or by proxy, without fear of losing their status. Although some nobles seized opportunities in overseas trade by participating in trading companies (Chaussinand-Nogaret, 1975), by the middle of the 18th century, the debate among French intellectuals on whether to have a commercial nobility remained lively. An example is the controversy stirred by the pamphlet "Commercial Nobility" (*La Noblesse Commerçante*) that appeared in 1756. The author, who was an influential writer at that time, rebutted Montesquieu's position, elaborated in *The Spirit of the Laws* (1748, Book XX, Ch. 21), that opposed the idea of having a commercial nobility (see Adam, 2003, and Grassby, 1960, for details on the controversy).

6 Multiple Equilibria

Thus far, I have considered economies with unique equilibria. It is well known that the KW environment may generate multiple steady states. Any of the rankings in Panel B of Table 8, which correspond to Model B of KW, may exhibit multiple steady states. Do these configurations of parameters add insights on the arrival of the commercial revolution? The short answer is no. In Model A, the commercial revolution was triggered by the emergence of a new trade link that magnified the trade frequencies and caused an income boom. In Model

Table 7: Average per Capita GDP in 1990 International GK Dollars

Year	1000	1500	1600	1700
Italy	450	1100	1100	1100
Italy (Center-North)	n.a.	1533	1363	1476
Netherlands	425	751	1381	2105
France	473	727	841	910

- Source: The figures for Center-North Italy are from Bold and van Zanden (2014). The remaining data are drawn from Maddison (2003).

Table 8: Steady State Equilibria, Strategies, and Money

Returns	F	S	Assets (F)	Assets (S)
Panel A				
R1 $r_3 < r_2 < r_1$	(0,1,0)	(1,1,0)	$\frac{1}{3}[1, \frac{1}{2}, 1]$	$\frac{1}{3}[a, b, 1]$
R2 $r_2 < r_1 < r_3$	(1,0,0)	(1,0,1)	$\frac{1}{3}[\frac{1}{2}, 1, 1]$	$\frac{1}{3}[b, 1, a]$
R3 $r_1 < r_3 < r_2$	(0,0,1)	(0,1,1)	$\frac{1}{3}[1, 1, \frac{1}{2}]$	$\frac{1}{3}[1, a, b]$
Panel B				
R4 $r_2 < r_3 < r_1$	(1,1,0)	(1,0,1)	$\frac{1}{3}[a, b, 1]$	$\frac{1}{3}[b, 1, a]$
R5 $r_3 < r_1 < r_2$	(0,1,1)	(1,1,0)	$\frac{1}{3}[1, a, b]$	$\frac{1}{3}[a, b, 1]$
R6 $r_1 < r_2 < r_3$	(1,0,1)	(0,1,1)	$\frac{1}{3}[b, 1, a]$	$\frac{1}{3}[1, a, b]$

- Note: $a = \frac{1}{2}\sqrt{2}$ and $b = \sqrt{2} - 1$. The F and S columns contain the triplet $(\theta_1, \theta_2, \theta_3)$ that describes the fundamental and the speculative steady-state strategy, respectively. The two columns that follow are the assets' stationary distributions: $[p_{1,2}, p_{2,3}, p_{3,1}]$. In rows R1 through R3, the equilibria are unique; only one type of agent plays speculative strategies in the S equilibrium. In rows R4-R6, the two equilibria may coexist; two types of agents play speculative strategies in the S equilibrium.

B, however, it is not possible to generate the emergence of an expanded trade network when going from a fundamental to a speculative steady-state equilibrium. Consider, for instance, the case in which $r_3 < r_1 < r_2$ (see R5, Panel B, Table 8). The higher nobility, lower nobility, and commoners, now correspond to type-2, type-3, and type-1 agents, respectively. A (0,1,1) steady-state equilibrium always exists. A (1,1,0) steady state exists (namely, type-1 and type-3 agents adopt speculative behavior) if the following two conditions are satisfied (a Technical Appendix details the derivations):

$$\frac{\sqrt{2} - 1}{3} > \frac{r_2 - r_3}{\alpha},$$

and

$$\frac{1}{\sqrt{2}} \frac{\sqrt{2} - 1}{3} > \frac{r_2 - r_1}{\alpha}.$$

Imagine that, initially, α is low such that at least one of the two above conditions is violated. A positive shock to α can precipitate a (0,1,1) equilibrium, which would coexist with the (1,1,0) equilibrium. Because there are multiple equilibria, agents can either maintain their current strategies or type-1 and type-3 agents will switch to speculative strategies. (Expectations, not history, determine the outcome). Nevertheless, neither of the two outcomes is able to capture the commercial revolution. Even if type-1 and type-3 agents react to the communication shock by turning to speculative strategies (the numerical algorithm suggests that this can occur under any reasonable discount rate), the frequency of aggregate trade is hardly affected. Indeed, in the fundamental equilibrium, one of the two upper classes is already engaged in indirect trade. If agents coordinate on the (1,1,0) the two upper classes simply switch strategies, with type-1 agents flipping from $\tau_1 = 0$ to $\tau_1 = 1$ and type-3 agents from $\tau_3 = 1$ to $\tau_3 = 0$.

7 Concluding Remarks

I explored the idea that an improvement in communication and transportation can direct a greater fraction of the population toward productive activities (Murphy et al., 1991) in an environment in which the frequency of trade drives the dynamics of production and consumption (Kiyotaki and Wright, 1998). The main result is that a communication shock is more likely to benefit a society when the gap between privileged and unprivileged individuals is not too large. To verify the plausibility of the result, I considered two major communication shocks: the medieval revival of the Mediterranean routes and the Colombian voyages. The main implication of the exercise is consistent with the characterization of the societies at the time of the two shocks. The commercial revolutions that followed the two communication

shocks occurred in places where the inequality in the distribution of privileges was limited, most notably in Northern Italy during the Middle Ages and in The Netherlands in the 16th and 17th centuries. In contrast, where society was highly stratified, as for example in pre-revolutionary France, the reaction to either communication shock was weak.

This paper implies that differences in the gains from trade across countries may reflect differences in the distribution of privileges. It also suggests that when a government is considering trade liberalization, it should first smooth the distribution of privileges.

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Technical Appendix [Not intended for publication]

Distribution of Inventories (Proposition 2)

For a given profile of strategies, the system (8)-(10) converges globally to the unique steady state reported in the following table:

<i>Strategies</i>	Assets Distribution	<i>Strategies</i>	Assets Distribution
$(0,1,0)$	$[\mu_1, \frac{\mu_2\mu_1}{\mu_3+\mu_1}, \mu_3]$	$(1,0,1)$	$[p_{1,2}^\#, \mu_2, p_{3,1}^\#]$
$(1,0,0)$	$[\frac{\mu_1\mu_3}{\mu_2+\mu_3}, \mu_2, \mu_3]$	$(0,1,1)$	$[\mu_1, p_{2,3}^{\boxtimes}, p_{3,2}^{\boxtimes}]$
$(0,0,1)$	$[\mu_1, \mu_2, \frac{\mu_1\mu_3}{\mu_1+\mu_2}]$	$(0,0,0)$	$[\mu_1, \mu_2, \mu_3]$
$(1,1,0)$	$[p_{1,2}^*, p_{2,3}^*, \mu_3]$		

Case (0,1,0). Eq. (8) reduces to $\dot{p}_{1,2} = \alpha p_{1,3} \mu_3$, implying that the line $p_{1,2} = \mu_1$ is globally attractive (henceforth, g.a.). Similarly, because Eq. (10) $\dot{p}_{3,1} = \alpha p_{3,2} [p_{1,3} + \mu_2]$, $p_{3,1} = \mu_3$ is g.a.. Finally, along these lines, the system collapses to

$$\dot{p}_{2,3} = (\mu_2 - p_{2,3})\mu_1 - p_{2,3}\mu_3,$$

which clearly converges globally to $\frac{\mu_2\mu_1}{\mu_3+\mu_1}$. In brief, under the profile of strategies (0,1,0), the distribution of inventories converges globally to the stationary distribution $[\mu_1, \frac{\mu_2\mu_1}{\mu_3+\mu_1}, \mu_3]$.

Case (0,0,1) and Case (1,0,0). One can verify that the stationary distribution converges to $[\mu_1, \mu_2, \frac{\mu_1\mu_3}{\mu_1+\mu_2}]$ and $[\frac{\mu_1\mu_3}{\mu_2+\mu_3}, \mu_2, \mu_3]$, respectively, using the same observations as in the previous case.

Case (1,1,0). Eq. (10) becomes $\dot{p}_{3,1} = \mu_2(\mu_3 - p_{3,1})$. Consequently, $\mu_3 = p_{3,1}$ is an invariant set. The Jacobian, J , of the system of the two remaining equations (8) and (9) along the line $\mu_3 = p_{3,1}$ is

$$J = \alpha \begin{bmatrix} -(\mu_3 + p_{2,3}) & -p_{1,2} \\ (\mu_2 - p_{2,3}) & -(\mu_3 + p_{1,2}) \end{bmatrix}.$$

The determinant is positive, and the trace is negative; therefore, both eigenvalues are negative, and the system is globally stable. To find the stationary distribution, set (8) and (9) to zero. They yield $p_{1,2} = \mu_1\mu_3/(\mu_3 + p_{2,3})$ and $p_{1,2} = \frac{\mu_3}{\mu_2/p_{2,3}-1}$, respectively. The two lines necessarily cross once and only once for $p_{2,3}$ in the interval $[0, \mu_3]$. The fixed point is $[p_{1,2}^*, p_{2,3}^*, \mu_3]$, where $p_{2,3}^* = \frac{1}{2}[-(\mu_1 + \mu_3) + \sqrt{(\mu_1 + \mu_3)^2 + 4\mu_1\mu_2}]$ and $p_{1,2}^* = \frac{\mu_1\mu_3}{\mu_3+p_{2,3}^*}$.

Cases (1,0,1) and (0,1,1). A Jacobian with similar properties can be obtained when the profiles of strategies are (1,0,1) or (0,1,1). The fixed point with (1,0,1) is $[p_{1,2}^\#, \mu_2, p_{3,1}^\#]$, where $p_{1,2}^\# = \frac{1}{2}[-(\mu_3 + \mu_2) + \sqrt{(\mu_3 + \mu_2)^2 + 4\mu_3\mu_1}]$ and $p_{3,1}^\# = \frac{\mu_1\mu_3}{p_{1,2}^\# + \mu_2}$. Similarly, under (0,1,1), the fixed point is $[\mu_1, p_{2,3}^\#, p_{3,2}^\#]$, where $p_{3,1}^\# = \frac{1}{2}[-(\mu_2 + \mu_1) + \sqrt{(\mu_2 + \mu_1)^2 + 4\mu_2\mu_3}]$ and $p_{2,3}^\# = \frac{\mu_1\mu_2}{p_{3,1}^\# + \mu_1}$.

Case (0,0,0). The system converges globally $\mathbf{p} = [\mu_1, \mu_2, \mu]$. In this stationary state, agents keep their production goods.

When the profile of strategies is (1,1,1), it is more difficult to characterize the properties of the Jacobian. This turns out not to be a Nash equilibrium, at least when the population is equally split across types.

Eq. (3) describes $V_{i,i+1}(t)$. The following does the same for $V_{i,i+2}(t)$

$$V_{i,i+2}(t) = \max_{\{\tau_i(s)\}_{s \geq t}} \int_t^\infty \alpha e^{-\alpha(s-t)} \left\{ e^{-\rho(s-t)} ([p_{i+1,i}(1 - \theta_{i+1}) + p_{i+2,i}](V_{i,i+1} + u_i) + \right. \quad (11)$$

$$\begin{aligned} & [p_{i,i+1}\theta_i + p_{i+2,i+1}](1 - \tau_i)V_{i,i+1} \\ & [1 - p_{i+1,i}(1 - \theta_{i+1}) - p_{i+2,i} - (p_{i,i+1}\theta_i + p_{i+2,i+1})(1 - \tau_i)]V_{i,i+2} + \\ & \left. \frac{1 - e^{-(s-t)\rho}}{\rho} r_{i+2} \right\} ds, \end{aligned}$$

Taking derivatives of eqs. (3) and (11) with respect to time yields

$$\begin{aligned} \dot{V}_{i,i+1} = & -\alpha([p_{i,i+2}\tau_i(1 - \theta_i) + p_{i+1,i+2}\tau_i]V_{i,i+2} + \quad (12) \\ & p + [1 - p_{i,i+2}\tau_i(1 - \theta_i) - p_{i+1,i+2}\tau_i]V_{i,i+1} + \\ & [p_{i+1,i} + p_{i+2,i}\theta_{i+2}]u_i) - r_{i+1} + (\alpha + \rho)V_{i,i+1}(s), \end{aligned}$$

$$\begin{aligned} \dot{V}_{i,i+2} = & -\alpha([p_{i+1,i}(1 - \theta_{i+1}) + p_{i+2,i}](V_{i,i+1} + u_i) + [p_{i,i+1}\theta_i + p_{i+2,i+1}](1 - \tau_i)V_{i,i+1} \\ & + [p_{i,i+1}\theta_i + p_{i+2,i+1}](1 - \tau_i)V_{i,i+1} + \\ & + [1 - p_{i+1,i}(1 - \theta_{i+1}) - p_{i+2,i} - (p_{i,i+1}\theta_i + p_{i+2,i+1})(1 - \tau_i)]V_{i,i+2}) + \\ & - r_{i+2} + (\alpha + \rho)V_{i,i+2}, \end{aligned}$$

respectively.

The last two expressions can also be written as

$$\begin{aligned} \dot{V}_{i,i+1} = & -\alpha([p_{i,i+2}\tau_i(1 - \theta_i) + p_{i+1,i+2}\tau_i](-\Delta_i) + V_{i,i+1} + \\ & [p_{i+1,i} + p_{i+2,i}\theta_{i+2}]u_i) - r_{i+1} + (\alpha + \rho)V_{i,i+1}(s), \end{aligned}$$

$$\begin{aligned}\dot{V}_{i,i+2} = & -\alpha([p_{i+1,i}(1-\theta_{i+1}) + p_{i+2,i} + [p_{i,i+1}\theta_i + p_{i+2,i+1}](1-\tau_i)]\Delta_i + [p_{i+1,i}(1-\theta_{i+1}) + p_{i+2,i}]u_i \\ & + V_{i,i+2}) - r_{i+2} + (\alpha + \rho)V_{i,i+2},\end{aligned}$$

Subtracting side-by-side, we obtain

$$\begin{aligned}\dot{\Delta}_i = & -\alpha([p_{i,i+2}\tau_i(1-\theta_i) + p_{i+1,i+2}\tau_i + p_{i+1,i}(1-\theta_{i+1}) + p_{i+2,i} + (p_{i,i+1}\theta_i + p_{i+2,i+1})(1-\tau_i)](-\Delta_i) + \\ & + \Delta_i + [p_{i+1,i}\theta_{i+1} - p_{i+2,i}(1-\theta_{i+2})]u_i) - r_{i+1} + r_{i+2} + (\alpha + \rho)\Delta_i\end{aligned}$$

Let

$$\chi_i \equiv p_{i,i+2}\tau_i(1-\theta_i) + p_{i+1,i+2}\tau_i + p_{i+1,i}(1-\theta_{i+1}) + p_{i+2,i} + (p_{i,i+1}\theta_i + p_{i+2,i+1})(1-\tau_i)$$

where $\phi_i \equiv [p_{i,i+2}\tau_i(1-\theta_i) + p_{i+1,i+2}\tau_i]$. Then the last two differential equations reduce to

$$\dot{\Delta}_i = -\alpha((1-\chi_i)\Delta_i + [p_{i+1,i}\theta_{i+1} - p_{i+2,i}(1-\theta_{i+2})]u_i) - r_{i+1} + r_{i+2} + (\alpha + \rho)\Delta_i, \quad (13)$$

Because $(\alpha + \rho) > 0$, for any given pattern of the asset distribution, the system is unstable. The Technical Appendix explains how this equation is used to verify whether a stationary distribution of assets is a Nash equilibrium.

Stationary Nash Equilibria (Proposition 3)

The stationary distribution of inventories is derived from (2) under the assumption that $\mu_1 = \mu_2 = \mu_3 = \frac{1}{3}$. The key condition for determining whether a stationary distribution is a NE is the sign of Δ_i . From (13), it follows that $\Delta_i > 0$ if

$$p_{i+1,i}\theta_{i+1} - p_{i+2,i}(1-\theta_{i+2}) > \frac{r_{i+2} - r_{i+1}}{\alpha u_i} \quad (14)$$

Consistency requires that $\theta_i = 0$ (1) with $\Delta_i > 0$ (< 0). This section reviews the consistency conditions (14) for the rankings listed in Table 8.

Model A ($r_3 < r_2 < r_1$). There are two unique NE: (0,1,0) and (1,1,0). The (0,1,0) equilibrium requires that

$$p_{2,1} - p_{3,1} > \frac{r_3 - r_2}{\alpha u_1} \quad (15)$$

$$-p_{1,2} < \frac{r_2 - r_1}{\alpha u_2}, \quad (16)$$

and

$$0 > \frac{r_2 - r_1}{\alpha u_3}, \quad (17)$$

with $p_{2,1} = \frac{1}{3} - p_{2,3}$, $p_{1,2} = \frac{1}{3}$, $p_{2,3} = \frac{1}{6}$, and $p_{3,1} = \frac{1}{3}$. Conditions (16) and (17) are clearly verified. From (15), it follows that the (0,1,0) equilibrium exists if $\frac{1}{6} > \frac{r_2 - r_3}{\alpha u_1}$. For the (1,1,0) equilibrium, the stationary distribution is $\mathbf{p} = \frac{1}{3}[a, b, 1]$. The above three conditions are replaced by

$$p_{2,1} - p_{3,1} < \frac{r_3 - r_2}{\alpha u_1},$$

$$0 < \frac{r_1 - r_3}{\alpha u_2},$$

and

$$p_{1,3} > \frac{r_2 - r_1}{\alpha u_3},$$

respectively, with $p_{2,1} = \frac{1}{3} - p_{2,3}$, $p_{1,2} = \frac{a}{3}$, $p_{2,3} = \frac{b}{6}$, and $p_{3,1} = \frac{1}{3}$. Again, the last two conditions are obviously satisfied. The first condition says that in a NE, type-1 agents play speculative if

$$-\frac{b}{6} < \frac{r_3 - r_2}{\alpha u_1}.$$

There are no other NE.

A similar proof applies for the rankings R2 and R3 in Panel A of Table 8. I turn now to rankings of Panel B of Table 8 that can give rise to multiple steady states.

Model B. One fundamental NE always exists. This could coexist with another equilibrium in which two types of agents play speculative strategies (multiple equilibria).

When $r_3 < r_1 < r_2$. The (0,1,1) is the fundamental NE. It exists for any set of parameters (for which the value functions are non-negative). Under the following two conditions, the (1,1,0) NE also exists:

$$p_{2,1} - p_{3,1} < \frac{r_3 - r_2}{\alpha u_1}$$

$$p_{1,3} > \frac{r_2 - r_1}{\alpha u_3}$$

with $p_{2,1} = \frac{1}{3}(1 - b)$, $p_{3,1} = \frac{1}{3}$ and $p_{1,3} = \frac{1}{3}(1 - a)$. The top one requires $-\frac{b}{3} > \frac{r_3 - r_2}{\alpha u_1}$ and the bottom one that $\frac{1}{3}(1 - a) > \frac{r_2 - r_1}{\alpha u_3}$.

A similar reasoning applies to scenarios R5 and R6.

Algorithm

The algorithm sets up an iteration on the profile of strategies, $\boldsymbol{\theta}(t)$, and on the distribution of assets, $\mathbf{p}(t)$. The value function, $V_{i,j}(t)$, of the representative agents i holding good j serves as device to update the guess on the profile of strategies and to determine when the algorithm has converged. As only pure strategies are considered,⁴ a representative agent i has a binary choice at each point in time. The algorithm seeks the convergence of representative agent i 's best response, $\tilde{\tau}_i(t; \boldsymbol{\theta}(t), \mathbf{p}_0)$, to the profile followed by the other individuals of her type, $\theta_i(t)$. When the representative individual i does not have an interest in deviating from a strategy that coincides with that followed by the rest of type- i agents, a Dynamic Nash Equilibrium is found. The algorithm works as follows.

Step 1. The distribution of inventories, $F(\mathbf{p}(t))$, is integrated forward in time beginning from some \mathbf{p}_0 , under a guess $\boldsymbol{\theta}^{(0)}(t)$. The integration is stopped at some time, T , that is sufficiently large that $|F(\mathbf{p}(T))| < 10^{-6}$. An obvious initial guess is $\boldsymbol{\theta}^{(0)}(t) = \boldsymbol{\theta}^{ss}$, where $\boldsymbol{\theta}^{ss}$ is the steady-state Nash profile of strategies. (For some $s > \bar{s}$ with \bar{s} sufficiently large, one can expect that $\tilde{\tau}_i(s) = \theta_i^{(0)}(s)$ for $s > \bar{s}$.) Let $\mathbf{p}^{(0)}(t)$ be the inventory solution under such a guess.

Step 2. The algorithm computes the best response of a representative agent i on the trajectory $\mathbf{p}^{(0)}(t)$. His Δ_i is computed integrating (18) backward in time, beginning from the initial condition $(\Delta_i(\boldsymbol{\theta}^{ss}, \mathbf{p}^{(0)}(T)), \mathbf{p}^{(0)}(T))$.⁵ At the end of this step, one obtains a trajectory, $\Delta^{(0)}(t)$, and more important, the corresponding best response $\tilde{\tau}_i^{(0)}(t)$ of the representative agent i .

Step 3. The consistency between $\theta_i^{(0)}(t)$ and $\tilde{\tau}_i^{(0)}(t)$ is verified. If these are different, $\tilde{\tau}_i^{(0)}(t)$ becomes the new guess on the next round, namely $\theta_i^{(1)}(t) = \tilde{\tau}_i^{(0)}(t)$, and the procedure restarts from step one. The method allows the profile of strategies to change at any point in time.

The iteration is repeated until convergence between $\theta_i^{(n+1)}(t)$ and $\tilde{\tau}_i^{(n)}(t)$ is achieved or until a maximum number of iterations is reached. If the iteration converges to a fixed point, say $\mathbf{p}^*(t)$ and $\boldsymbol{\theta}^*(t)$, then $\mathbf{p}^*(t)$ and $\boldsymbol{\theta}^*(t)$ are the distribution of assets and the trading strategies, respectively, of a Markov-perfect Nash equilibrium. The procedure verifies that at such a fixed point, the value function of any agent is at its maximum value, given the actions of the rest of the agents.

Forward and Backward Integration

This section examines the dynamic properties of the value functions of the representative

⁴Kehoe et al. (1993) build cyclical equilibria in a similar environment under sets of parameters that do not admit pure strategy equilibria.

⁵ $\Delta_i(\boldsymbol{\theta}^{ss}, \mathbf{p}^{ss})$ could also be used as the initial point. In principle, on $\Delta_i(\boldsymbol{\theta}^{ss}, \mathbf{p}^{ss})$, the system stays still, but if computed numerically, there is always a small machine error that allows the integration to begin. In the experiments, the difference – in norm – between the two points is smaller than 10^{-5} when $|F(\mathbf{p}(t))| < 10^{-6}$.

agent i . These properties are important because they will be used by the algorithm to verify whether a particular distribution of assets and of strategies is an open-loop Nash equilibrium. According to (4), what matters for representative i 's decision is only the *sign* of Δ_i . After some algebra, one obtains,

$$\dot{\Delta}_i = (\alpha\chi_i + \rho)\Delta_i + \omega_i, \quad (18)$$

where $\chi_i \equiv p_{i,i+2}\tau_i(1-\theta_i) + p_{i+1,i+2}\tau_i + p_{i+1,i}(1-\theta_{i+1}) + p_{i+2,i} + (p_{i,i+1}\theta_i + p_{i+2,i+1})(1-\tau_i) > 0$ and $\omega_i \equiv -\alpha[p_{i+1,i}\theta_{i+1} - p_{i+2,i}(1-\theta_{i+2})]u_i + (r_{i+2} - r_{i+1})$.

For a given pattern of χ_i , the solution of Δ_i can be obtained numerically by integrating (18) backward in time,⁶ starting from a neighborhood of the steady state Δ_i^* , where this satisfies $(\alpha\chi_i + \rho)\Delta_i^* + \omega_i = 0$.

It is important to recognize that the distribution of inventories, $\mathbf{p}(t)$, and the value functions in differences, $\Delta_i(t)$, could be studied jointly as a whole system. In particular, it is possible to generate equilibrium trajectories by simply applying the principle of backward induction as long as the population's profile of strategies is kept consistent with the sign of $\Delta_i(t)$ at each point in time. The problem with this procedure is that it is difficult to construct a trajectory that goes through a designated point in the space of the asset distribution. In models with a unidimensional state variable, backward induction works well— at least when the dynamics are not cyclical or chaotic — because it can be stopped when the state variable reaches a desired level. However, as the dimension of the manifold expands, guiding the system toward a particular point on the state space (that is, on the initial condition) becomes a challenge. In fact, if the system has Liapunov exponents of a different order of magnitude, some regions of the manifold cannot even be reached. Conversely, the method proposed here offers total control over the initial condition, a feature that turns out to be essential for most interesting macroeconomic experiments.

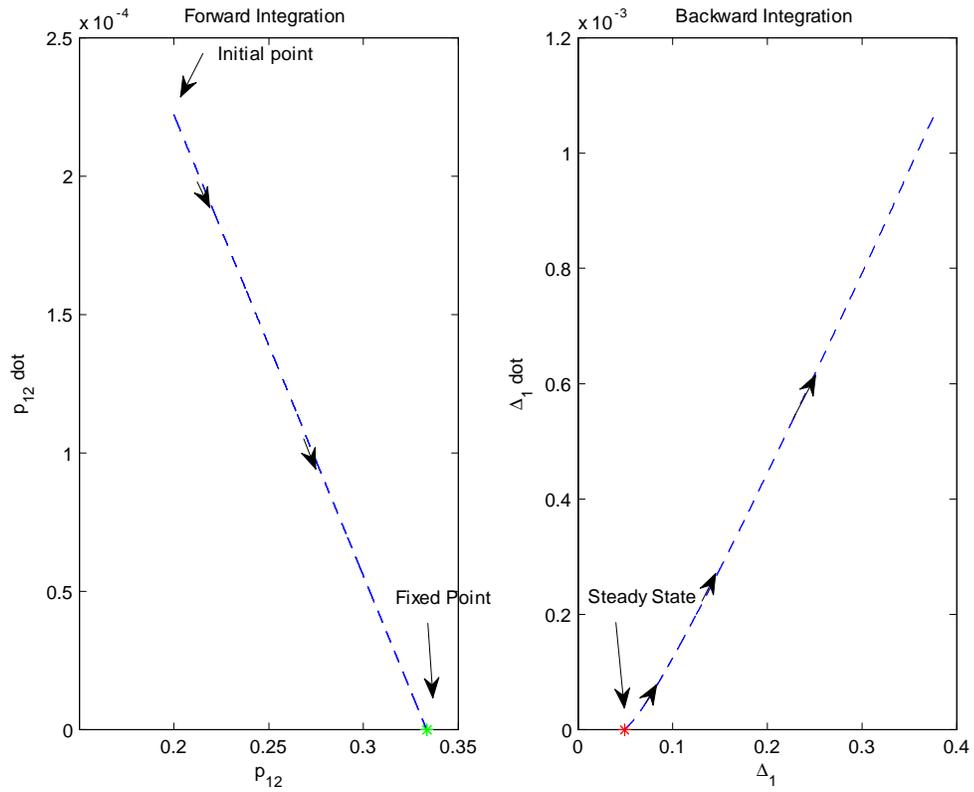
The right plot of Fig. 4 illustrates a solution of (18) for $i = 1$ for a particular pattern of χ_1 . This pattern is obtained by integrating forward in time (8)-(10), beginning from some arbitrary initial $\mathbf{p}(0)$, under an exogenous profile of strategies.

Matlab Codes

All the programming is done in Matlab. The main running file, called 'dynamics_KW', sets the parameters, specifies the initial distribution of inventories and the initial guess of the strategies, launches the iteration solution procedure, and generates the plots. The iteration procedure is based on the interaction between two files, triggered within the file 'dynamics_KW': 'backward_ode_kw89' and 'forward_ode_kw89'. The 'backward' file gives instructions to integrate the distribution of inventories forward. The resulting distribution

⁶The mechanism is illustrated in a figure contained in the Technical Appendix.

Figure 4: Combining Forward and Backward Integration



Note - For parameters see Table 2, with $r_2 = 0.1$. The initial value of $p_{1,2}$ is 60 percent of its steady-state value. The slope of the right plot is $\alpha\chi_1(t) + \delta$ with $\tilde{\tau}_1(t) = 0$ and $\boldsymbol{\theta}(t) = [0, 1, 0]$. The path of $\mathbf{p}(t)$ in $\chi_1(t)$ is obtained by integrating the system (8)-(10) forward in time (left plot).

of inventories (reversed with respect to time) is saved and used as an input for in the 'backward' file. This integrates the value functions using, as the initial condition, their steady state values. The 'backward' file delivers the three value functions (in differences) of the representative agents $i = 1, 2, 3$ and a profile of strategies. The 'dynamics_KW' file saves this profile and uses it as a new initial guess for the overall population. After each iteration, the 'dynamics_KW' file computes the differences between vector $\theta_i(t)$ and $\tau_i(t)$. When no discrepancy is noticed, the iteration is stopped, the resulting trajectory of strategies and of the assets distribution is recorded as an equilibrium, and all remaining variables (acceptability, consumption, etc.) are computed along such a trajectory. In both the backward and forward files, the ordinary differential equations are computed at a fixed 0.0001 time-step of a year. The Runge-Kutta approximation method with adjustable steps, readily available in Matlab, is not used because it would require an additional layer of coding to synchronize the timing in the 'backward_ode_kw89' and 'forward_ode_kw89' files.