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Probably Too Little, Certainly Too Late. An Assessment of the Juncker Investment Plan*

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Abstract

This paper aims at quantifying the impact of a stimulus plan based on a public investment push, within a dynamic stochastic general equilibrium model of the Eurozone economy. We estimate an extension of Leeper et al.'s (2010) model with public capital and time-to-build, to quantify the impact of the European Commission's Investment Plan for Europe (the "Juncker plan"). The public investment push is assessed in normal times and starting from the zero lower bound, making different hypotheses on private investment leverage and on capital productivity. Then, in order to assess the effectiveness of the Juncker plan, we compare it with the stimulus plan implemented by the Obama administration in 2009. The main conclusion of the paper is that, had it been implemented at the beginning of the crisis, the Juncker plan would have had a significant positive impact. But as it is being launched very late in the crisis, to be effective the plan should be significantly larger in size.

Keywords: public investment, leverage, Juncker plan, zero lower bound
JEL Codes: E22, E32, E65

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1 Introduction

Mario Draghi's speech at the central bankers' gathering in Jackson Hole, in August 2014 (Draghi, 2014), marked a change in the Eurozone crisis narrative. For the first time, in fact, a senior EMU figure explicitly mentioned insufficient investment as one of the causes of both short-term sluggish growth and longer term potential growth stagnation. Investment in Europe is today still largely below the levels reached before the crisis. In 2014, it was 12.6% and 16% lower than in 2007 for the EU and the EMU respectively. Uncertainty about the recovery's strength, the imbalances that persist within the euro area, and even doubts about the ability of the latter to overcome the Greek crisis, prevent a clear recovery of private investment, despite favorable external conditions (such as expansionary monetary policy and energy prices).

The purpose of the Juncker plan, announced in November 2014, is to overcome the reluctance of private investors by relieving them, through co-financing, of part of the financial risk associated with infrastructure projects. The plan, adopted by the European Parliament in June 2015, establishes a European Fund for Strategic Investments (EFSI), endowed by the Commission and by the European Investment Bank (EIB). The fund is now co-financing private sector investment projects for an amount that the Commission hopes to reach €315bn in 3 years. A detailed description of the mechanism is left for section 4.1.

The Juncker plan is long overdue. Infrastructure expenditure has been decreasing in European countries over the last 30 years, with tangible consequence on their efficiency (IMF, 2014, ch. 3; DIW, 2013). This is the direct result of the reduction in public investment expenditure, progressively eroded since the 1980s (see Figure 1)[1]. The low level of public capital, and in many cases its poor condition, imply that there is room for productive public investment.

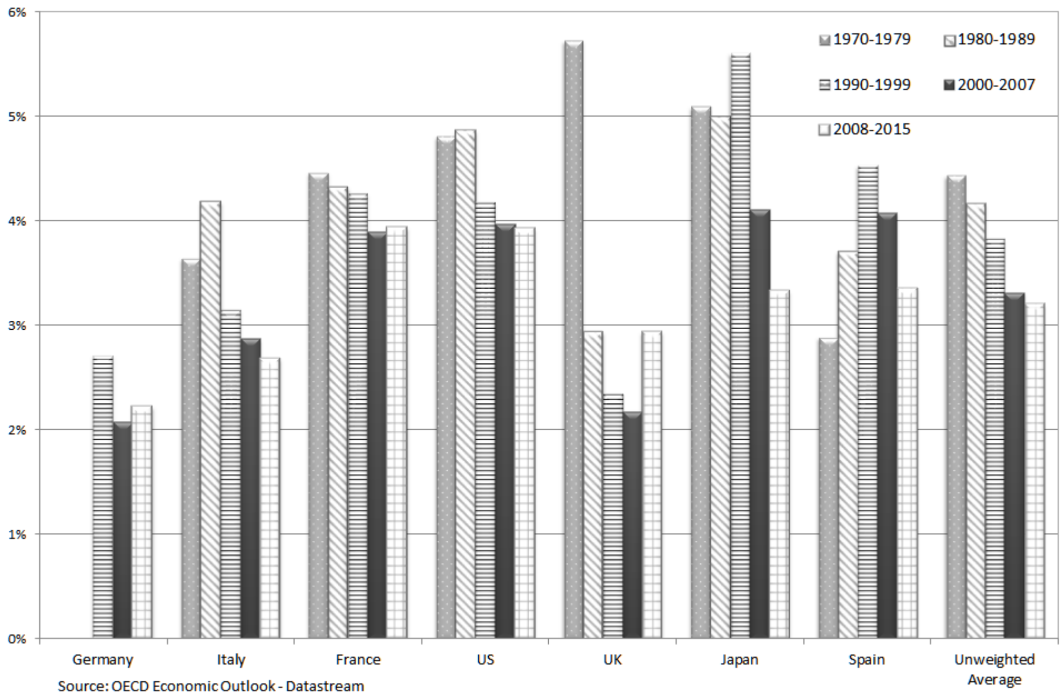

Figure 1: Public Investment as % of GDP in Selected OECD Countries

The current crisis has furthermore reunited a number of additional conditions, that make

[1]The crisis further reduced the ratio between public investment and GDP. This is not surprising, as fiscal consolidation affected public investment more than current expenditure.

public investment desirable. First, interest rates, nominal as well as real, are historically low, which facilitates financing and increases the range for profitable investment projects (*i.e.* having an internal rate of return larger than the interest rate). Second, private investment paid a high toll to the crisis and, because of the complementarity between public and private capital, a revival of public investment is likely to generate a rebound in private investment with a positive effect on economic activity. Finally, in a situation of insufficient aggregate demand, and with a productive apparatus burdened by competitiveness problems, public investment has the advantage of simultaneously stimulating demand in the short-term and potential growth in the long run. These positive effects are largely confirmed by empirical estimates (see *e.g.* Valla et al., 2014 for the case of Europe, and Blinder and Zandi, 2010 for the US) and model simulations (Leeper et al., 2010; Bouakez et al., 2014). There are reasons to believe, in short, that the multiplier of public investment in this moment could be far larger than one (especially if the projects are selected carefully).

Thus, in spite of the shaky situation of public finances in a number of European countries, this combination of structural and contingent factors led some (IMF, 2014) to talk about a "free lunch," in the sense that public investment projects would broadly finance themselves with hardly any effect, if any, on public debt ratios.

The structure of the paper is as follows. In section 2, we begin by shortly reviewing the literature on public investment and growth. Then, section 3 introduces an extension of Leeper et al.'s (2010) model with public capital and time-to-build. In section 4, we use an estimated version of the model to assess the impact of the Juncker plan in normal times and. In section 5, we analyze the impact of the time in a context of crisis caracterized by a zero lower bound (ZLB thereafter). Section 6 compares the Juncker plan with the stimulus plan implemented by the Obama administration in 2009. These two sections show that, had it been implemented at the beginning of the crisis, the Juncker plan would have had a significant positive impact. But as it was put in place only in 2015 the plan should have been significantly larger in size in order to lift the eurozone out of the ZLB. In section 7, we perform a sensitivity analysis of our result to other hypotheses on key parameters. Section 8 concludes.

2 Public Investment and Growth

The Juncker plan relies on both the direct public financing of investment projects and the expected crowding in of private investment, a topic that has been largely studied.

The literature on public investment and growth can be traced back at least to Aschauer's 1989b seminal work on the US, for which he found a strong positive impact of public capital on production (the estimated elasticity ranges from 0.24 for base infrastructures to 0.39 for broader measures). Since then, the literature can be divided in two large categories (Romp and De Haan, 2007). The first group treats public capital as an input of the aggregate production function, and estimates either a production function or a cost function (the latter has the advantage that it needs no hypothesis on factor substitutability). Most of the pioneering work following Aschauer's article (Tatom, 1991; Hulten and Schwab, 1991; Holtz-Eakin, 1994; Evans and Karras, 1994) used variants of this method, and found significantly smaller elasticities. A number of subsequent studies do not confirm Aschauer's results, finding virtually no impact of public capital on production. Eberts (1986) finds a significant but small value (0.03), that increases only slightly in later work (0.1 in Eberts, 1990). The comparison between public and private elasticities also became a topic for investigation. Ratner (1983) found that US output elasticity with respect to public capital was positive but smaller than private capital (close to 0.06, whereas the output elasticity with respect to private capital was 0.22). Other authors insist on the similarity between public and private capital elasticities. Munnell (1990), for example, finds, for the two a value ranging from 0.06 to 0.15 depending on the constraints she imposes to the model. These values are close to those found by Leeper et al. (2010) (0.05 and 0.1), or by IMF (2014) (0.17).

The second group of contributions surveyed by Romp and De Haan (2007) uses VAR (or VECM) models including public capital; the advantage of this approach is that, by explicitly taking into account the dynamic links among variables, it allows to disentangle possible reverse causation (*i.e.* from output to capital and investment) and to differentiate the short run and long run relationships between public investment and GDP, or between public investment and private investment. Furthermore, this method allows to give an assessment of causal relationships that go beyond the production side of the economy.

Thus, VAR estimations enlarge the field of analysis, and allow also to investigate the impact of public capital on private investment and productivity. This is particularly relevant for the present study, because the success of the Juncker Plan will crucially depend on its capacity to leverage, or crowd in, private investment. Early work on the correlation between public and private investment can be traced, once more, to Aschauer (1989a) who showed that while public investment crowds out private investment, this effect is counterbalanced by the positive impact of public capital on the return to private capital in the production function. Erenburg (1993) finds instead that public investment crowds-in private investment. Erenburg and Wohar (1995) also uses US data, and finds strong feedback effects between public and private capital. Pereira (2000) estimates an annual VAR in first differences for the US, finding long-run positive output level effects of a temporary increase in the growth rate of public investment. On the contrary, Voss (2002) studies the impact of public investment on private investment in the US and Canada, and weakly concludes for crowding out effect.

In a widely cited working paper, Perotti (2004) estimates a structural VAR in levels for 5 countries (Australia, Canada, West Germany, the UK and the US). His model contains 6 variables: government current and investment spending, GDP, net taxes, interest rate and inflation. He uses institutional features to restrict some instantaneous cross-elasticities at zero and estimates some others. Perotti's conclusion is not only that investment seems to have limited effects on GDP, but also that these effects are smaller than those of current spending. A possible explanation that Perotti offers for these puzzling findings is that the level of public capital is so large in the countries considered, that public investment is not productive enough. The crowding out of private investment hence more than compensates the direct effect on aggregate demand. A number of more recent contributions nevertheless challenged Perotti's results. For example, Creel et al. (2009) find, in the case of the UK, a robust and positive impact of public investment on growth; an impact that is even stronger in the period of application of the so-called "Golden Rule of Public Finances".

More recently Afonso and St Aubyn (2008) estimate VAR models for 17 developed countries and show that crowding in effects go in both directions, from public to private investment and the other way round. The former effect varies across countries whereas the latter is more homogeneous. Creel et al. (2015) are more agnostic, as their estimation for four OECD countries (France, the US, the UK and Germany) yields weak evidence of crowding in for France, and of crowding out for the US. Furthermore, by computing dynamic correlation they show that the relation is unstable over time for all the countries.

Using Japanese data, Fujii et al. (2013) also argue that on aggregate crowding out dominates, but they introduce a sectoral dimension and show that the effect of public investment on private capital accumulation varies according to the sector.

In conclusion, the empirical literature on public investment and growth yields results that vary greatly depending on the methodology, the period, and the countries considered. A useful effort of systematization has been recently been made by the surveys of Romp and De Haan (2007) and Pereira and Andraz (2013), as well as by the meta-analyses of Nijkamp and Poot (2004) and Bom and Ligthart (2014). All these surveys agree on the broad conclusion that public investment (or capital) has a positive but mild impact on GDP and growth. They further agree on the fact that in more recent studies the elasticity tends to be larger, a finding that we believe to be linked to the trend, mentioned in the introduction, towards a decreasing (and insufficient) public capital stock.

3 The Model

For assessing the Juncker plan we adopt a quantitative approach, making use of an estimated DSGE model of the Eurozone economy. We are of course aware of the limitations of our exercise, due to the simple model we use and to the uncertainty surrounding several parameters (especially the elasticity of output with respect to public capital); but we believe nevertheless that it provides a quantitative insight on the expected impact of this plan, especially in comparison with the US stimulus plan of 2009.

The model is built upon a standard New Keynesian core, with two categories of households (savers and borrowers), a productive sector that uses three factors of production (labor, private capital and public capital), and a government that conducts fiscal and monetary policies. The government has the possibility to make discretionary investment decisions, that will affect the stock of public capital. In order to model the co-financing of public investment projects by the private sector (through public-private partnership), we assume that, for every euro invested by the government, households will contribute a given number of euros to the project (independently from their investment decision on the stock of private capital, which is driven by an optimizing behavior).

3.1 Households

We consider a continuum of households of mass 1, indexed by $i \in [0,1]$, divided in two categories: a mass $1-n$ of patient households, who have a discount factor $\beta(i) = \beta^S$ and are savers in equilibrium; and a mass n of impatient households, who are borrowers in equilibrium, with a discount factor $\beta(i) = \beta^B < \beta^S$.

Each household i maximizes a standard inter-temporal utility function $u(C_t(i), L_t(i))$, with external habit formation and preference for leisure:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta(i)^t \mu_t \left[\frac{(C_t(i) - h\, C_{t-1})^{1-\sigma}}{1-\sigma} - \chi \frac{L_t(i)^{1+\epsilon}}{1+\epsilon} \right] \tag{1}$$

where $C_t(i)$ is the household's consumption level, C_{t-1} the per capita average level of consumption in the previous period (across both types of agents), h the degree of habit formation in consumption, σ the coefficient of relative risk aversion, $L_t(i)$ the household's labor supply, ϵ the Frisch elasticity of labor, and χ the relative weight of labor disutility.

We assume the existence of a preference shock μ_t, common to all households, and with mean unity. The law of motion of μ_t, and of the other shocks, is described in appendix B.

The nominal debt of each household is denoted by $B_t(i)$ (which becomes negative if the household is a net saver). Households are subject to a credit constraint on their debt, which is specified in real terms, as in Eggertsson and Krugman (2012):

$$\frac{B_t(i)}{P_t} \leq D > 0 \tag{2}$$

with P_t the final good price. Because of the difference of time preference across household types, the credit constraint will never be binding for patient households, while it will always be binding for impatient ones.

Patient Households (Savers)

Patient households have access to financial markets so that they can hold bonds issued by the government, or lend to firms investing in private capital. They also own the (intermediate good) firms, whose profits Π_t are part of their income.

Each household's real labor income is given by $(1-\tau_t^w)w_t(i)L_t(i)$ where $w_t(i)$ is the real wage the household will ask for its type of labor $L_t(i)$ on the labor market, and τ_t^w is the government's tax rate on labor income. As patient households choose the nominal wage they will demand on the intermediate labor market, they endure an adjustment cost when the nominal wage deviates from the steady state path (along which wage inflation $\pi_t^w(i)$ is assumed to be zero). This Rotemberg-type adjustment cost is assumed to be $\frac{\gamma^w}{2}\pi_t^w(i)^2 w_t$ where $\pi_t^w(i) = \frac{w_t(i)}{w_{t-1}(i)} - 1$ is the wage inflation.

Patient households can invest in (per capita) private physical capital $\bar{K}_t^S(i)$, which depreciates at rate δ^k. Their real investment in private physical capital is $I_t^S(i)$. Patient households also decide the utilization intensity of capital $u_t(i)$ in order to turn physical capital into capital services $K_t^S(i)$ that will then be used by firms:

$$K_t^S(i) = u_t(i)\bar{K}_t^S(i).$$

Capital services have a return rate r_t^k, which is taxed at rate τ_t^k. Patient households endure an adjustment cost if the capital utilization rate deviates from its steady state value (equal to 1). This cost is described by an increasing and convex function (per unit of capital):

$$\psi(u_t(i)) = r^{k*}\left(1-\tau^{k*}\right)\sigma_u\left[\exp\left(\frac{u_t(i)-u^*}{\sigma_u}\right)-1\right]$$

where $\sigma_u = \frac{\psi'(u_t(i))}{\psi''(u_t(i))}$ is the elasticity of capital utilization rate with regard to the rental rate of capital services, and r^{k*} (resp. τ^{k*}) is the steady state value of the capital rental rate (resp. of the capital income taxation rate).

Patient households can also invest in the public capital stock, for a per capita amount I_t^{GS}. In order to model the private co-financing of public investment, we assume that this investment is not the result of optimization, but follows an *ad hoc* behavioral rule, as explained in section 3.4. Moreover, households don't obtain any direct return on their investment in the public capital stock (though they benefit from it through the externality effect described in section 3.3).

To finance the gap between revenues and expenditures, households contract a debt $B_t^S(i)$ according to the credit constraint (2) and pay a risk free nominal interest rate i_t. Since patient households are net savers in equilibrium, one will typically have $B_t^S(i) < 0$.

The budget constraint of a patient household is written, in real terms:

$$(1-\tau_t^w)w_t(i)L_t(i) + (1-\tau_t^k)r_t^k K_{t-1}^S(i) + \frac{B_t^S(i)}{P_t} + \Pi_t =$$
$$(1+\tau_t^c)C_t^S(i) + I_t^S(i) + I_t^{GS} + (1+i_{t-1})\frac{B_{t-1}^S(i)}{P_t} + \psi(u_t(i))\,\bar{K}_{t-1}^S(i) + \frac{\gamma^w}{2}\pi_t^w(i)^2 w_t(i) \quad (3)$$

Note that real consumption expenditures $C_t^S(i)$ are taxed at rate τ_t^c, and that for the capital stock we use an end-of-period timing convention (*i.e.* K_t^S is the capital accumulated at the end of period t, available for production in $t+1$).

The law of motion of private physical capital is given by:

$$\bar{K}_t^S(i) = (1-\delta^k)\bar{K}_{t-1}^S(i) + \kappa_t \left[1 - S\left(\frac{I_t^S(i)}{I_{t-1}^S(i)}\right)\right] I_t^S(i) \tag{4}$$

as in Christiano et al. (2005), $S\left(\frac{I_t^S(i)}{I_{t-1}^S(i)}\right) = \frac{\gamma^I}{2}\left(\frac{I_t^S(i)}{I_{t-1}^S(i)} - 1\right)^2$ is the adjustment cost on investment effort. κ_t is an investment specific technological shock with unit mean, affecting the efficiency with which consumption goods are transformed into capital.

Impatient households (Borrowers)

Impatient households have access to the financial markets in order to contract a debt or save, but cannot invest in private capital. Their budget constraint therefore simplifies to:

$$(1-\tau_t^w)w_t(i)L_t(i) + \frac{B_t^B(i)}{P_t} = (1+\tau_t^c)C_t^B(i) + (1+i_{t-1})\frac{B_{t-1}^B(i)}{P_t} \tag{5}$$

Because these agents are less patients than the savers, they borrow up to their credit constraint, so that (2) is always binding ($B_t^B(i) = D$). As a consequence, the budget constraint simplifies to:

$$(1-\tau_t^w)w_t(i)L_t(i) = (1+\tau_t^c)C_t^B(i) + (\frac{1+i_{t-1}}{1+\pi_t} - 1)D \tag{6}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the final good price inflation.

3.2 Labor Market

The labor market is monopolistically competitive, and equilibrium employment is demand-determined. A perfectly competitive "labor-packer" buys the differentiated individual labor services $L_t(i)$ supplied by households (of both types) and transforms them into a homogeneous composite labor input L_t that in turn is sold at rate w_t to good-producing firms. This "labor-packer" is a CES aggregator which solves:

$$\begin{aligned} &\max_{L_t(i)} \quad w_t L_t - \int_0^1 w_t(i)L_t(i)\,\mathrm{d}i \\ &\text{s.t.} \quad L_t = \left(\int_0^1 L_t(i)^{\frac{\theta_t^w - 1}{\theta_t^w}}\,\mathrm{d}i\right)^{\frac{\theta_t^w}{\theta_t^w - 1}} \end{aligned} \tag{7}$$

The first order condition (see appendix A) gives the demand for each kind of differentiated labor service:

$$L_t(i) = \left(\frac{w_t(i)}{w_t}\right)^{-\theta_t^w} L_t \tag{8}$$

Patient households solve the inter-temporal problem of setting optimally a wage $w_t(i)$ for their labor of type i that would maximize their utility (1), given their budget constraint (3) (which includes the costs of misalignments of wage growth from steady state inflation), and the labor demand constraint (8).

For impatient households, we make the simplifying assumption that they set their wage at the level of the average wage of patient households. As a consequence, since all households face the same aggregate labor demand, the wage rate and the labor supply will be equalized among all households.[2]

[2]This is the same assumption as in Forni et al. (2009). They show that, in the context of their model (which is

3.3 Production

The perfectly competitive final good sector produces for consumption, private investment and public investment. Inputs come from a monopolistically competitive intermediate sector, made of a continuum of firms of mass 1. The intermediate sector drives the demand for labor, taking real wages as given.

Final goods

The representative final good producer purchases a variety of differentiated intermediate goods, indexed by $j \in [0,1]$. Its maximization program is:

$$\max_{y_t(j)} P_t Y_t - \int_0^1 p_t(j) y_t(j)\,\mathrm{d}j \tag{9}$$

$$\text{s.t. } Y_t = \left(\int_0^1 y_t(j)^{\frac{\theta_t^p - 1}{\theta_t^p}} \mathrm{d}j \right)^{\frac{\theta_t^p}{\theta_t^p - 1}} \tag{10}$$

where Y_t is the final good production, $y_t(j)$ the demand for intermediate good j, $p_t(j)$ the price of intermediate good j, and θ_t^p the elasticity of substitution between intermediate goods in the production function of the final good sector. Note that a positive shock to θ_t^p, that can be assimilated to a shock of demand from final good producers, or increased competition on the intermediate good market, would result in a decrease of the price mark-up. Maximization of (9) yields:

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\theta_t^p} Y_t \tag{11}$$

Intermediate goods

Each firm j in the intermediate good sector produces a differentiated good using the following technology:

$$y_t(j) = F(K_{t-1}(j), L_t(j), K_{t-1}^G) = z_t K_{t-1}(j)^{\alpha} L_t(j)^{1-\alpha} \left(K_{t-1}^G\right)^{\nu} \tag{12}$$

where $K_t(j)$ is the private capital stock of firm j, $L_t(j)$ its labor demand, K_t^G the aggregate public capital stock, α the share of capital among the private factors, ν the elasticity of production to public capital and z_t an exogenous common productivity shock.

We assume that intermediate firms face Rotemberg nominal price rigidities: they are subject to a quadratic nominal prices adjustment cost, measured in terms of the final good and given by $\frac{\gamma^p}{2}\pi_t(j)^2 Y_t$, where $\pi_t(j) = \frac{p_t(j)}{p_{t-1}(j)} - 1$ is the price inflation specific to firm j.

At each period t, firm j chooses inputs (private capital and labor) in order to minimize its cost function, for a given level of output $y_t(j)$. The optimization program is:

$$C(y_t(j)) = \min_{K_{t-1}(j), L_t(j)} w_t L_t(j) + r_t^k K_{t-1}(j) \tag{13}$$

$$\text{s.t. } y_t(j) \leq F(K_{t-1}(j), L_t(j), K_{t-1}^G) \tag{14}$$

Firm j chooses its price in order to maximize the sum of its discounted profits, taking into account the demand of the final good sector:

$$\max_{p_s(j)} \mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta^S\right)^{s-t} \frac{\lambda_s^S}{\lambda_t^S} \left[\frac{p_s(j)}{P_s} y_s(j) - C(y_s(j)) - \frac{\gamma^p}{2} \pi_s(j)^2 Y_s \right] \tag{15}$$

very similar to ours), this simplification delivers results which are not substantially different from the case where impatient households optimally choose their wage rate.

$$\text{s.t. } y_s(j) = \left(\frac{p_s(j)}{P_s}\right)^{-\theta_t^p} Y_s$$

where λ_t^S is the Lagrange multiplier on the budget constraint of patient households (the i index is dropped because in equilibrium all patient households behave identically).

3.4 Government

Government expenditure is split between consumption G_t and investment in public capital I_t^G. Expenditure is financed through taxes (on consumption, returns on capital, and labor income), and (nominal) debt B_t^G contracted with households. The government real budget constraint therefore can be written as:

$$T_t + \frac{B_t^G}{P_t} = G_t + I_t^G + \frac{1+i_{t-1}}{1+\pi_t}\frac{B_{t-1}^G}{P_{t-1}} \tag{16}$$

The total tax revenue T_t is defined by:

$$T_t = \tau_t^c\left((1-n)C_t^S + n\,C_t^B\right) + \tau_t^w w_t L_t + \tau_t^k r_t^k (1-n) K_{t-1}^S \tag{17}$$

where C_t^S, C_t^B, K_{t-1}^S denote per capita averages within the respective household types, and government consumption expenditure G_t is assumed to be exogenous.

Fiscal Policy

We mimic the European *Fiscal Compact* by assuming that the government is tied to a fiscal rule forcing the primary structural deficit to adjust in order to stabilize the long run debt-to-GDP ratio. Taxes are therefore adjusted following:

$$\Delta_t - r_t\frac{B_{t-1}^G}{P_{t-1}} = \Phi\left(\frac{B_{t-1}^G}{P_{t-1}} - b^{G*}\right) - \varepsilon_t^G \tag{18}$$

where Δ_t is the government's structural primary balance, r_t is the real rate of interest on public debt, b^{G^*} is the steady state target for the stock of real debt, $\Phi > 0$ governs the speed of fiscal consolidation and ε_t^G is a zero-mean i.i.d. fiscal shock. The fiscal shock ε_t^G allows for temporary deviations from the fiscal rule: when it is positive, the government adopts a more expansionary policy stance.

The structural primary balance is defined as:

$$\begin{aligned}\Delta_t = \tau_t^c\left((1-n)C^{S^*} + n\,C^{B^*}\right) + \tau_t^w w^* L^* \\ +\tau_t^k r^{k^*}(1-n)K^{S^*} - G_t - I^{G^*}\end{aligned}$$

where C^{S^*}, C^{B^*}, w^*, L^*, K^{S^*} and I^{G^*} are steady state values of their dynamic counterparts. Note that here we assume that the so-called "golden rule" applies: public investment is excluded from the computation of the primary structural balance.

Finally, we assume that each tax component is a constant fraction of total tax revenues:

$$\tau_t^c((1-n)C_t^S + n\,C_t^B) = \alpha_c T_t \tag{19}$$

$$\tau_t^w w_t L_t = \alpha_w T_t \tag{20}$$

$$\tau_t^k r_t^k (1-n) K_{t-1}^S = (1-\alpha_c-\alpha_w) T_t \tag{21}$$

where α_c and α_w are the proportionality coefficients.

Public investment

Following Leeper et al. (2010), we assume that public investment turns into public capital through a "time-to-build" process, reflecting the lags between project initiation and completion. Both the government and the patient households can make public investment decisions that contribute to the stock of public capital.

We denote by N the number of quarters it takes to complete an investment project once it has been launched. Consequently, if the government decides at date t to launch a new public investment project of size A_t, then this will induce an increase of the productive public capital stock at date $t + N$. Similarly, patient households contribute to the stock of public capital by making (per capita) investment decisions A_t^S at date t that increase the productive public capital stock N periods later.

The law of motion for public capital is therefore:

$$K_t^G = (1 - \delta^G)K_{t-1}^G + A_{t-(N-1)} + (1 - n)A_{t-(N-1)}^S \tag{22}$$

with δ^G the depreciation rate of public capital.

Investment spending on a given project is spread over N periods, with the $\{\phi_0, \phi_1, ..., \phi_{N-1}\}$ being the spending rates (such that $\sum_{s=0}^{N-1} \phi_s = 1$). If a government project of size A_t is launched at date t, then $\phi_0 A_t$ will be effectively spent by the government in t, $\phi_1 A_t$ will be spent in $t+1$, $\phi_2 A_t$ in $t+2$, and so on until the project is finalized by date $t+N$. Public investment spending at date t is therefore a fraction of all the projects launched in the last N periods, so that:

$$I_t^G = \sum_{s=0}^{N-1} \phi_s A_{t-s} \tag{23}$$

Similarly, the public investment spending by patient households is:

$$I_t^{GS} = \sum_{s=0}^{N-1} \phi_s A_{t-s}^S \tag{24}$$

The public investment decision by the government A_t is assumed to be exogenous. The public investment decision by patient households A_t^S is meant to capture the private leverage effect on public investment projects decided by the government. Making this decision endogenous, as the result of an optimization done by the patient households, would have added too much complexity to the model. What we are interested here is to assess the effect of the Juncker plan for different values of the private leverage effect. We therefore make the hypothesis that:

$$(1 - n)A_t^S = (\xi - 1)(A_t - A^*) \tag{25}$$

where $\xi \geq 1$ is the private leverage factor.

This means that, for every euro spent by the government in public investment (above the steady state value), the private sector will contribute $\xi - 1$ additional euros to the project (so that the total contribution is ξ times the public contribution).[3]

Note that in the particular case where there is no time-to-build ($N = 1$), one has $I_t^G = A_t$ and $I_t^{GS} = A_t^S$, so that the law of motion of public capital reduces to $K_t^G = (1 - \delta^G)K_{t-1}^G + I_t^G + (1 - n)I_t^{GS}$ and the private leverage effect becomes $(1 - n)I_t^{GS} = (\xi - 1)(I_t^G - I^{G*})$.

[3]Note that we will never consider cases where $A_t < A^*$ in our simulations, so that A_t^S and I_t^{GS} will never be negative.

Monetary policy

The monetary authority follows a classical Taylor Rule, subject to a ZLB constraint:

$$1 + i_t = \max\left((1 + i_{t-1})^{\rho^i}\left(\frac{1 + \pi_t}{1 + \pi^*}\right)^{(1-\rho^i)\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{(1-\rho^i)\phi_Y}(1 + \varepsilon_t^i)\ , 1\right) \tag{26}$$

where $\rho^i \in [0, 1)$ is the interest rate smoothing parameter, $\phi_\pi > 0$ (resp. $\phi_Y \geq 0$) captures the central bank reaction to inflation (resp. growth), and ε_t^i is a monetary policy shock.

3.5 Market Clearing

The equilibrium on the final good market is given by

$$\begin{aligned} Y_t = (1 - n)C_t^S + nC_t^B + (1 - n)(I_t^S + I_t^{GS}) + G + I_t^G \\ + \int_0^1 \frac{\gamma^p}{2}\pi_t(j)^2 Y_t \mathrm{d}j + \int_0^{1-n}\left[\frac{\gamma^w}{2}\pi_t^w(i)^2 w_t(i) + \psi(u_t(i))\bar{K}_{t-1}^S(i)\right]\mathrm{d}i \end{aligned} \tag{27}$$

where I_t^S is the average per capita private investment of patient households. Market clearing on markets for debt, private capital and labor, implies:

$$B_t^G + n\,D + \int_0^{1-n} B_t^S(i)\mathrm{d}i = 0 \tag{28}$$

$$(1 - n)K_t^S = \int_0^1 K_t(j)\mathrm{d}j \tag{29}$$

$$L_t = \int_0^1 L_t(j)\mathrm{d}j \tag{30}$$

4 Simulating the Juncker Plan

We now consider the possibility of simulating a public investment plan as proposed by the European Commission in 2014. We first estimate the model using EMU quarterly data. Then we introduce *ad hoc* shocks to mimic as faithfully as possible the impact of the Juncker plan.

4.1 How does the Juncker Plan Work?

At the time of writing (March 2016) the EFSI is moving its first steps, with an endowment of €21bn, coming in part from the European budget (€16bn), and for the rest from the European Investment Bank. The EIB is in fact the driving force, as the EFSI is in fact just a label for the EIB assets linked to the plan (Claeys, 2015). The Fund endowment should allow bond issuance allowing to increase available funding to €60bn. These resources will be topped by voluntary pledges from Member countries (that will not be counted as deficit within the Stability Pact framework). Third-party countries will also be allowed to contribute (China was the first to announce its contribution, in September 2015). This amount will then be used to co-finance private investment projects for a total expected to reach €315bn over the three years 2015-2017. Expected leverage is therefore rather large. It is 5 if we take as a basis the €60bn the Fund should raise on markets; but it climbs to a staggering 15 if we use as a basis for calculation the initial endowment of €21bn. Figure 2, borrowed from Claeys (2015), details the plan's structure.

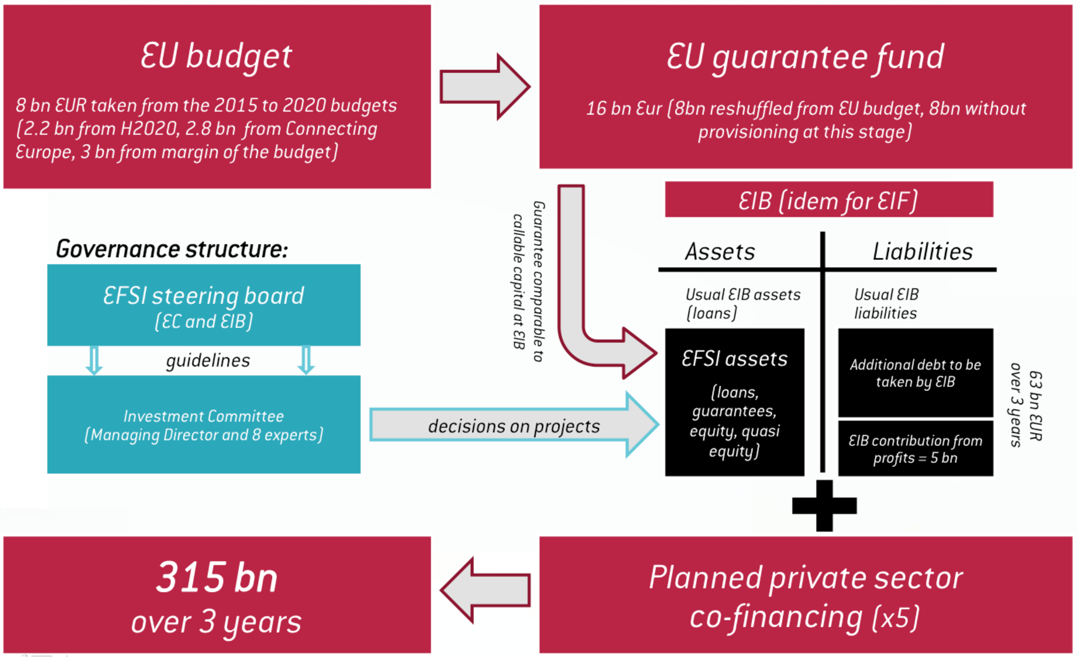

Figure 2: The Working of the Juncker Plan

4.2 Calibration and estimation

We calibrate our model to quarterly frequency. The list of the parameters that are calibrated is given in Table 1.

The share of borrowers is set to 34%, as in Forni et al. (2009). The fiscal rule targets a debt-to-GDP ratio, in annual terms, of 60%, as in the Stability and Growth pact, while the speed of convergence to that objective is calibrated to match the debt criterion.

The most controversial parameter is the production elasticity to the stock of public capital, denoted by ν in our model. The empirical literature failed to reach a consensus on its value. The already cited work by Aschauer (1989b) found an elasticity that could be as large as 0.4. Subsequent estimates were smaller (sometimes significantly so). The benchmark value we used in our simulation is $\nu = 0.1$, which is in the middle of the range, and very close to the average output elasticity of public capital $\nu = 0.106$ emerging from the meta-analysis of Bom and Ligthart (2014). In section 7 we test for the sensitivity of our results to this parameter.

The depreciation rate of public capital is set at 5% per year[4] This is lower than the depreciation rate of private capital which is set at 10% per year.

The time-to-build for public capital is set at 3 years (other values are explored in the sensitivity analysis). The spending profile is linear, *i.e.* the amount allocated for a project is spent uniformly over the 12 quarters of the time-to-build.

Some parameters of the model are estimated using Bayesian techniques on the quarterly Eurozone dataset "Area Wide Model".[5] We use data from 1970 to 2005 (we exclude from the sample the period starting with the crisis of 2008). Our estimation (Table 2) is rather close

[4]This is the value used by the Quest III model of the European Commission, see Ratto et al. (2008). Leeper et al. (2010) use a higher value of 8% per year, while the simulations of IMF (2014) are based on the GIMF model (Kumhof et al., 2010) which uses a lower value of 4% per year.

[5]This dataset provides a number of macroeconomic series for the Eurozone going back to 1970. Data for current members of the single currency are aggregated backwards. Thus our dataset includes the current member states. For more details see Fagan et al. (2001) or http://www.eabcn.org. The estimation is performed using Dynare, see Adjemian et al. (2011) or http://www.dynare.org.

Table 1: Calibrated parameters

Parameter	Symbol	Value
Share of borrowers	n	0.34
Private leverage factor of public investment	ξ	5
Preferences		
Discount rate of savers	β^S	0.995
Discount rate of borrowers	β^B	0.99
Disutility of labor	χ	1
Persistence of time rate preference	ρ^μ	0.75
Production		
Private capital depreciation rate	δ^k	0.025
Public capital depreciation rate	δ^G	0.0125
Private capital share in production	α	0.36
Public capital influence in production	ν	0.1
Private capital utilization rate (steady state)	u^*	0.85
Price and wage stickiness		
Market power (goods, at steady state)	θ^{p*}	6
Market power (labor, at steady state)	θ^{w*}	6.2
Monetary policy		
Inflation (steady state)	π^*	0
Fiscal policy		
Speed of fiscal consolidation	Φ	$\frac{1}{80}$
Debt target	b^{G*}	$2.4Y^*$
Consumption tax (steady state)	τ^{c*}	0.2
Capital income tax (steady state)	τ^{k*}	0.184
Time to build of public investment	N	12
Time profile of public investment	ϕ_s	$\frac{1}{N}$
Government consumption (steady state)	G^*	$0.25Y^*$
Public investment (steady state)	A^*	$0.02Y^*$
Debt constraint of borrowers	D	$0.125Y^*$

Note: the steady state tax rates (τ^{c*}, τ^{k*}) are not directly calibrated, but are targets that are matched by calibrating α_c et α_w accordingly.

to Forni et al. (2009). This is not surprising, as we use a similar model, exhibiting the same type of rigidities as well as the existence of non-Ricardian agents, and use the same dataset to estimate the model.

4.3 Assessing the Impact of a Public Investment Push

First we used the estimated model to simulate the impact of a public investment push. Our benchmark simulation has an elasticity $\nu = 0.1$, a three year time-to-build lag, and a private leverage of 5. We assume a public investment shock of 0.5% of (annual) GDP, which corresponds to the approximate size of the Juncker plan before private sector co-financing, that will be spread over three years because of the time-to-build.

The simulations show two distinct phases following the shock. During the first phase, corresponding to the construction of public capital, public investment sustains aggregate demand,

Table 2: Estimated parameters

Parameter	Symbol	Prior Type	Prior Mean	Prior St. Dev	Posterior mode
Preferences					
Frisch elasticity of labor	ε	Γ	2	0.25	1.9200
Relative risk aversion	σ	Γ	1.75	0.5	1.8951
Habit formation in consumption	h	β	0.5	0.2	0.8715
Production					
Adjustment cost on private investment	γ^I	N	5	0.25	5.1858
Elasticity of capacity utilization rate	σ_u	N	5	0.1	4.9879
Persistence of investment shock	ρ^κ	β	0.5	0.2	0.9042
Persistence of productivity shock	ρ^z	β	0.5	0.2	0.8476
Price and wage stickiness					
Adjustment cost on wages	γ^w	Γ	110	100	353.5216
Adjustment cost on prices	γ^P	Γ	300	100	83.5008
Persistence of price markup shock	ρ^p	β	0.5	0.2	0.8972
Persistence of wage markup shock	ρ^w	β	0.5	0.2	0.1187
Monetary policy					
Persistence of interest rate	ρ^i	β	0.8	0.1	0.8065
Sensitivity to inflation	ϕ_π	Γ	1.7	0.1	1.7292
Sensitivity to GDP	ϕ_Y	N	0.125	0.05	0.1766

thus leading to an increase of production (Figure 3), and of inflation (Figure 4). This triggers a reaction of monetary policy, and an increase of the nominal interest rate that, after a few periods, yields an increase of the real rate as well. The increase of real interest rates in turn leads to a crowding out of private investment, that dampens the expansionary effect of the initial shock. This sequence of effects explains the U-shaped GDP reaction function between quarters 1 and 12 following the shock.

Once the investment push is over, the demand effect vanishes (explaining the sharp drop at $t = 12$, when GDP shortly becomes negative), but it is then replaced by the supply-side impact of the new infrastructures that become productive. Production then remains for a very long time above its steady state value, as long as the new public capital is not completely depreciated. The increase of supply also leads to lower inflation, and therefore to a drop of the nominal interest rate.

It should nevertheless be noted that the impact is quantitatively weak, though not insignificant; the peak of production, reached in the first period of the plan, is about 1.2% above the long run equilibrium level. This is not surprising given the limited size of the investment push (0.5% of annual GDP, multiplied by an optimistic leverage of 5, but spread over three years). While limited in size, nevertheless, the shock is very persistent thanks to the supply-side effect of public investment. Ten years after the investment push, production is still more than 0.6% above the long term trend.

Turning to public debt (Figure 5), our simulations show a decrease of the debt-to-GDP ratio, in spite the fact that (because of the fiscal rule we adopted) investment is mostly financed with debt. The ratio decreases therefore thanks to the growth impact of public investment, that more than compensates the increase of debt. Our simulation is therefore consistent with

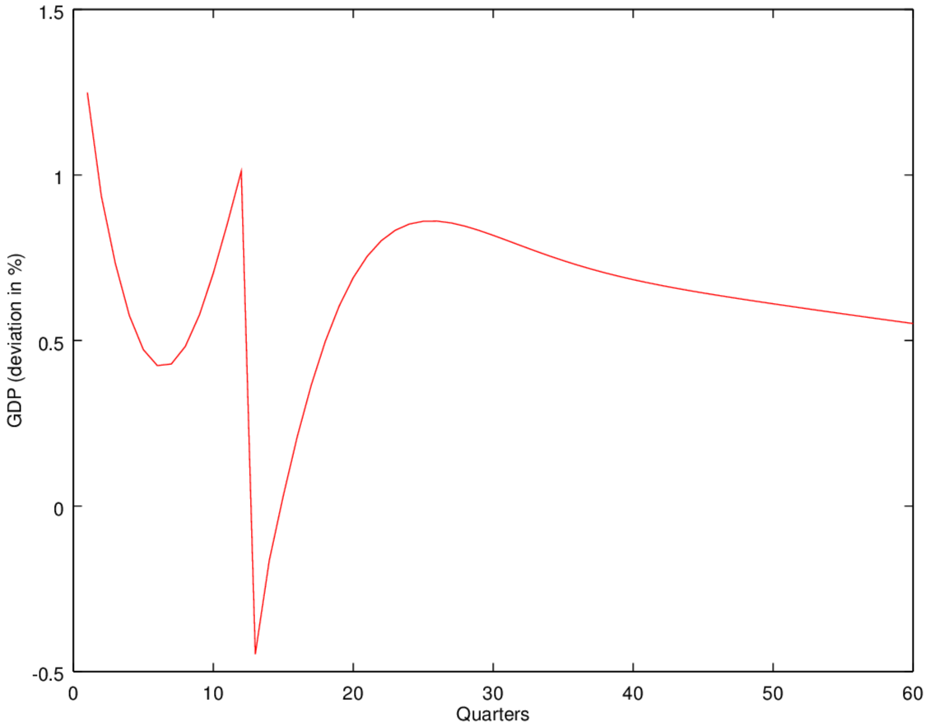

Figure 3: Baseline scenario, GDP deviation from steady state

the widely discussed findings reported in a recent IMF *World Economic Outlook*, (IMF, 2014, p. 83), and more recently by OECD (2016), which both show that on average, in the current conditions, a public investment push eventually leads to decreasing public debt ratios.

As for private investment (Figure 6), in the short run crowding out dominates. The financing of public investment reduces savings available for private capital, through the interest rate channel. In the medium run, *i.e.* once the time-to-build period is over, crowding in kicks in, and eventually ends up dominating. The larger stock of public capital enhances private investment productivity, encouraging its accumulation. This also explains the persistence of the shock.

To fully grasp the impact of the a public investment shock, we computed the dynamic fiscal multipliers. At a given time horizon, the dynamic multiplier is defined as the integral of GDP deviations from steady state, divided by the integral of investment expenditure deviations over the same period. In our framework such a computation is complex because of the existence of leverage, that tends to magnify to a great extent the impact of public expenditure shocks. In other words, multipliers embed a part of private expenditure. This is why in Table 3 we report the multiplier of simulations with and without leverage.

The results show that during the construction phase (up to year 3), the multipliers without leverage are close to one, hence not particularly large. In fact, during this phase, as the new capital is not yet operational, the shock is equivalent to a public consumption one. It is also worth remembering that monetary policy tightens in response to the shock. With a more accommodating policy, multipliers would be larger (this will be the case in the following section, where the ZLB binds). In the second phase, when public capital becomes productive, multipliers increase significantly, and reach 4.1 at the horizon of 20 years.

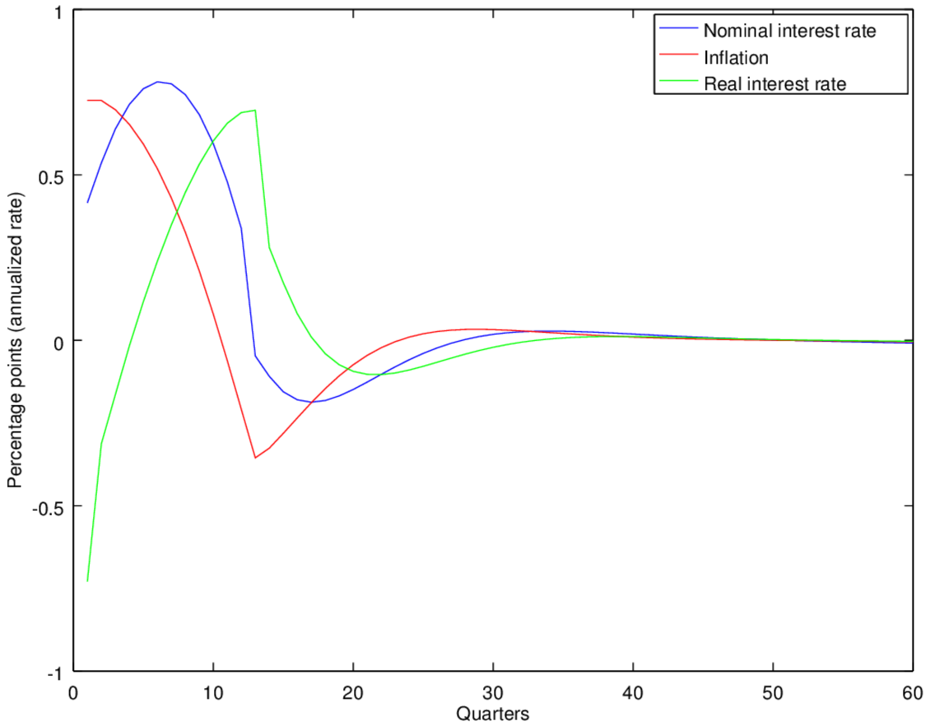

Figure 4: Baseline scenario, inflation and interest rates

Table 3: Dynamic multipliers for the baseline calibration

	1 year	*3 years*	*10 years*	*20 years*
Without leverage	1.0	0.8	2.2	4.1
With leverage of 5	5.2	4.2	13.0	24.0

With the leverage effect the multiplier follows the same pattern, but obviously has a much larger size, even more than five times the multiplier without leverage. This can be explained by the increasing returns to scale of the production function.

The conclusion of this first exercise is that in spite of large cumulative multipliers, an investment push of the size of the Juncker Plan has, in normal times, rather limited effects. True, the plan pays for itself, but even if expected leverage materialized, the short term impact of the shock is modest.

The plan has not been conceived in normal times, though. The Eurozone seems unable, after 8 years of crisis, to go back to a normal growth path. The economy is stuck in a liquidity trap with the ECB that is constrained by interest rates that are at the ZLB and needs to resort to unconventional monetary policies. The Quantitative Easing program, that will continue at least until the Spring of 2017, is precisely aimed at pulling the economy out of this deflationary situation. But it is increasingly clear that unconventional monetary policies will not be enough if not supported by fiscal policy, and hence much hope is put in the working of the Juncker Plan (even if, as OECD, 2016 points out, actual expenditure is running behind schedule). The next

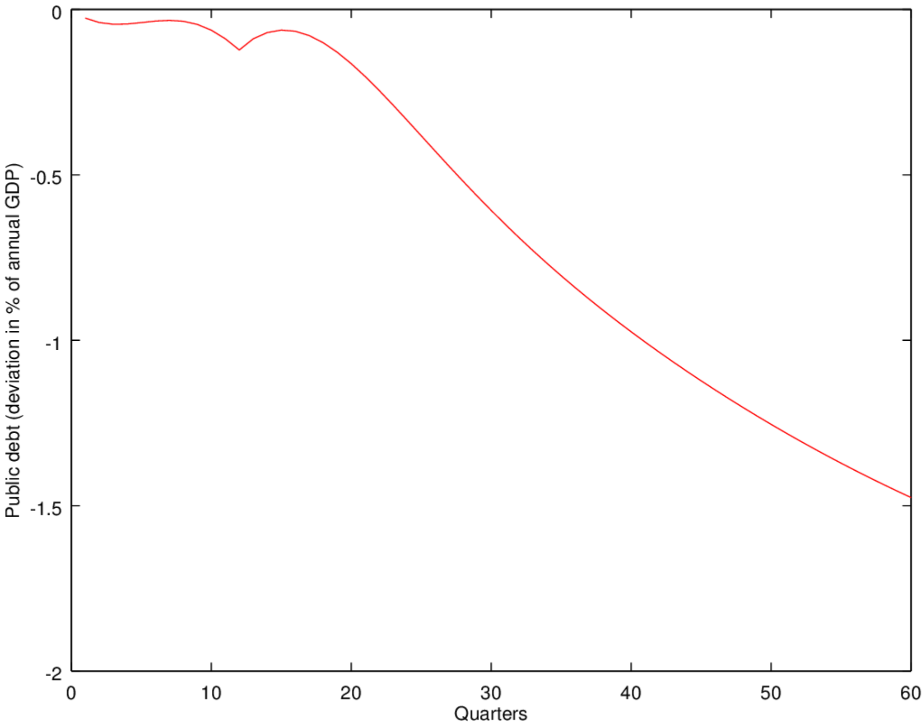

Figure 5: Baseline scenario, public debt-to-GDP ratio (deviation from steady state)

section tries to assess whether this hope is justified.

5 The Zero Lower Bound and the Juncker Plan

Given that the Euro area currently stands in a liquidity trap, where nominal interest rates are close or even below zero, our quantification of the impact of the Juncker plan needs to take that context into account. In particular, an interesting question to ask is whether the Juncker plan has the potential to pull the Euro area out of the trap.

In order to model the actual liquidity trap situation, we have applied an important negative demand shock to our model, which consists in a strong but temporary fall of the rate of time preference. This amounts to lowering the natural real rate of the economy, and therefore echoes the revival of the secular stagnation theories as put forward by *e.g.* Larry Summers (2014).

Using a time preference rate shock to generate a depression that pushes the economy into a liquidity trap is a strategy that is often used in the quantitative literature (see *e.g.* Christiano et al., 2011). Even though it is not entirely satisfactory from a methodological point of view, because it is too simple a representation of the sequence of events that led the Euro area where it currently stands, it can nevertheless help us to understand the interactions between the liquidity trap and an investment shock and deliver some quantitative insights.

The shock is calibrated in such a way as to make the ZLB bind a sufficiently long time. In our model, it is implemented through a temporary negative shock to μ_t, that leads the agent to discount the future less relatively to the present than it usually does. More precisely, we use the value $\varepsilon_1^\mu = -0.6$ (while $\varepsilon_t^\mu = 0$ for $t \geq 2$). A shock of this magnitude, absent any fiscal reaction from the government, pushes the economy to the ZLB for several quarters. We do not impose

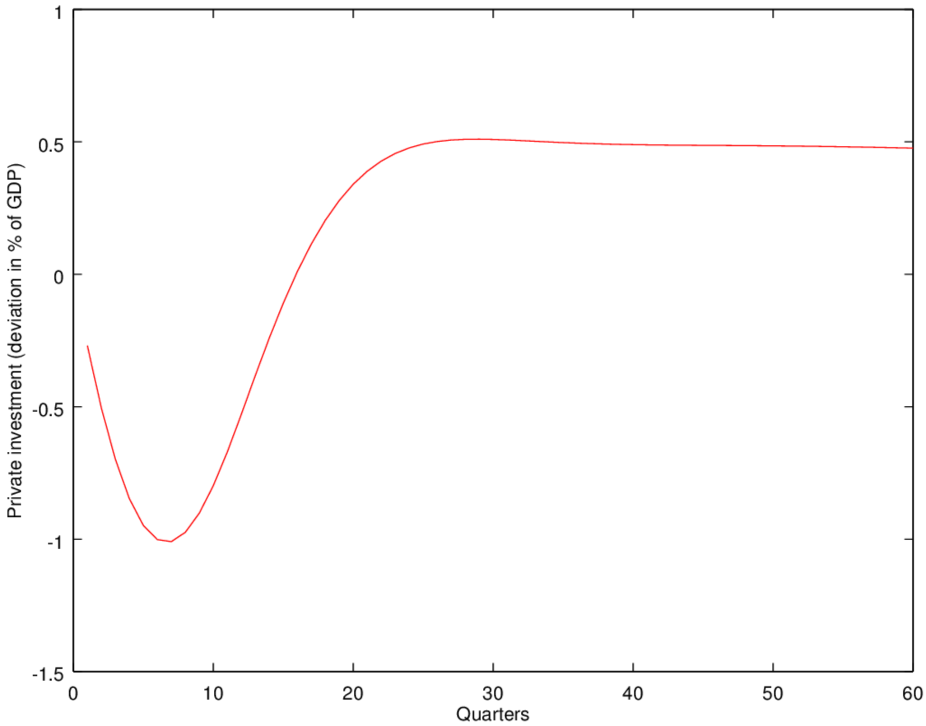

Figure 6: Baseline scenario, private investment (deviation from steady state)

a priori the duration of the ZLB: it is determined endogenously by the agents' behavior. We solve the model using the extended path method (Fair and Taylor, 1983): it ensures that all nonlinearities (including the zero bound on the Taylor rule) are fully taken into account, while allowing a reasonable treatment of expectations (agents are surprised by shocks at every period, and behave as if no shock will happen in the future).[6]

As is well-known in the literature, new Keynesian models with an interest rate rule subject to a ZLB can exhibit multiple equilibria (see Benhabib et al., 2001). This property emerges because, in addition to the steady state with a positive nominal interest rate, there exists another steady state where the ZLB binds and where, consequently, monetary policy is passive (*i.e.* it does not react more than on to one to inflation). The usual stability conditions (the so-called Blanchard and Kahn, 1980 conditions) are therefore not satisfied at this second steady state, and this can give rise to a multiplicity of equilibrium paths.

In the scenario that we consider, multiple equilibria are a concrete issue, not just a theoretical curiosity. Two equilibrium paths turn out to be consistent with the demand shock that we consider (in the case where there is no further government intervention): on the first equilibrium path, the ZLB binds during 14 quarters, and the maximum GDP deviation from steady state is -12.0%; on the second equilibrium path, the ZLB lasts 7 quarters and the lowest point of GDP is -3.1%.

Our equilibrium selection criterion is that we want to construct a scenario that resembles what the Euro area has been going through. In 2009-2010, at the worst of the European crisis,

[6] This contrasts in particular with a perfect foresight setup where agents are surprised only at the first period, and know from there all future shocks. The two solutions methods are by construction identical for a scenario with a shock in the first period but no shock thereafter.

GDP was 6% below its long term trend, despite the fact that national governments had just started to implement stimulus packages. This leads us to select the simulated path where GDP goes as low as 12% under its pre-crisis level (and this point is reached 5 quarters after the shock). Note that it makes sense to select a scenario in which GDP goes below what actual values, because we are for the time being considering a case with no discretionary government intervention. If European authorities had not reacted the way they did, the Euro area would probably have undergone a more violent contraction.[7]

From a practical point of view, we therefore make the hypothesis that in our simulations, agents coordinate on the worst equilibrium in the first period(s) of the simulation, and continue to do so as long as there is no unexpected investment shock. Note that this does not preclude the possibility that multiple equilibria arise again after the public investment shock.

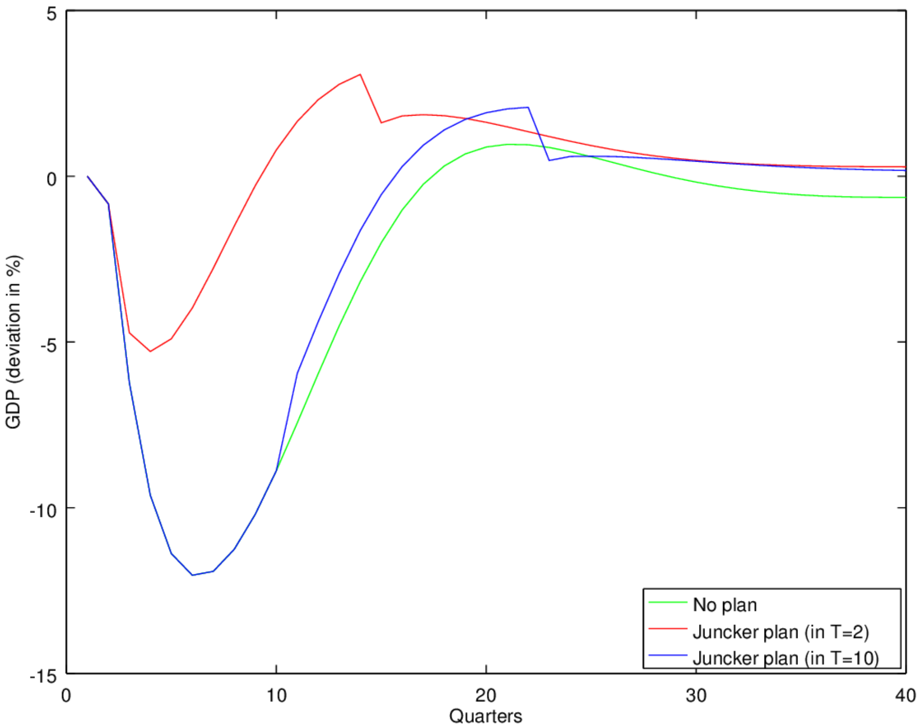

Figure 7: Juncker plan in ZLB, GDP deviation from steady state

Figure 7 shows the path of GDP, following the demand shock (that happens in period 1), depending on whether and when the Juncker plan is implemented. Figure 8 shows the path of the nominal interest rate for the same scenarios.

It appears that the impact of the Juncker plan, if it occurs in period 10, is very limited. It speeds the recovery a little bit, but not much. The ZLB is shortened by 2 quarters, and the return of GDP to its pre-crisis level still takes about 6 quarters after the beginning of the plan.

Had the plan occurred in period 2, things would have been quite different. The ZLB would have been much shorter (7 periods in total), though this means that the economy would have

[7]See in particular Albonico et al. (2016) for a quantitative investigation of the role of euro area fiscal policy during the Great Recession. Even though the authors conclude that fiscal policy supported growth in 2008-2009, they show that discretionary fiscal shocks were actually of small size and that monetary policy played a much more important role.

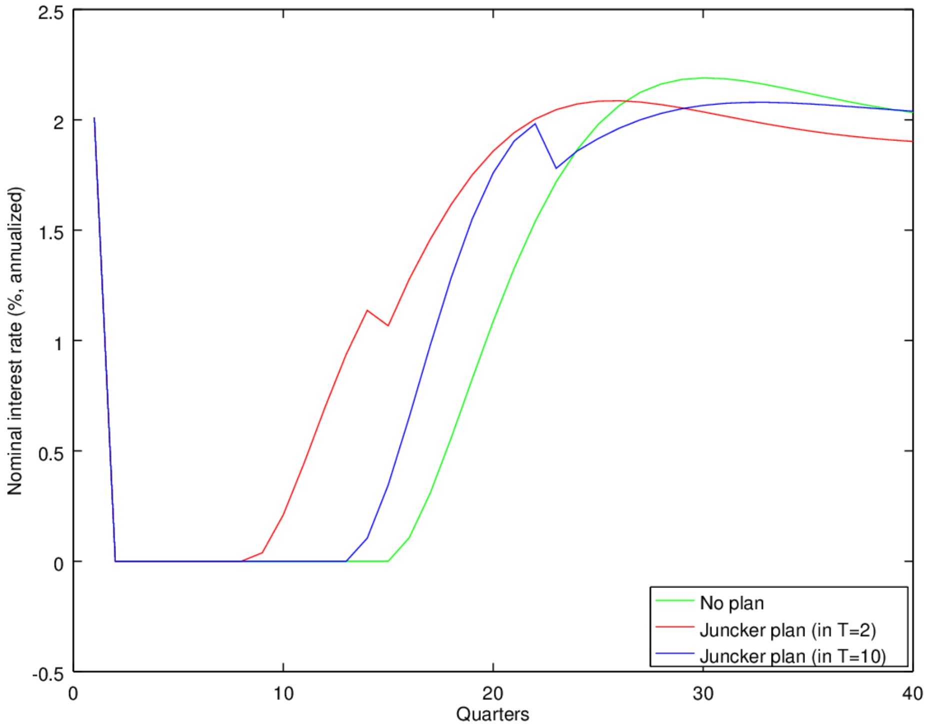

Figure 8: Juncker plan in ZLB, nominal interest rate

spent 5 more quarters in ZLB after the Juncker plan. The economic downturn would also have been much less violent, with a GDP through about 5% below the pre-crisis level, and a complete recovery 2 years after the beginning of the crisis (followed by a small over-shooting due to the enhanced productivity effect).

Why such a difference in performance? The reason lies in the behaviour of private investment. Once the economy settles in the ZLB, investment drops, and the private capital stock is reduced. The expected impact of a public investment push is therefore lower (remember that public capital in our model behaves as a productivity enhancer), and the crowding in effect less important. If the public investment shock happens when private capital is not yet depleted, its expected impact will be larger, and the economy rebounds more easily. This mechanism is consistent with recent empirical and theoretical work highlighting that facing large recessionary shocks it is better to “over react” than to be too cautious and let the economy sink (Reifschneider et al., 2013; Fatas and Summers, 2015).

Note that in our simulations, there are no multiple equilibria after the plan has been implemented: only one path post-plan satisfies the equilibrium conditions. But this is a matter of size: we could determine that, if the plan had been 4 times smaller, there would have been two equilibria for the case where it occurs in period 2 (one in which the ZLB is shortened, and the other in which the situation is even worsened compared to the no-plan scenario, probably because of the deflationary impact of supply side of the public investment shock).

From these simulations, we draw the conclusion that the Juncker plan comes far too late given the state of the European economy. It has started in the last quarter of 2015, 10 quarters after the ECB main rate went below 0.5% (in the second quarter of 2013). In these circumstances, it is unable to make a difference and to significantly accelerate the recovery, nor does it definitely dispel the deflationary risk. If European policy makers had been reactive and implemented the same plan 2 years before, then it would have made a significant difference, by shortening and downsizing the slump.

In those conditions, what is the right macroeconomic policy to be put in place today? More precisely, would it be possible to conceive a stimulus package that would be ambitious enough for pulling the economy out of the liquidity trap? This is the question that we try to answer in the following section, where we evaluate the impact of a more ambitious plan, that was launched in the United States by President Obama in the wake of the financial crisis of 2007-2008.

6 A Comparison with the US Recovery and Reinvestment Act of 2009

Section 5 showed that the Juncker Plan may turn out to be of limited impact on the economy because it came too late in the crisis. Had it been implemented earlier, it would have been much more effective in lifting the economy out of the liquidity trap. One can then ask whether such a late intervention could be effective, were it larger in size. In other words, besides being "too late", is the Juncker Plan also "too little"? We ask this question by comparing the EU plan with the stimulus package implemented in 2009 by the newly elected Obama administration in the United States.

The *American Recovery and Reinvestment Act* (ARRA) was implemented starting in February 2009, with the objective of sustaining the economic activity and saving between three and four millions jobs. The ARRA amounted to 789 USD billions, or 5.5% of 2009 US GDP. Broadly speaking, it was composed of tax breaks for around 4% of GDP (over two years), and for the rest (1.5% of GDP) of a public investment plan aiming at long term growth. The Obama administration had in mind the importance of supporting short term economic activity, and therefore it privileged "shovel-ready" projects for which the decision and implementation lags were minimized. Disbursements began as early as the summer 2009, a remarkably quick achievement. For our simulation exercise we model the ARRA through two shocks:

- A shock on public spending amounting to 4% of US GDP, spread uniformly over 8 quarters (2 years). The reader should be aware that the Obama plan relied mostly on tax reductions. We could therefore have simulated a shock on tax revenues, rather than on public expenditure. We chose nevertheless the latter, because as the EU does not have a single tax system, the comparison between the two stimulus plans would have been harder.
- A public investment shock amounting to 1.5% of US GDP. We set to zero the decision lag (the "shovel-ready" effect), and we impose a construction period of 12 quarters, three years, for the sake of comparison with the Juncker Plan.

The ARRA is different from the Juncker plan in two respects. First, it was larger in size. In our simulations we focused just on the public investment part of the plan, that at 1.5% of GDP is more than three times larger than the Juncker Plan. Second, the Obama administration acted far more quickly than the EU authorities.

We compare the two plans in the ZLB case. We first look at what happens when the two public investment shocks are put in place as soon as the economy hits the ZLB. In other words, we ask what would have happened, had the EU authorities been as reactive as the US administration. Figure 9 shows that size matters. While a timely intervention of the kind of the Juncker plan would have considerably shortened the crisis (as we have seen in Figure 7), it would not have avoided it altogether. On the contrary, a bold investment push, like the ARRA, would have avoided the recession altogether.

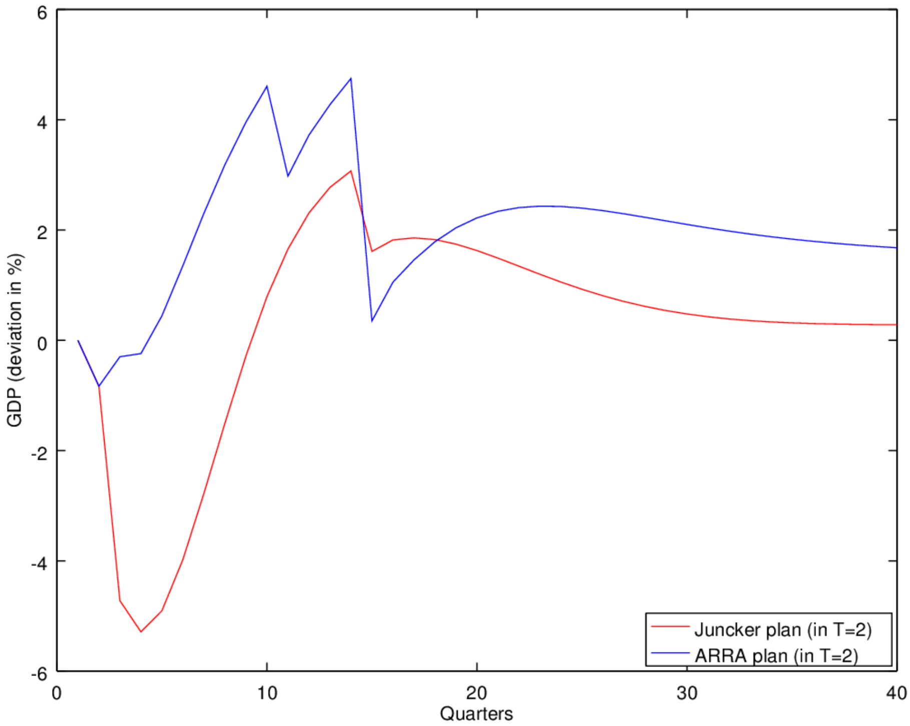

Figure 9: Impact of Juncker and ARRA plans in ZLB (plan in period 2)

We then ask the complementary question of what would have happened had the Obama administration waited as long as the EU authorities, and implemented its investment program only after ten periods, but larger in size than the Juncker Plan. The results can be found in Figure 10. In this case the results are different as well, even if less so than in the previous case.

While as we saw in section 5 (Figure 7) the Juncker plan fails to display significant positive effects, the ARRA, even if implemented late, would have immediately lifted the economy out of the ZLB. The positive demand shock brings the economy back to steady state in the very same period, and then yields a reaction of the private sector (anticipating higher future productivity) that further pushes GDP upwards. The Juncker plan is clearly too small in size to yield the initial aggregate demand shock.

Our simulation of the Juncker plan at the ZLB, in comparison with the Obama stimulus plan, yields the basic message that the Juncker plan is insufficient in size. In the current situation, with the economy well installed in a ZLB, the plan only marginally accelerates the return to the steady state. Even a more timely implementation, on the other hand, while making the recession milder, could not avoid it altogether. In comparison, the ARRA put in place by the Obama administration, was of such a size that it would boost aggregate demand, and therefore lift the economy out of the ZLB, regardless of the timing of its implementation.

7 Sensitivity analysis

The quantitative conclusions that we draw from our simulations are strongly dependent on the calibration chosen for some of the parameters that we did not estimate. In particular, it is important to underscore that our results are obtained using optimistic hypotheses for the

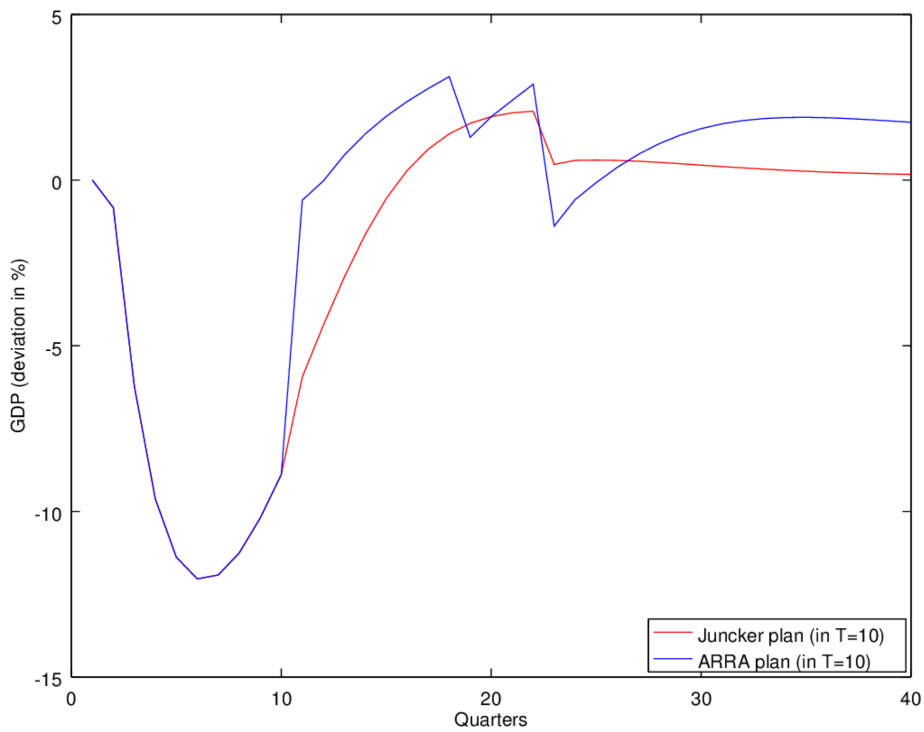

Figure 10: Impact of Juncker and ARRA plans in ZLB (plan in period 10)

elasticity of production to public capital (10% in our baseline calibration) and on the private leverage effect (the European Commission's estimation of a leverage of 5 being quite debatable).

In this section, we discuss the sensitivity of the results to these two parameters as well as to the duration of the time to build.

7.1 Elasticity of production to public capital

The elasticity of production to public capital (ν) determines the size of the response of the economy to a public investment package. Table 4 gives the sensitivity of the dynamic fiscal multipliers following a public investment, assuming no private leverage effect (hence, the line corresponding to the baseline calibration $\nu = 0.1$ is the same as the first line of Table 3).

Table 4: Dynamic multipliers as a function of the elasticity of production to public capital (assuming no private leverage)

Elasticity (ν)	*1 year*	*3 years*	*10 years*	*20 years*
0	1.12	0.79	0.24	-0.08
0.05	1.07	0.78	1.21	1.98
0.10*	1.02	0.77	2.19	4.05
0.15	0.97	0.76	3.17	6.13
0.17	0.95	0.76	3.57	6.96

*Baseline calibration.

One can see that short term fiscal multipliers (3 years or less, *i.e.* during the duration of the time-to-build) are slightly decreasing as the elasticity grows, while they become strongly increasing in the long term (because of the productive impact of public capital).

It is interesting to note that short term multipliers for an improductive public capital ($\nu = 0$) are not very different from those with a productive public capital. This confirms that the demand effect dominates in the short run. On the contrary, in the long run, multipliers become small or even negative, in the case of non-productive expenditure, because the supply effect is not present; the dominant effect is the crowding out of private capital and the distortionary impact of taxes.

This last point is important because it qualifies our conclusions on the expected impact of a public investment package such as the Juncker plan: the public investment projects chosen by the EFSI must exhibit strong positive externalities on the productivity of private factors if the Juncker plan is to be effective in pulling the Euro area out of the crisis. The architecture of the political process through which the projects are chosen is therefore crucial (and this is related to the problematically low reactivity of the European authorities mentioned above).

Also note that the long run crowding in of private investment is an increasing function of the productivity of public capital. In particular, in the case of an improductive public capital, one only observes a crowding *out* effect that is very slow to disappear.

7.2 Time-to-build

The time-to-build process is important because it governs the relative timing of the two dimensions embedded in the public investment shock: the demand effect, in the short run; and the supply effect, in the longer run.

In particular, if there is no time-to-build, the demand shock is very short-lived, while the supply shock comes sooner. In that case, the peak of GDP is reached just after the launch of the public investment plan, while the short run crowding out effect on private investment disappears: only the crowding in effect is observed.

As noted by Bouakez et al. (2014), the time-to-build has another effect in the case where the ZLB binds. Because the short run demand effect is inflationary, while the longer run supply effect is deflationary, a long time-to-build has the desired property of postponing the deflationary effect, hence making an anticipated exit from the ZLB more likely.

Indeed, this property can be verified through our numerical exercises. If the simulations of the Juncker plan in a ZLB context (section 5) are rerun with a time-to-build of 1 quarter, then it takes longer to exit from the ZLB compared to the baseline case with a 3 year time-to-build. If the Juncker plan intervenes just after the demand shock ($T = 2$), the time-to-build is critical in delivering a quick exit from ZLB: compared to our baseline, if there is no time-to-build, then the ZLB lasts 6 quarters more, and the recession is much more severe (through of -10.5% compared to -5.3%). In the case where the Juncker plan intervenes later ($T = 10$), the effect is still there, though it is less spectacular: without the time-to-build, the ZLB lasts one quarter more.

7.3 Private leverage

The impact of the private leverage effect (ξ) on GDP is the expected one: the fiscal multipliers on public investment are a quasi-linear function of the leverage coefficient (*i.e.* a doubling of the leverage coefficient will approximatively double the size of the shock to GDP).

However, the impact on the debt-to-GDP dynamics is not linear. Absent any leverage effect ($\xi = 1$), if other parameters are kept at the baseline calibration, there is an increase in the debt-to-GDP ratio following a public investment shock (the numerator effect dominates the denominator effect). But for a higher leverage, such as 3 or 5 (the baseline value), it's the opposite: the positive shock to GDP is more important, while the debt increase is the same, so that the debt-to-GDP ratio diminishes.

In the case where the economy has fallen into a liquidity trap, the private leverage effect plays an important role in pulling the economy out of the ZLB. In particular, if the Juncker plan intervenes just after the demand shock ($T = 2$) but that there is no private leverage effect ($\xi = 1$), then multiple equilibria arise again: in the good equilibrium, compared to the baseline case ($\xi = 5$), the economy stays in the ZLB two more quarters, which is already significant but not dramatic; but there is also a bad equilibrium, in which the economy stays in the ZLB for seven more quarters, with a much more violent recession (GDP through of -12.3% compared to -5.3%). The hypothesis of a significant private leverage is therefore critical for making the plan efficient in a ZLB context, and a bigger public involvment would of course be needed were the private support be smaller than hoped.

8 Conclusion

This paper presented a quantitative assessment of the public investment plan currently being implemented by the European Union and the EIB.

Using a simple inter-temporal stochastic general equilibrium model with public capital and non-Ricardian agents, we evaluated the capacity of the Juncker plan to support the economy, through enhanced private capital productivity, and its ability to address the Eurozone liquidity trap.

Our exercise has inherent limitations, such as the difficulty, which we share with the literature on the subject, to model the economy at the zero lower bound (where, for example, multiple equilibria may arise); or the difficulty of embedding in a simple model the multiple aspects of complex packages like the Juncker and the Obama plans. In spite of these limitations, we believe that our simulations allow a quantitative and qualitative assessment that is consistent with the literature on public investment and growth.

Since it was announced, the Juncker plan was criticized as being "too little and too late." Our simulations confirm this criticism, and help making it more precise. We found that had the Juncker plan been implemented in a timely manner, it would have helped to significantly shorten the recession. True, had it been larger in size, as was the ARRA plan put in place by the US in 2009, the positive impact would have been even larger. But it remains that even the small investment shock agreed upon by the EU authorities could have had a positive impact if implemented earlier.

But once the political process delayed the intervention, then EU authorities should have implemented a much bolder plan. As it is, the Juncker plan is likely not going to be effective at all. Thus, we could rephrase the criticism of the plan as "probably too little, and certainly too late."

Our analysis confirms, indirectly, that a major flaw in European economic governance is its inability to respond quickly to shocks hitting the economy. An institutional architecture and decision-making process that could be adapted to the so-called "Great Moderation" (if it ever existed), are certainly not fit for the new era of instability which began in 2008.

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Appendix

A First order conditions of the model

A.1 Households

For every household i, the slackness condition on the credit constraint is:

$$\eta_t(i) \geq 0 \qquad D - \frac{B_t(i)}{P_t} \geq 0 \qquad \eta_t(i)\left(D - \frac{B_t(i)}{P_t}\right) = 0 \tag{31}$$

where $\eta_t(i)$ is the Lagrange multiplier on the credit constraint (2).

The constraint will never be binding for patient households ($\eta_t^S(i) = 0$), while it will always be binding for impatient ones ($\eta_t^B(i) > 0$).

Maximizing (1) subject to (2), (3) and (4) yields the first-order conditions for patient households (in the following, we drop the i index because the equilibrium is symmetric):

$$\text{for } C_t^S : \lambda_t^S = \frac{\left(C_t^S - hC_{t-1}\right)^{-\sigma}}{1+\tau_t^c} \tag{32}$$

$$\text{for } B_t^S : \lambda_t^S = \beta^S\,\mathbb{E}_t\left\{\lambda_{t+1}^S \frac{\mu_{t+1}}{\mu_t}(1+i_t)\frac{P_t}{P_{t+1}} + \eta_t^S\right\} \tag{33}$$

$$\text{for } \bar{K}_t^S : Q_t = \beta^S\,\mathbb{E}_t\left\{\frac{\lambda_{t+1}^S}{\lambda_t^S}\frac{\mu_{t+1}}{\mu_t}\left[(1-\delta^k)Q_{t+1} + (1-\tau_{t+1}^k)r_{t+1}^k u_{t+1} - \psi(u_{t+1})\right]\right\} \tag{34}$$

$$\text{for } I_t^S : 1 = \kappa_t Q_t \left[1 - \frac{\gamma^I}{2}\left(\frac{I_t^S}{I_{t-1}^S} - 1\right)^2 - \gamma^I\left(\frac{I_t^S}{I_{t-1}^S}\right)\left(\frac{I_t^S}{I_{t-1}^S} - 1\right)\right] \tag{35}$$

$$+ \beta^S\,\mathbb{E}_t\left\{\frac{\lambda_{t+1}^S}{\lambda_t^S}\frac{\mu_{t+1}}{\mu_t}\kappa_{t+1}Q_{t+1}\gamma^I\left(\frac{I_{t+1}^S}{I_t^S}\right)^2\left(\frac{I_{t+1}^S}{I_t^S} - 1\right)\right\}$$

$$\text{for } u_t : (1-\tau_t^k)r_t^k = \left(1-\tau^{k*}\right)r^{k*}\exp\left(\frac{u_t - u^*}{\sigma_u}\right) \tag{36}$$

with λ_t^S the Lagrange multiplier on (3) and $\lambda_t^S Q_t$ the Lagrange multiplier on the capital accumulation equation (4). Thus, λ_t^S represents the marginal utility of real income, whereas Q_t represent the shadow price of an additional unit of physical capital (Tobin's Q).

Because we assume that at steady state the government will also hold a positive debt (see (16)), the market clearing condition on the debt market (28) imposes that patient households are net creditors to both impatient households and the government. Thus, they cannot hit their credit constraint (as their debt is negative). Euler equation (33) therefore implies that at steady state:

$$\frac{1+i^*}{1+\pi^*} = \frac{1}{\beta^S}$$

A.2 Labor market

The wage index satisfies:

$$w_t = \left(\int_0^1 w_t(i)^{1-\theta_t^w}\,\mathrm{d}i\right)^{\frac{1}{1-\theta_t^w}} \tag{37}$$

The first order condition on wage demand by patient households is:

$$\chi\theta_t^w \frac{L_t^{1+\epsilon}}{w_t}\left(\frac{w_t(i)}{w_t}\right)^{-\theta_t^w(1+\epsilon)-1} + \lambda_t^S(1-\theta_t^w)L_t(1-\tau_t^w)\left(\frac{w_t(i)}{w_t}\right)^{-\theta_t^w}$$
$$+ \beta^S\frac{\mu_{t+1}}{\mu_t}\lambda_{t+1}^S\gamma^w\pi_{t+1}^w\frac{w_{t+1}}{w_t(i)}\left(1+\pi_{t+1}^w\right) = \lambda_t^S\gamma^w\pi_t^w\frac{w_t}{w_{t-1}(i)}$$

Since all households face the same labor demand, both the wage rate and hours worked will be equal for every agent in the economy (recall that impatient households do not have an inter-temporal optimizing behavior concerning their wage rate, and choose the same wage as patient households). Hence we are in a symmetric equilibrium ($w_t(i) = w_t$), and the first order condition driving wage rate demand is:

$$\chi\theta_t^w \frac{L_t^{1+\epsilon}}{w_t} + \lambda_t^S(1-\theta_t^w)L_t(1-\tau_t^w) + \beta\frac{\mu_{t+1}}{\mu_t}\lambda_{t+1}^S\gamma^w\pi_{t+1}^w\left(1+\pi_{t+1}^w\right)^2$$
$$= \lambda_t^S\gamma^w\pi_t^w\left(1+\pi_t^w\right) \tag{38}$$

A.3 Production

In the final good sector, the price index satisfies:

$$P_t = \left(\int_0^1 p_t(j)^{1-\theta_t^p} \, \mathrm{d}j \right)^{\frac{1}{1-\theta_t^p}} \tag{39}$$

In the intermediate good sector, let's denote $\lambda_t^F(j)$ the Lagrange multiplier associated with constraint (14). First order conditions of the cost minimization program (13) are:

$$w_t = \lambda_t^F(j) \cdot F_L(K_{t-1}(j), L_t(j), K_{t-1}^G)$$

$$r_t^k = \lambda_t^F(j) \cdot F_K(K_{t-1}(j), L_t(j), K_{t-1}^G)$$

By the envelope theorem we have that $\lambda_t^F(j)$ is the real marginal cost $MC_t(j)$ of firm j:

$$w_t = MC_t(j) \cdot F_L(K_{t-1}(j), L_t(j), K_{t-1}^G) \tag{40}$$

$$r_t^k = MC_t(j) \cdot F_K(K_{t-1}(j), L_t(j), K_{t-1}^G) \tag{41}$$

The first order condition of profit maximization program (15) is:

$$1 - \gamma^p \pi_t(j)(1+\pi_t(j)) \frac{P_t}{p_t(j)} \frac{Y_t}{y_t(j)} + \beta^S \gamma^p \frac{P_t}{p_t(j)} \mathbb{E}_t \left[\frac{\lambda_{t+1}^S}{\lambda_t^S} \pi_{t+1}(j)(1+\pi_{t+1}(j)) \frac{Y_{t+1}}{y_t(j)} \right] = \theta_t^p \left(1 - MC_t(j) \frac{P_t}{p_t(j)} \right) \tag{42}$$

In a symmetric equilibrium all firms choose the same price at each period of time, and we have:

$$p_t(j) = P_t$$

$$\pi_t(j) = \pi_t$$

$$y_t(j) = Y_t$$

$$K_t(j) = K_t$$

$$L_t(j) = L_t$$

Aggregate production is therefore given by

$$Y_t = F(K_{t-1}, L_t, K_{t-1}^G) \tag{43}$$

Equations (40), (41) and (42) become:

$$w_t = MC_t \cdot F_L(K_{t-1}, L_t, K_{t-1}^G) \tag{44}$$

$$r_t^k = MC_t \cdot F_K(K_{t-1}, L_t, K_{t-1}^G) \tag{45}$$

$$1 - \gamma^p \pi_t(1+\pi_t) + \beta^S \gamma^p \, \mathbb{E}_t \left[\frac{\lambda_{t+1}^S}{\lambda_t^S} \pi_{t+1}(1+\pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] = \theta_t^p \, (1 - MC_t) \tag{46}$$

B Laws of motion of the shocks

The preference shock μ_t has the following law of motion:

$$\log(\mu_t) = \rho^{\mu} \log(\mu_{t-1}) + \varepsilon_t^{\mu}$$

with $\rho^{\mu} \in [0, 1)$ and ε_t^{μ} a zero-mean i.i.d. innovation.

The investment specific technological shock κ_t follows the exogenous stochastic process:

$$\log(\kappa_t) = \rho^{\kappa} \log(\kappa_{t-1}) + \varepsilon_t^{\kappa}$$

with $\rho^{\kappa} \in [0, 1)$ and ε_t^{κ} a zero-mean i.i.d. innovation.

The elasticity of substitution between different individual labor supplies (θ_t^w) follows the stochastic process:

$$\log(\theta_t^w) = (1 - \rho^w) \log(\theta^{w*}) + \rho^w \log(\theta_{t-1}^w) + \varepsilon_t^w$$

with θ^{w*} the steady state level of the elasticity of substitution between individual supplies of labor, $\rho^w \in [0, 1)$ and ε_t^w a zero-mean i.i.d. innovation. A positive shock on θ_t^w would imply more competition between the workers.

The elasticity of substitution between intermediate goods θ_t^p in the final good production function is subject to exogenous shocks:

$$\log(\theta_t^p) = (1 - \rho^p) \log(\theta^{p*}) + \rho^p \log(\theta_{t-1}^p) + \varepsilon_t^p$$

with θ^{p*} the steady state level of substitution between intermediate goods, $\rho^p \in [0, 1)$ and ε_t^p a zero-mean i.i.d. innovation.

The productivity shock z_t follows:

$$\log(z_t) = (1 - \rho^z) \log(z^*) + \rho^z \log(z_{t-1}) + \varepsilon_t^z$$

where $z^* = 1$ is the steady state productivity level, $\rho^z \in [0, 1)$ and ε_t^z a zero-mean i.i.d. innovation.

Government consumption expenditure fluctuates around a constant:

$$G_t = G^* + \varepsilon_t^G$$

where ε_t^G is an exogenous i.i.d. shock.