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WHY FISCAL REGIMES MATTER FOR FISCAL SUSTAINABILITY ANALYSIS: AN APPLICATION TO FRANCE

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Abstract

This paper introduces a Regime-Switching Model-Based Sustainability test allowing for periodic (or local) violations of Bohn (1998, QJE)'s sustainability condition. We assume a Markov-switching fiscal policy rule whose parameters stochastically switch between sustainable and unsustainable regimes. We demonstrate that long-run fiscal sustainability not only depends on regime-specific feedback coefficients of the fiscal policy rule but also on the average durations of fiscal regimes. Evidence on French data suggests that the No-Ponzi game condition weakly holds in the long run, when accounting for regime switches. Accounting for a potential fiscal limit, we test whether estimated Markov-switching fiscal policy rule fulfills a debt-stabilizing condition depending on two measures of the interest rate on public debt. With the average apparent rate, fiscal data rejects the null hypothesis of an explosive public debt-to-GDP ratio. Still, we are unable to reject it using the average market rate, thus suggesting unstable dynamics of the public debt-to-GDP ratio.

JEL: E6, H6
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1 Introduction

Fiscal policy rules describing the reaction of primary balance to the initial level of public debt have been widely used to analyze fiscal sustainability. According to Bohn (1998)'s seminal contribution, primary public balance must increase after an increase of public debt-to-GDP ratio to ensure the sustainability of public finance, as defined by the government intertemporal budget constraint. This paper is motivated by the numerous empirical evidence of fiscal regimes during which public debt-to-GDP becomes explosive without any improvement in primary public balance. It raises critical questions regarding long-run fiscal sustainability since it appears that fiscal policy periodically violates Bohn's sustainability condition. Is a periodically unsustainable fiscal policy a threat to long-run sustainability of public finance? How long can fiscal policy be periodically unsustainable without violating its sustainability constraints in the long-run?

To our knowledge, only a few papers have addressed these difficulties about a regime-switching (or time-varying) fiscal policy rule and proposed a testing framework for long-run sustainability. In their seminal contribution Canzoneri et al. (2001) consider a time-varying fiscal policy rule and derive a necessary and sufficient condition such that the government intertemporal budget constraint holds in the long-run. Davig (2005) extends Wilcox (1989)'s unit-root testing procedure to a Markov-switching framework in which discounted debt can be periodically expanding. Finally, there is a literature on regime-switching monetary and fiscal policy rules that has successfully identified *local* equilibria in the data where fiscal policy (and monetary policy) is either "active" or "passive", following Leeper (1991). Still, these works cannot test whether fiscal policy *globally* satisfies its intertemporal budget constraint or the debt-stabilizing criterion in the long-run, given estimated regimes' transition probabilities and/or expected durations. Based on a Markov-switching monetary policy rule, Davig and Leeper (2007b) have proposed a Long-Run Taylor Principle such that the price-level is *globally* determined despite periodic violations of the Taylor monetary principle; but there is no equivalent proposition for a *globally* sustainable fiscal policy.

This paper introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes. We define unsustainable regimes by periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violating Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, so that fiscal regimes matter for global (in opposition with local) fiscal sustainability analysis.

The paper addresses the two usual concepts of long-run fiscal sustainability: the No-Ponzi game condition (related to the transversality condition) and the debt-stabilizing condition (related to the stationarity of the debt-to-GDP ratio). For each concept of fiscal sustainability, we demonstrate that long-run (or global) fiscal sustainability not only depends on regime-specific feedback coefficients of the Markov-switching fiscal policy rule but also on expected durations (or persistence, equivalently) of fiscal regimes. We derive necessary and sufficient conditions, for both No-Ponzi Game and debt-stabilizing conditions, such that sustainable regimes balance unsustainable regimes in the long-run. Consequently, fiscal policy can be *locally* unsustainable, with a periodically explosive public-debt-to-GDP ratio, and still be *globally* sustainable. The situation of a locally-explosive debt, which does not lead to global unsustainability or default can be related to the recent theoretical result by Blot et al. (2016).

We apply this test to France. As a Euro Area member state, France has neither a domestic monetary policy nor a lender of last resort. Both features make fiscal sustainability issues very acute. First, the French government cannot expect a domestic accommodative monetary policy when or after it implements a non-Ricardian fiscal policy. Second, sustainability issues cannot be disregarded and left to the management of the lender of last resort. As a result, we focus exclusively on Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price-level.[1] Based on a Markov-switching fiscal policy rule, estimated on annual data lying between 1963 and 2013, we identify sub-periods of unsustainability and sustainability of France's fiscal policy. Evidence suggests that the No-Ponzi game condition weakly holds in the long run: a unilateral test rejects the null of a Ponzi scheme at slightly above 5% whereas a bilateral test rejects the null only at slightly above 10%. Accounting for a potential fiscal limit, we test whether estimated Markov-switching fiscal policy rule fulfills a debt-stabilizing condition. The rejection of the null of an explosive debt-to-GDP ratio depends on the measure of the growth-adjusted real interest rate, hence on the choice of the interest rate on public debt: the apparent rate or the market rate. With the average apparent rate, fiscal data rejects the null hypothesis. On the contrary, using the average market rate, we cannot reject the hypothesis of an explosive debt-to-GDP ratio. Another outcome concerns the European institutional fiscal framework and how it relates to fiscal sustainability in France: despite being under an excessive deficit procedure between 2003 and 2007, findings of this paper report that fiscal policy was sustainable.

The rest of the paper is organised as follows. Section 2 briefly reviews the related literature on fiscal sustainability. Section 3 presents the extension of the Model-based approach of sustainability to regime switches and develops a new condition for fiscal sustainability. Section 4 deals with an application of the empirical methodology to French data. Section 5 concludes.

2 Related literature

The starting point for the analysis of fiscal rules is the government budget constraint: government noninterest spending G_t is financed by tax revenues T_t or by issuing debt B_t at price $(1+r_t)^{-1}$; thus government faces the following flow budget constraint:

$$(1+r_t)^{-1} B_t = (G_t - T_t) + B_{t-1} \tag{1}$$

where B_t is the end-of-period stock of public debt. Denote the price of a bond that matures j−periods ahead of t by $(1+r_{t,j})^{-1} = \prod_{i=1}^{j}(1+r_{t+i-1})^{-1}$; thus $(1+r_{t,1})^{-1} = (1+r_t)^{-1}$. Then, iterating on equation (1), we obtain:

$$B_{t-1} = S_t + \sum_{i=1}^{+\infty} \mathbb{E}_t\left[\frac{S_{t+i}}{1+r_{t,i}}\right] + \lim_{T\to+\infty} \mathbb{E}_t\left[\frac{B_{t+T}}{1+r_{t,T+1}}\right] \tag{2}$$

where $S_t \equiv T_t - G_t$ is the primary budget surplus.

A sustainable fiscal policy must satisfy the standard solvency condition according to which the initial stock of public debt must be backed by future expected present-value primary budget surpluses. The Present-Value Budget Constraint (PVBC henceforth) can be written as:

$$B_{t-1} \leq S_t + \sum_{i=1}^{+\infty} \mathbb{E}_t\left[\frac{S_{t+i}}{1+r_{t,i}}\right] \tag{3}$$

[1] Another consequence of the first feature is methodological: the Leeper (1991) and Davig and Leeper (2011)'s policy interaction framework is not applicable to France.

Following equation (2), the PVBC can be equivalent to a transversality condition on the expected present-value stock of public debt:

$$\lim_{T\to+\infty} \mathbb{E}_t\left[\frac{B_{t+T}}{1+r_{t,T+1}}\right] \leq 0 \tag{4}$$

This latest condition is the No-Ponzi Game condition (NPG). At equilibrium, both the PVBC (3) and the Transversality Condition (4) must hold with equality, preventing both lenders and government from playing a Ponzi scheme against each other, see Bohn (1995).

Seminal empirical investigations on fiscal sustainability proposed a testing framework based on the PVBC and the transversality condition, drawing on stationarity or cointegration properties of fiscal data (Hamilton and Flavin, 1986; Trehan and Walsh, 1988, 1991; Wilcox, 1989; Wickens and Uctum, 1993; Quintos, 1995). Still, the econometric analysis of fiscal sustainability has raised a number of issues and led to important criticisms by Bohn (1995, 1998, 2007). First, it usually involves restrictive assumptions on the real discounting rate. Bohn (1995) shows that in a stochastic economy the PVBC should require the introduction of the stochastic discount factor which is the common pricing kernel for all financial assets, under the complete market hypothesis. Bohn argues that these tests of fiscal sustainability with a constant safe interest rate (Hamilton and Flavin, 1986) or the actual interest rate (Wilcox, 1989) rather than the stochastic discount factor are misleading since they assume that lenders are risk-neutral and/or that there is no uncertainty. Second, based on Barro (1979) tax-smoothing model, Bohn (1998) argues that ignoring cyclical components of primary surplus (like output gap and government cyclical spending) induces a bias in unit-root test. Controlling for cyclical components, Bohn (1998) provides evidence that U.S. public debt-to-GDP ratio is actually mean-reverting. Third, Bohn (2007) formulates a general criticism: usual econometric restrictions[2] are not necessary for the PVBC to hold[3] and lead to "absurdly weak" sustainability because debt-to-GDP would violate any upper bound implied by the existence of a fiscal limit. As a result, Bohn (2007) acknowledges that an upper bound on primary surplus requires a stationary public debt-to-GDP for fiscal sustainability to hold; but this condition would stem from additional economic considerations, such as the existence of distortionary taxation, implying a dynamic "Laffer curve" and a fiscal limit, but not from the PVBC *per se*. [4]

As an alternative to the econometric analyses *à la* Hamilton-Flavin, Bohn (1998) suggests analyzing fiscal sustainability through the lens of fiscal policy rules, or fiscal reaction functions, in a simple general equilibrium model. Basically, he assumes the following framework composed of a linear fiscal rule (5), the government one-period budget constraint (6) and the Euler equation (7) derived from the consumer's optimal choice:

$$s_t = \gamma b_{t-1} + \mu_t \tag{5}$$

$$b_t = \frac{1+r_t}{1+y_t} b_{t-1} - (1+r_t)s_t \tag{6}$$

$$(1+r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \tag{7}$$

[2]Such that the public debt must be stationary (Hamilton and Flavin, 1986), or difference-stationary, implying cointegration between revenues and spending (Trehan and Walsh, 1988, 1991), or at least integrated of order two (Quintos, 1995).

[3]Bohn (2007) shows that a m-th order integrated debt-to-GDP process is actually sufficient for the PVBC and TC to hold, for any arbitrary high m. Proof can be summarized as follows: for any discount factor $\rho < 1$, the transversality condition is exponential in the time horizon n and the conditional expectation of a m-th order integrated variable is at most a polynomial of order m. Since exponential growth dominate polynomial growth in the long-run, given $\rho < 1$, then the transversality condition holds, see Bohn (2007).

[4]Research about the upper-bound of primary surplus has been recently explored by Bi (2012); Bi and Traum (2012); Davig et al. (2011); Daniel and Shiamptanis (2013). Daniel and Shiamptanis show that stationarity and cointegration restrictions are necessary for fiscal sustainability when assuming existence of a fiscal limit. Existence of a fiscal limit (i.e. an upper bound on primary balance-to-GDP and on public debt-to-GDP) requires a sustainability criterion ensuring that public debt must be stable around a long-run value compatible with fiscal limit.

where s_t is the primary surplus-to-GDP ratio, b_t is the end-of-period public debt-to-GDP ratio and finally μ_t is a vector including all cyclical components of primary surplus (e.g. output gap, temporary public spending), plus a constant and an error term. The utility function $u(.)$ is assumed to be stricly increasing ($u'(.) > 0$) and concave ($u''(.) < 0$); β is the subjective discount factor and C_t is the level of consumption. Thus, Bohn seeks a condition on the fiscal rule feedback parameter γ such that the No-Ponzi Game (NPG) condition holds. He argues that if there is a strictly positive feedback effect of public debt on primary surplus, that is

$$\gamma > 0 \tag{8}$$

then fiscal policy satisfies the NPG condition (4). If one considers a stricter sustainability concept, such as a debt-stabilizing fiscal policy rule, then this feedback effect should be larger than the growth-adjusted real average interest rate on public debt [5] such that

$$\gamma > r - y \tag{9}$$

with r the average real interest rate and y the average growth rate of real GDP. Hence, for a debt-stabilizing fiscal policy, after substituting (5) into (6) and taking the expectation, public debt converges to its unconditional mean:

$$\mathbb{E}[b_t] \simeq \frac{-(1+r)\bar{\mu}}{(1+r)\gamma - \frac{r-y}{1+y}} \tag{10}$$

where $\bar{\mu} < 0$ is the mean value of μ_t. Bohn's framework for fiscal sustainability analysis is often labeled "Model-Based Sustainability" (henceforth MBS) because it relies on general equilibrium conditions (i.e. Euler equation) and on an explicit fiscal policy rule, see Bohn (1995, 1998). MBS analysis has been shown to be empirically powerful in the case of US fiscal policy on long-run data (Bohn, 1998, 2008). On international panel data, Mendoza and Ostry (2008) find evidence that fiscal policy is "responsible" (i.e. evidence of a strictly positive feedback rule).

Recent empirical research has introduced two types of nonlinear specifications. One strand of the literature specifies fiscal policy rules as polynomial functions of public debt-to-GDP ratio, i.e. including quadratic and cubic terms (Bohn, 1998). This specification is motivated by the idea that primary surplus may either react more to lagged public debt or on the contrary may become "flatter", at higher public debt levels. This approach has been followed by Ghosh et al. (2013a,b) to account for "fiscal fatigue" where they derive fiscal limits as the maximum level of public debt beyond which primary balance can no longer adjust to stabilize debt. Another strand of empirical research considers time-varying fiscal policy rules. The assumption that simple linear policy rules (either monetary or fiscal) are constant overtime seems not convincing regarding empirical evidence, given multiple evidence of "structural breaks" or "regime changes", thus motivating a regime-switching approach in empirical studies.[6] In particular, empirical literature on regime-switching fiscal policy rules has produced evidence that fiscal policy rules vary overtime and may be better described by "fiscal regimes", see Favero and Monacelli (2005); Chung et al. (2007); Davig and Leeper (2007a, 2011); Bianchi (2012); Burger and Marinkov (2012); Afonso and Toffano (2013). This literature generally identifies sub-periods during which fiscal policy does not stabilize public debt by increasing the primary surplus after an increase in public debt, and sometimes even displays a negative feedback effect of initial public debt on primary surplus.[7]

[5] If one considers a fiscal rule with variables in absolute level rather than as share of GDP, then this feedback should be larger than the real average interest rate on public debt. This is basically what Leeper (1991) finds when describing the stability conditions of an active monetary/passive fiscal regime.

[6] For monetary policy, see Clarida et al. (2000); Auerbach (2002); Lubik and Schorfheide (2004), among others.

[7] In some specifications, the fiscal rule is modelled as the reaction of the aggregate tax rate, rather than the primary surplus, as in Davig and Leeper (2011) for example.

The former literature on regime-switching monetary and fiscal policy rules builds on Leeper (1991)'s seminal contribution, which developed a set of conditions for *local* equilibrium determinacy stemming from the properties of the monetary and fiscal rules. Fiscal policy is *passive* under the debt-stabilizing condition (9), *active* otherwise[8]. Recent research on fiscal policy (Bi, 2012; Bi and Leeper, 2013) has explored consequences of regime-switching fiscal policy to derive an endogenous and stochastic fiscal limit. This literature analyzes fiscal sustainability as the sovereign default probability, computed from the fiscal limit distribution[9], rather than as generalized conditions on the regime-switching fiscal rule. Davig and Leeper (2007b) define the long-run Taylor monetary principle, based on a Markov-switching Taylor rule, allowing for periodic (or local) violations of the Taylor principle. But, to our knowledge, none has proposed and tested analogous conditions on a regime-switching fiscal rule, such that NPG and debt-stabilizing conditions hold in the long-run. In this respect, this paper's motivation is similar to Davig and Leeper (2007b).

Finally, this paper is motivated by two important contributions in the field of fiscal sustainability analysis. Canzoneri et al. (2001) study a particular time-varying fiscal policy rule in which public debt feedback effect on primary surplus is positive or null. They show primary surplus only has to react positively to public debt on an infrequent basis but "infinitely often" in order to satisfy the government intertemporal budget constraint. This analysis is restrictive in at least two respects. First, assuming primary surplus does not react negatively to initial public debt is a critical assumption, at odds with some empirical evidence on regime-switching policy rules (Favero and Monacelli, 2005; Davig and Leeper, 2007a, 2011; Afonso and Toffano, 2013). Second, the sustainability condition does not ensure a stationary public debt-to-GDP ratio, which is probably the relevant fiscal sustainability condition when the economy faces a fiscal limit. In contrast, we propose a necessary and sufficient condition on the regime-switching fiscal policy rule such that fiscal policy stabilizes public debt-to-GDP ratio in the long-run. Alternatively, Davig (2005) proposes a modified version of Wilcox (1989)'s unit-root testing framework using a Markov-switching model, which takes account for episodes of periodically expanding discounted public debt. Still, this approach is inherently subject to the criticisms addressed by Bohn (1995, 2007) to the econometric analysis of fiscal sustainability. In particular, unit-root testing does not provide any information about fiscal policy behavior since it does not involve an explicit model of fiscal policy.

3 Theory: Regime-Switching Model-Based Sustainability

We assume a stochastic real endowment and cashless economy composed of a representative rational consumer and a government. By assuming a real cashless economy, we implicitly assume that monetary policy has full control over the price-level and inflation dynamics. Using the terminology of the Fiscal Theory of Price-Level (Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2005), we only consider Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price-level.

Total output Y_t is following a unit-root with drift:

$$\ln Y_t = y + \ln Y_{t-1} + \varepsilon_t^y \tag{11}$$

[8] Condition on monetary policy is the Taylor Principle: monetary policy is labeled "active" when it reacts agressively to inflation (i.e. the Taylor principle holds) and "passive" otherwise. From these two conditions, Leeper (1991) identify four local regimes: Monetary regime (AM/PF), Fiscal regime (PM/AF), Indeterminacy regime (PM/PF) and an Explosive regime (AM/AF).

[9] Fiscal limit distributions are obtained by numerical approximation of the decision rule in calibrated or sometimes estimated RBC models.

where $y > 0$ is the long-run growth rate of output and ε_t^y is a random shock to the growth rate.

Representative consumer's preferences are represented by the utility function $u(.)$ which is strictly increasing ($u'(.) > 0$) and concave ($u''(.) < 0$) and a subjective discount factor β. At each period, consumer chooses consumption C_t and buys public bond B_t at a price $(1+r_t)^{-1}$ in order to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the following budget constraint:

$$C_t + (1+r_t)^{-1} B_t = B_{t-1} + Y_t - T_t$$

and transversality condition:

$$\lim_{T \to +\infty} \mathbb{E}_t \frac{B_{t+T}}{1 + r_{t,T+1}} \geq 0$$

with $(1+r_{t,T+1})^{-1}$ being the T-period ahead real interest rate. First order conditions of the representative consumer's maximization program yield the standard Euler equation:

$$(1+r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \tag{12}$$

Equation (12) defines the stochastic discount factor $Q_{t,1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$ which is the common pricing kernel of any asset in the economy. Hence, a j−period public bond has a price $(1+r_{t,j})^{-1} = \mathbb{E}_t Q_{t,j}$ with $Q_{t,j} = \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$.

Government spends G_t and collects lump-sum taxes T_t. At each start of period t, government carries one-period public bonds B_{t-1} and it will issue B_t at a price $(1+r_t)^{-1}$ at end of period. Thus, government faces the following one-period budget constraint:

$$(1+r_t)^{-1} B_t = (G_t - T_t) + B_{t-1} \tag{13}$$

with $S_t \equiv G_t - T_t$ representing the primary budget balance. Under balanced growth, all variables in level grow at rate y_t, thus we rewrite the government budget constraint in terms of output ratios:

$$b_t = \frac{1+r_t}{1+y_t} b_{t-1} - (1+r_t) s_t \tag{14}$$

where b_t is the end-of-period debt-output ratio, s_t is the primary surplus-output ratio, r_t and y_t are respectively the real interest rate and the growth rate of real output.

Preventing government from running a Ponzi scheme against its creditor implies the following Present-Value Budget Constraint (PVBC). Following Bohn (1995), we write the PVBC using the stochastic discount factor in order to account for uncertainty and consumer's risk-aversion:

$$B_t = \sum_{i=0}^{+\infty} \mathbb{E}_t \left[Q_{t,i} S_{t+i} \right] \tag{15}$$

which is equivalent to the following transversality condition (TC):

$$\lim_{T \to +\infty} \mathbb{E}_t \left[Q_{t,T+1} B_{t+T} \right] = 0 \tag{16}$$

Both the PVBC and TC must hold with equality since the representative consumer cannot run a Ponzi Scheme against government.

We assume the following Markov-switching fiscal policy rule:

$$s_t = \gamma(z_t)b_{t-1} + \mu_t(z_t) \tag{17}$$

Regime-switching parameter $\gamma(z_t)$ represents the feedback effect of the initial public debt-output ratio b_{t-1} on primary surplus-output ratio *conditional on* fiscal regime z_t. Fiscal regimes are then defined as:

$$\gamma(z_t) = \begin{cases} \gamma_S > 0 & \text{if} \quad z_t = 1 \quad \text{(Sustainable Regime)} \\ \gamma_{NS} \leq 0 & \text{if} \quad z_t = 0 \quad \text{(Unsustainable Regime)} \end{cases} \tag{18}$$

During sustainable regimes ($\gamma_S > 0$) primary balance improves following a debt increase while it does not improve or even worsen during unsustainable regimes ($\gamma_{NS} \leq 0$)[10]. Finally, we define $\mu_t(z_t)$ by:

$$\mu_t(z_t) = \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon_t^s \tag{19}$$

where $\hat{y}_t$ is the output gap, $\hat{g}_t$ is temporary public spending, $\alpha(z_t)$ is a regime-switching constant, $\sigma(z_t)$ is the regime switching standard-error associated to an i.i.d distributed shock $\varepsilon_t^s \sim \mathcal{N}(0,1)$.

We assume regime-switching to be stochastic and exogenous, following a hidden two-state Markov process z_t describing fiscal regimes. The use of a Markov-switching model rather than endogenous or threshold-switching models represents an agnostic way of modelling regime changes of fiscal policy without making any critical assumption about what drives fiscal regime shifts.

Define $\gamma = (\gamma_S \ \gamma_{NS})$ a row-vector containing regime-specific parameters and $Z_t = (z_t \ 1 - z_t)^T$ a column-vector associated to the Markov process z_t. Hence, we can define the scalar $\gamma(z_t)$ by:

$$\gamma(z_t) \equiv \gamma Z_t = \begin{pmatrix} \gamma_S & \gamma_{NS} \end{pmatrix} \times \begin{pmatrix} z_t \\ 1 - z_t \end{pmatrix} \tag{20}$$

Markov process z_t is associated to a transition matrix P which elements are $p_{ij} \equiv \mathbb{P}(z_t = i | z_{t-1} = j)$ for all $(i,j) \in \{0,1\}$ such that:

$$Z_t = PZ_{t-1} + v_t \quad \text{with} \quad v_t \equiv Z_t - \mathbb{E}_{t-1}[Z_t] \tag{21}$$

We assume z_t to be an ergodic Markov process[11] implying that $\mathbb{E}_t Z_{t+j} = P^j Z_t$ converges to a unique ergodic distribution π:

$$P^j Z_t \underset{j \to +\infty}{\longrightarrow} \pi \tag{22}$$

where $\pi = (\pi_S \ \pi_{NS})^T$ is the column-vector of ergodic probabilities associated to each fiscal regime. Ergodic probabilities are defined by:

$$\pi_i = \frac{1 - p_{jj}}{(1 - p_{ii}) + (1 - p_{jj})} \tag{23}$$

[10] Canzoneri et al. (2010, p. 959) discuss empirical results of Davig and Leeper (2007a, 2011) and note that a negative coefficient on lagged debt in the fiscal rule may be difficult to interpret since "regardless of whether the fiscal rule is Ricardian or non-Ricardian, we would expect a positive estimated coefficient". Indeed, Cochrane (2001) shows there exist a positive correlation between primary surplus and initial debt at *equilibrium*, even when fiscal policy is active (with primary surplus following an AR(1) process). Still, empirical research on regime-switching fiscal policy rules provides some evidence of periodic *negative* feedback effect, see Davig and Leeper (2011) and Afonso and Toffano (2013) for instance; these empirical results motivate our specification of unsustainable fiscal regimes by $\gamma_{NS} \leq 0$.

[11] Any Markov process is ergodic as long as $p_{ii} < 1$ and $p_{ii} + p_{jj} > 0$ for all $(i,j) \in \{0,1\}$ (Hamilton, 1994, Chap. 22), meaning there is no absorbing state.

for all $(i, j) \in \{0, 1\}$. Hence, using equations (20) and (22), the conditional expectation at time t of feedback parameter $\gamma(z_t)$ converges toward its *unconditional* expectation, i.e. ergodic (or long-run) value:

$$\mathbb{E}_t[\gamma(z_t + j)] = \gamma P^j Z_t \underset{j\to+\infty}{\longrightarrow} \gamma\pi \tag{24}$$

Following Bohn (1998) and Aldama (2015), we derive a necessary and sufficient condition on the sequence $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$ such that (15) and (16) hold. Denoting the j-periods growth-adjusted stochastic discount factor by

$$\tilde{Q}_{t,j} \equiv Q_{t,j} \prod_{i=0}^{j-1} (1 + y_{t+i}) \tag{25}$$

then (16) can be expressed in terms of debt-output ratio by:

$$\lim_{T\to+\infty} \mathbb{E}_t\Big[\tilde{Q}_{t,T+1} b_{t+T}\Big] = 0 \tag{26}$$

In a dynamically efficient economy and provided that innovations μ_t are bounded, a necessary and sufficient condition such that transversality condition (26) holds is:

$$\gamma\pi > 0 \tag{27}$$

with $\gamma\pi \equiv \gamma_S\pi_S + \gamma_{NS}\pi_{NS}$ being the unconditional expectation of $\gamma(z_t)$. Proof follows from Bohn (1998) and Aldama (2015) by using (17) and iterating on (14). To understand this condition, it is useful to consider the following approximation of tranversality condition when $T \to +\infty$:

$$\mathbb{E}_t\Big[\tilde{Q}_{t,T+1} b_T\Big] \approx (1 - (1 + y)\gamma\pi)^T b_t \tag{28}$$

Following Bohn (2008), consider a Ponzi Scheme such that $\{s_t\}_{t=0}^{\infty} = 0$. This Ponzi Scheme implies debt-output ratio growing at a rate $\frac{r_t - y_t}{1+y_t}$. As a consequence the limit value of future discounted debt-output ratio is equal to initial debt-output ratio (which violates the transversality condition):

$$\mathbb{E}_t\Big[\tilde{Q}_{t,T+1} b_{t+T}\Big] = b_t \tag{29}$$

Thus, $\gamma\pi > 0$ implies the reduction of $\mathbb{E}_t\Big[\tilde{Q}_{t,T+1} b_{t+T}\Big]$ by a factor $(1 - (1 + y)\gamma\pi)^T$ relative to a Ponzi Scheme. Saying it differently: the average growth rate of debt-income ratio is reduced by a factor $(1 - (1 + y)\gamma\pi) > 0$. See appendix A.1 for detailed proof of condition (27).

Condition (27) states that a regime-switching fiscal policy has to satisfy the NPG condition *on average*, that is, sustainable regimes have to be frequent enough to balance unsustainable regimes in the long-run. Using the definition of ergodic probabilities (23) and denoting expected duration of regimes by $d_i = \frac{1}{1-p_{ii}}$, we can express condition (27) by:

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} \tag{30}$$

Ruling out a Ponzi Scheme means that the longer unsustainable regimes are *vis-à-vis* duration of sustainable regimes, the larger primary deficits are during unsustainable regimes, then the larger the required reaction of primary surplus to debt has to be during sustainable regimes. Still, provided (30) holds, fiscal policy can be periodically unsustainable *and* satisfying its PVBC.

A stronger constraint on fiscal policy would require that debt-output ratio must be stationary. Assuming an upper-bound on the primary surplus-output ratio such that $s_t \leq s^{max}$[12] implies the existence of a maximum level of debt-output ratio, i.e. a fiscal limit, such that:

$$b^{max} = s^{max} \sum_{i=0}^{+\infty} \mathbb{E}_t \Big[Q_{t,i} \Big] \tag{31}$$

Thus, for $b_t > b^{max}$ fiscal policy would be necessarily running a Ponzi Scheme against creditors. Since condition (30) does not rule out explosive path for debt-output ratio, a necessary and sufficient condition for fiscal sustainability would be a debt-stabilizing rule around a deterministic steady-state level below the fiscal limit. A regime-switching fiscal rule implies that debt-output ratio follows a Markov-switching autoregressive process, defined by equations (17) and (14):

$$b_t = \phi(z_t) b_{t-1} + u_t(z_t) \tag{32}$$

where $\phi(z_t) = \frac{1+r_t}{1+y_t}\Big(1 - (1+y_t)\gamma(z_t)\Big)$ and $u_t(z_t) = -(1+r_t)\mu_t(z_t)$. A necessary and sufficient condition for strict stationarity of process (32) requires that:

$$\gamma\pi > r - y \tag{33}$$

See appendix A.2 for a detailed proof of condition (33). Thus, to stabilize public debt in the long-run, a regime-switching fiscal policy must verify:

$$\gamma_S > \left|\gamma_{NS}\right| \frac{d_{NS}}{d_S} + (r-y)\frac{d_S + d_{NS}}{d_S} \tag{34}$$

Provided conditions (33) or (34) hold, then public debt-output ratio converges to its unconditional mean:

$$\mathbb{E}[b_t] = \frac{-(1+r)\mathbb{E}[\alpha(z_t)]}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \tag{35}$$

where $\mathbb{E}[\alpha(z_t)] < 0$ is the ergodic value of $\alpha(z_t)$. As long as growth-adjusted real interest rate is positive (under a dynamically-efficient economy), a debt-stabilizing condition is stricter than the NPG condition. During sustainable regimes, the required reaction of primary surplus to initial debt must be large enough to compensate for both primary deficits during unsustainable regimes, weighted by the ratio of expected durations, and the growth-adjusted real interest rate, weighted by the inverse fraction of (expected) time spent in sustainable regimes. On the contrary, when $r < y$, condition (34) could eventually imply government is violating NPG condition (30) which is the minimum requirement for fiscal sustainability. Since history provides numerous examples of $r < y$, this illustrates why testing stationarity of debt-output ratio may sometimes be misleading as a test of fiscal sustainability. As a result, NPG condition and debt-stabilizing condition would be complements rather than substitutes: a stationary public debt-output ratio does not always rule out Ponzi Schemes.

The assumption of the existence of different fiscal regimes may, in general, imply that public debt-output ratio can periodically follow an explosive path. To see why, let us consider the example of Canzoneri et al. (2001) and assume $\gamma_{NS} = 0$. We find exactly the same proposition they made: based on equation (30), any infrequent $\gamma_S > 0$ would be sufficient to rule out Ponzi Schemes. Yet this equilibrium does not ensure a bounded debt-output ratio, that is public debt is $I(1)$. For a bounded debt-output ratio, assuming $r - y > 0$ and $\gamma_{NS} = 0$, a regime-switching fiscal policy must satisfy the following condition,

[12] This assumption would be justified by the existence of distortionary taxation and the top of a Laffer curve, or even by the inability to reduce public spending below some level for political reasons, see Bi (2012); Bi and Leeper (2013).

from equation (34):

$$\gamma_S > (r-y)\frac{d_S + d_{NS}}{d_S} \tag{36}$$

For any $\gamma_{NS} < 0$ the condition on γ_S would be stronger. Under a regime-switching debt-stabilizing fiscal policy, debt-output ratio becomes periodically explosive, and explosive regimes can be really frequent without necessarily implying debt-output is globally non-stationary.

Periodic explosive dynamics of public debt, implied by existence of fiscal regimes, has critical consequences on regime-switching policy rules, in particular on estimates of the constant $\alpha(z_t)$. Rewriting equation (17) in terms of deviations of primary balance and public debt from their respective target values $s^*(z_t) = (s^*_S, s^*_{NS})$ and $b^*(z_t) = (b^*_S, b^*_{NS})$:

$$s_t - s^*(z_t) = \gamma(z_t)\Big(b_{t-1} - b^*(z_t)\Big) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon^s_t \tag{37}$$

gives $\alpha(z_t)$ equal to:

$$\alpha(z_t) = s^*(z_t) - \gamma(z_t)b^*(z_t) \tag{38}$$

From the existence of periodic explosive dynamics of public debt, we can deduce that α_{NS} (i.e. the value of the constant during unsustainable regimes) would be expected to be equal to zero, in the case $\gamma_{NS} < 0$. First, unsustainable fiscal regime and implied explosive debt-output ratio dynamics are not compatible with any debt-output target value, hence b^*_{NS} must disappear from equation (37), which is mathematically equivalent to $b^*_{NS} = 0$. Second, during unsustainable regimes and when $\gamma_{NS} < 0$, primary balance is necessarily non-stationary, since it is (negatively) correlated to $\{b_t\}$ which is itself non-stationary. Thus, s^*_{NS} would be equal to zero as well, in the case of a strictly negative γ_{NS}; otherwise, if $\gamma_{NS} = 0$, $\{s_t\}$ is $I(0)$ and s^*_{NS} could be significantly different from zero. During sustainable regimes, both s^*_S and b^*_S would be expected to be different from zero. In particular, under a debt-stabilizing fiscal rule, we would expect s^*_S to be equal to the debt-stabilizing primary surplus ratio, for a stationary debt-output target ratio b^*_S:

$$s^*_S = (r-y)b^*_S \tag{39}$$

which implies:

$$\alpha_S = \Big((r-y) - \gamma_S\Big)b^*_S < 0 \tag{40}$$

provided that condition (33) holds, which would account for negative estimates of α_S but also $\mathbb{E}[\alpha(z_t)] = \pi_S\alpha_S < 0$ if $\gamma_{NS} < 0$.

4 Empirical analysis

We apply Regime-Switching MBS analysis to French data. Recent empirical investigation about the fiscal sustainability of French public finances has given rise to contradictory outcomes: some recent papers (Afonso, 2005; Lamé et al., 2014; Schoder, 2014) either did not find evidence of a sustainable fiscal policy in France, or reached mixed evidence (Afonso and Jalles, 2016; Chen, 2014; Fincke and Greiner, 2012; Weichenrieder and Zimmer, 2014); an earlier study even found that fiscal policy in France was sustainable (Greiner et al., 2007). The observation that French sovereign interest rates have been historically low during the European sovereign-debt crisis also conveys some information about lenders' seemingly expectations that France's fiscal policy is on a sustainable path.

We argue that this apparent contradiction can be attributed to the lack of account for regime-switching fiscal policies in empirical papers. To assess our argument, we develop a two-stage empirical strategy.

First, we estimate various fiscal rules following Bohn's MBS tests. From these tests, we conclude that French public debt is not sustainable. Second, we estimate a Markov-switching fiscal policy rule and perform a Regime-Switching MBS test. The latter outcomes challenge the former results obtained with standard techniques. We conclude that omitting fiscal regime-switches may lead to reject mistakenly French sustainability. A broader, model-based and non-linear, approach, would better fit the reality of fiscal policies in different states of nature and bring two informations: not only overall (un)sustainability but also dated sub-periods of (possible) unsustainability.

4.1 Dataset

The choice of annual data is guided by two arguments. First, fiscal sustainability can only be appreciated in the long-run since governments are (almost surely) infinitely-lived agents, unlike households, firms or private financial institutions. PVBC or stationary debt-to-GDP ratios might only be satisfied in the long-run – over half a century, or more. Regarding data availability for France, we are forced to renounce using *true* quarterly data which are available from 1995-Q4 only for public debt. It is possible to build a quarterly measure for public debt using interpolation methods and quarterly government budget balance[13]. Still, a second argument prevents us from using quarterly data: fiscal policy decisions are taken on an annual basis, despite some infra-annual adjustments. Thus, using quarterly data may result in spurious results as it may add noise to the *true* response of primary budget balance to initial stock of debt.

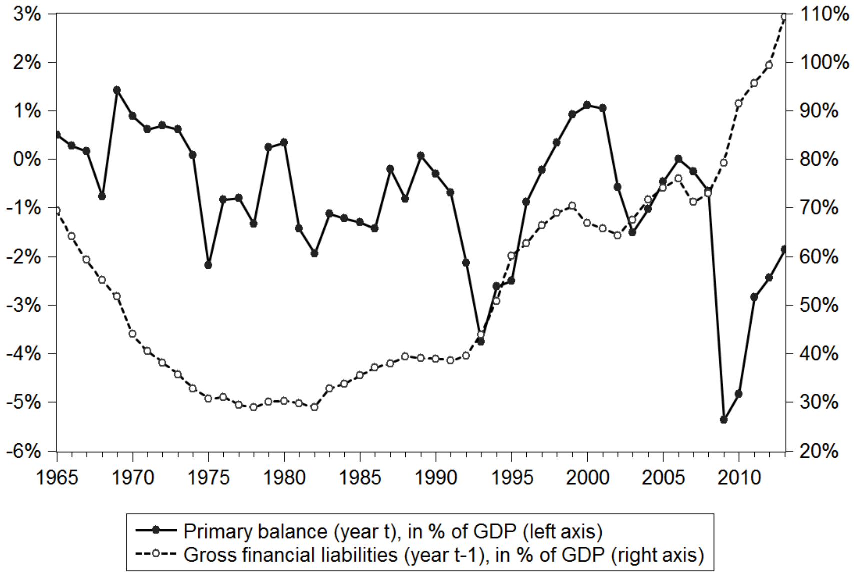
This paper uses the longest time series available for French public debt. Indeed, because of changes in national accounts systems, it is relatively hard to find historical data on French public debt. Most of available time series (in particular, those using Maastricht debt definitions) start by 1978. The IMF Historical Public Debt Database (HPDD) proposes a long-run time series for public debt, but still with missing observations for years 1978 and 1979, because of national accounting issues. Thus, regarding public debt, we use the OECD government total gross financial liabilities rather than the Maastricht definition of gross public debt since the OECD series goes back to 1969. Second, we complete this series by backward induction using the government overall budget balance which is approximatively equal to the variation of gross financial liabilities. Regarding time convention in national accounts, public debt stock is the *end-of-period* stock of debt.

Overall budget balance and primary budget balance (budget balance *minus* interests paid) are taken from OECD database for years 1977-2013; observations for years 1963 to 1977 are completed using data collected by Creel and Le Bihan (2006), from French National Institute for Statistics and Economic Studies (INSEE). We build time series for output gap and temporary government spending by detrending and removing the cyclical component of real GDP and real government spending using the HP filter. Regarding the estimation of output gaps, many competing techniques are available and their relative strengths and weaknesses still discussed (see Cotis et al. (2005), for a survey of estimation methods). Our choice of the HP-filtered method has been motivated by its easiness, fastness and recent use by Fincke and Greiner (2012) and, with more sophistication, by Borio et al. (2014). Finally, our dataset covers 51 years of annual data, from 1963 (1962, for gross public debt) to 2013.

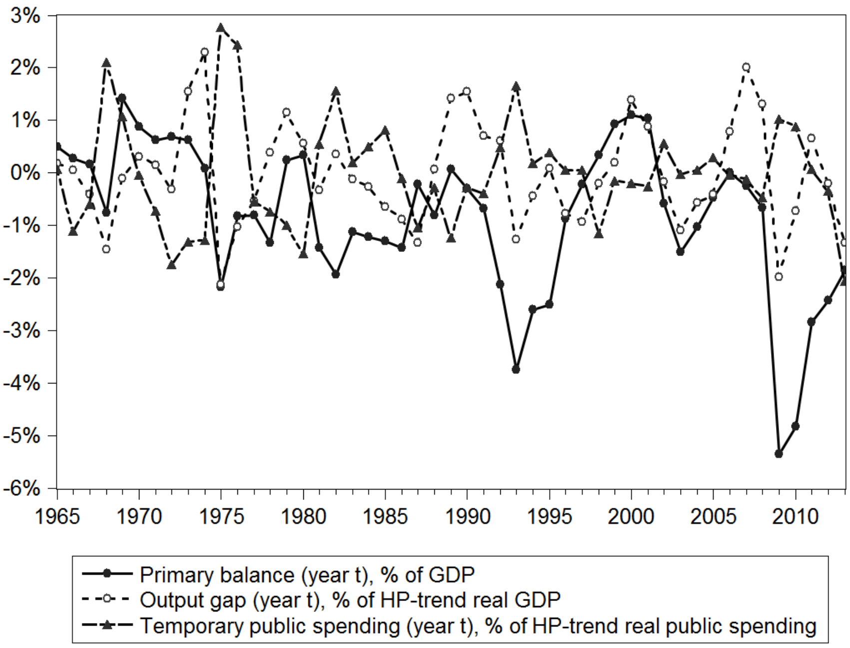
[13] Lamé et al. (2014) report the use of recalculated quarterly data of net French public debt, though on a shorter time span (1980Q1-2007Q4) than the one used in this paper.

Figure 1: Dataset overview, France (1965-2013)

(a) Primary balance and initial public debt



(b) Primary balance, output gap and temporary public spending



4.2 Model-Based Sustainability tests

We estimate various specifications of a standard fiscal policy rule searching for a strictly positive and significant feedback effect between lagged level of public debt and primary surplus, in percent of GDP. We produce OLS estimates as a benchmark for comparison with Regime-Switching estimates. We specify the following fiscal rule, based on equation (5):

$$s_t = \gamma b_{t-1} + \beta X_t + \varepsilon_t \tag{41}$$

where X_t is a vector of control variables. It includes a constant, output gap $\hat{y}_t$, cyclical government spending $\hat{g}_t$, as suggested by Bohn (1998). Then, we include a dummy variable FinCrisis_t equal to one for years 2008–2013 in order to account for severe recession years. We also intend to account specifically for years of increasing debt-to-GDP with a dummy variable RisingDebt_t and an interaction term $\text{RisingDebt}_t \times b_t$. Therefore, we account for potential non-linearities regarding the level of debt and its evolution, as Bohn (1998) and Mendoza and Ostry (2008). Finally, we account for a potential deterministic time trend, as suggested by previous unit-root and stationarity tests. In presence of serial correlation in the residuals, we correct for serially correlated residuals of order one or two, depending on the model estimated.

Table 1: Constant-parameters Fiscal policy rules, OLS estimates

Dependent variable: s_t	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Initial debt b_{t-1}	-0.0121	-0.0058	-0.0160	-0.0191	-0.0066	0.0300*	0.0457
	(-0.71)	(-0.35)	(-0.89)	(-1.01)	(-0.30)	(-1.72)	(1.31)
Constant	-0.0025	-0.0052	-0.0028	-0.0009	-0.0069	-0.0065	-0.0064
	(-0.24)	(-0.57)	(-0.26)	(-0.08)	(-0.57)	(-0.96)	(-0.59)
Output gap $\hat{y}_t$	0.4190***	0.3807***	0.4263***	0.4387***	-0.3003***	0.4527***	0.4449***
	(3 .38)	(3.23)	(3.47)	(3.50)	(-3.12)	(-3.56)	(3.40)
Temporary spending $\hat{g}_t$	-0.4053***	-0.3667***	-0.3975***	-0.3910***	-0.4117***	-0.3763***	-0.3684***
	(-3.18)	(-3.09)	(-3.17)	(-3.06)	(-3.20)	(-2.91)	(-2.87)
FinCrisis_t	.	-0.0179***	.	.	.	-0.0160**	.
		(-2.95)				(-2.25)	
RisingDebt_{t-1}	.	.	0.0035	.	0.0084	.	0.0123
			(1.33)		(-1.16)		(1.68)
$\text{RisingDebt}_{t-1} \times b_{t-1}$	.	.	.	0.0051	-0.0109	.	-0.0219
				(0.95)	(-0.74)		(-1.44)
Trend	.	.	.	.	.	-0.0006**	-0.0009
						(-2.16)	(-1.49)
DW	1.98	1.99	1.95	1.94	1.99	1.87	1.79
Adj. R^2	0.70	0.75	0.70	0.70	0.70	0.73	0.72
Observations	49	49	48	48	48	50	49

Notes: t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). Models (1)–(5) are controlling for second-order serial correlation in the residuals. Models (6) and (7) control for first-order serial correlation in the residuals.

Table 1 presents the results. Based on OLS estimation of constant-parameters fiscal policy rules, we find no evidence in favor of fiscal sustainability[14]. Models (1)–(5) give no positive feedback effect, but rather negative though non-significant estimates for γ. Unit-root and stationarity tests (available upon request) conclude to the presence of deterministic time trends respectively negative in s_t and positive in b_t. Thus we control for a deterministic trend in equation (41), in models (6) and (7). Model (6) shows a positive, significant at 10% level, feedback response of primary surplus to initial debt. But γ is no longer significant when including the dummy RisingDebt_t and the interaction term $\text{RisingDebt}_t \times b_t$.

[14]This result contrasts with Fincke and Greiner (2012) who find a significant positive reaction of the primary surplus to debt. Two differences with our approach are worth mentioning. First, Fincke and Greiner do not strictly reproduce Bohn' fiscal rule: they limit cyclical public spending to spending related to the social insurance system though some of these expenditures may be structural; second, their sample is shorter (1970-2008) than ours.

Moreover, deterministic trends enter negatively in both equations which implies $\lim_{t\to+\infty} s_t = -\infty$, thus obviously violating the PVBC.

4.3 Regime-switching Model-Based Sustainability test

We estimate the following Markov-switching fiscal rule by direct maximisation of the log likelihood (Hamilton, 1989):

$$s_t = \gamma(z_t)b_{t-1} + \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + u_t \quad (42)$$

where s_t is primary balance-to-GDP ratio, b_{t-1} is public debt-to-GDP ratio at end of period $t-1$, $\hat{y}_t$ and $\hat{g}_t$ are measures of output gap and temporary real government spending. All parameters can periodically shift between two values, according to a hidden two-state Markov-process z_t[15].

Numerical optimization of the log likelihood function is usually difficult, raising identification issues, so we choose the following estimation strategy. First, we estimate the most general model, allowing *all parameters, including error variance* to switch between regimes 1 and 2, thus being agnostic on the true structural form of the regime-dependent fiscal rule. At this stage, if the maximization algorithm converges, we can already appreciate how precise the resulting estimates are, both across regimes and in the long-run through the computation of the ergodic value of each parameter. This can be achieved through basic t-statistics and F-statistics analysis. Using regime-switching techniques, we also look carefully at estimated regimes, especially if they are consistent with historical knowlegde on fiscal policy shifts, and if they are sufficiently persistent, regarding the timing of fiscal policy.

If any subset of parameters were non significantly different from zero in *both regimes* it would be a strong motivation to estimate a restricted model in which this subset of parameters would be regime-invariant. Thus, if any restricted model can be successfully estimated, that is, if the maximization algorithm successfully converges, then the same procedure as before can be applied: checking if results are significant across regimes, in the long-run and if estimated regimes are both enough consistent with historical knowlegde on fiscal policy and sufficiently persistent.

Given the short length of the sample, we acknowledge that MLE estimates must be considered with caution. Still, in the presence of unit-root in the debt-to-GDP ratio, with stationary primary balance-to-GDP ratio, OLS estimates of a *constant* fiscal policy rule would be equally dubious. Apart from our baseline Markov-switching regression, we estimate restricted specifications of equation (42), for instance by removing regime-specific error variance or allowing for subset of regime-invariant parameters. Still, none of these models leads to successful and satisfying estimates, according to our estimation strategy. As a consequence, the baseline unrestricted regime-switching equation seems to fit the data best as far as convergence of MLE numerical algorithm, significance tests and estimated regimes are concerned.

Table 2 presents estimation results of equation (42). We report estimated parameters for each regime and we also compute implied long-run estimates using ergodic probabilities; standard deviations of long-run estimates are obtained using standard deviations and covariance of regime-specific parameters. For any regime-switching parameter $\alpha(z_t)$ which takes two values (α_1, α_2), with associated standard deviations $(\sigma_{\alpha_1}, \sigma_{\alpha_2})$ and covariance $cov(\alpha_1, \alpha_2)$, we compute the long-run (ergodic) estimate α using ergodic probabilities (π_1, π_2) by:

$$\alpha \equiv \alpha_1\pi_1 + \alpha_2\pi_2$$

[15] To account for second-order serial correlation in the data, we assume: $\big(1-\Theta(L)\big)u_t = \sigma(z_t)\varepsilon_t$ with an i.i.d error term $\varepsilon_t \sim \mathcal{N}(0,1)$ and $\Theta(L)$ a second-order polynomial of the lag-operator L

and with standard deviation:

$$\sigma_\alpha \equiv \sqrt{\pi_1 \sigma^2_{\alpha_1} + \pi_2 \sigma^2_{\alpha_2} + 2\pi_1\pi_2 cov(\alpha_1, \alpha_2)}$$

The results raise some comments. First, France's fiscal policy is well described by a two-state Markov-switching policy rule. One regime is sustainable with a strong positive correlation between primary balance and initial debt, while the other one shows a strong negative correlation. As expected, the constant is significantly negative in the sustainable regime and non-significantly different from zero in the unsustainable regime which is consistent with the fact that the debt-output ratio is non-stationary in the unsustainable regime, as explained in section 3.2. Second, both regimes appear to be strongly persistent with expected durations of 7.2 and 6.4 years, respectively for sustainable and unsustainable regimes. This would explain why OLS estimates were inconclusive about the long-run correlation between primary surplus and initial debt in table 1. Estimates of α_y and α_g might be respectively upward and downward biased because of a simultaneity bias. Still, if we instrumented these variables using lags for instance, we would probably underestimate the cyclical component of s_t which is precisely the *instantaneous* effect of output gap and temporary public spending at time t. Moreover, IV methods are not well established for regime-switching models. Third, regarding long-run estimate of $\gamma(z_t)$, primary balance seems positively correlated with initial debt, though with a p-value above the 10% level.

Table 2: Estimated Markov-switching fiscal rule for France (1965–2013)

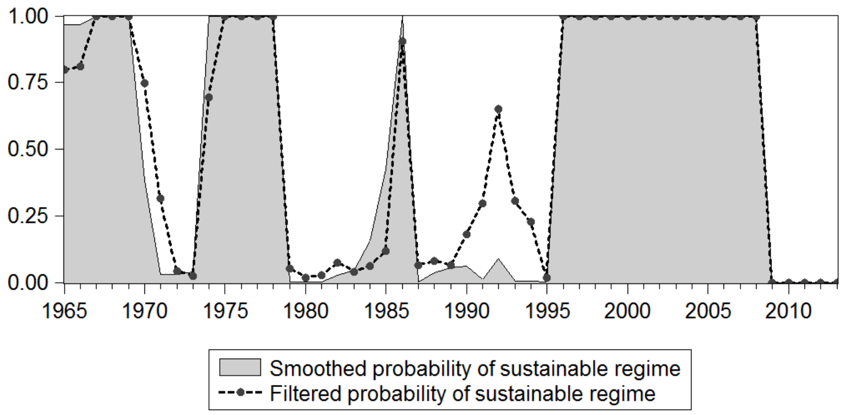
Dependent variable: s_t	Regime 1	Regime 2	Long-run estimates
Initial debt b_{t-1}	0.0540***	-0.0358***	0.0118
	(4.92)	(-6.72)	(1.67)
Constant	-0.0351***	0.0032	-0.0171***
	(-5.89)	(0.98)	(4.07)
Output gap $\hat{y}_t$	0.5826***	0.2788***	0.4398***
	(5.25)	(6.30)	(6.31)
Temporary spending $\hat{g}_t$	-0.0514	-0.7399***	-0.3750***
	(-0.37)	(-13.17)	(-5.53)
Standard-error σ	0.00571***	0.00232***	0.0041***
	(4.17)	(3.91)	(5.66)
Transition probabilities	$\mathbb{P}(z_t = i \mid z_{t-1} = j)$		
	j=1	j=2	
i=1	0.86	0.14	
i=2	0.16	0.84	
Ergodic probability π_i	0.533	0.467	
Expected duration d_i (years)	7.22	6.40	
DW	1.77		
Number of observations	49		

Notes: t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). We control for regime-invariant second-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log\hat{\sigma}$: consequently, standard erros and t-statistics are obtained applying the Delta method.

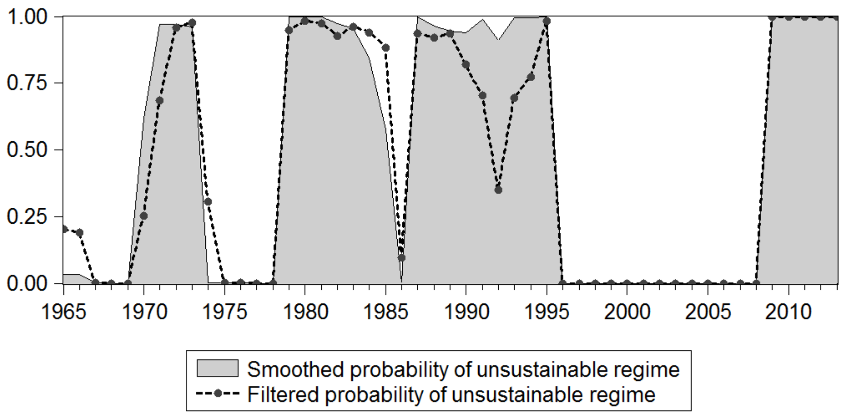
Figure 2 represents estimated smoothed and filtered probabilities for each regime. Results show a succession of periods of unsustainable or sustainable fiscal policies with very marked decades. Public finances in the 1970s were sustainable over the most part, but it is noteworthy that despite a low and declining debt-to-GDP ratio, France underwent a short period of unsustainability in the early 1970s. In sharp contrast, France's fiscal policy has been mostly unsustainable during the 1980s, except during the so-called "Tournant de la rigueur" of 1983-1986 when the Socialist government turned to disinflation and deficit-reduction policies. Overall, results are consistent with a comprehensive and historical analysis of France's fiscal policy. In the 1990s, results report that France's fiscal policy became gradually sustainable (or passive to use Leeper's terminology) and actually so by 1996, until 2008 and the advent of the Great Recession. This finding supports the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, despite it being under an Excessive Deficit Procedure from 2003 to 2007. In contrast with Weichenrieder and Zimmer (2014) who show that Euro membership of France has reduced the responsiveness of the primary surplus to debt, our results show that the 1999-2011 period (Euro membership years in Weichenrieder and Zimmer) was heterogeneous as regards fiscal responsiveness: it was positive until 2008 and then negative.

Figure 2: Estimated Fiscal Policy Regimes, France (1965-2013)

(a) Regime 1: *Sustainable regime*



(b) Regime 2: *Unsustainable regime*



The long-term estimate of $\gamma\pi$ is positive, equal to 0.01 but not significant at the 10% level. Thus, results are *a priori* inconclusive regarding *long-run sustainability* of France's fiscal policy. Yet, significance tests are not appropriate to test for NPG and debt-stabilizing conditions on $\gamma\pi$ since they are *bilateral* tests.

Table 3: Regime-Switching MBS: unilateral versus bilateral tests

Student tests for...	t-stat	Bilateral test p-value	Unilateral test p-value
No-Ponzi Game condition (30)	1.6693	0.1040	0.0520
Bounded long-run debt-to-GDP ratio (34)			
...using apparent rate (average, 1963-2013): $r-y=-3.11\%$	5.9765	0.0000	0.0000
...using market rate (average, 1963-2013): $r-y=0.32\%$	1.2297	0.2270	0.1135

On the contrary, conditions (30) and (34) call for *unilateral* tests for which critical values are lower with respect to bilateral tests.[16]

Beyond the appropriateness of the bilateral vs. unilateral test, a question arises: what is the appropriate measure for the growth-adjusted real interest rate? Should we use the *market value* of long-term real interest rate or the *apparent* real interest rate on public debt? Using the market value of long-term interest rate yields a mean growth-adjusted real interest rate equal to 0.3% over the sample while using apparent interest rate yields a growth-adjusted real interest rate equal to -3.1%. This difference mainly comes from the strong divergence between these two rates[17] before the late 1980s, see figures 3: the apparent rate was much smaller than the market rate. Figure 3 shows that France benefited from *large* and *negative* growth-adjusted real interest rates until 1980, if we consider the market long-term rate, and until 1987 if we consider the apparent rate. This snowball effect in reverse explains *other things equal* the rapid and persistent decrease in France's public debt-output ratio during the 1960s and the 1970s. Using the apparent interest rate rather than the long-term rate dramatically changes the result of the bounded debt-to-GDP ratio test, see table 3. Consequently, we propose to test for a bounded debt-output ratio using two different measures of the growth-adjusted real interest rate on public debt: the average apparent real rate and the average market long-term real rate.

We find weak but significant evidence that France's fiscal policy satisfies the NPG condition (30). In contrast with a bilateral Student test which rejects the null of a Ponzi Scheme with p-value equal to 10.4%, a unilateral Student test rejects Ponzi Scheme at the 10% level, with a p-value equal to 5.2%. Tests of condition (34) yield different results according to the measure of $r-y$ which we use. With the average apparent rate between 1963 and 2013, we strongly reject the null of explosive debt-output ratio dynamics, with a high level of confidence; since the growth-adjusted apparent real interest rate is strongly negative, debt-output ratio is automatically stable[18]. On the contrary, using the growth-adjusted market long-term real rate, the test cannot reject the null of explosive debt-ratio dynamics (with a p-value equal to 11.4%). As a result, even a small and positive growth adjusted real rate increases the risk that France's public debt may be on an explosive path.

Finally, using point estimates reported in table 2 and historical average for the real interest rate and real GDP growth rate, table 4 reports the computed debt-to-GDP ratios under two alternative scenarios. In scenario 1, we suppose sustainable regimes last longer and we increase their expected duration (or persistence) while keeping the expected duration of unsustainable regimes constant and equal to their estimated value. In scenario 2, we suppose unsustainable regimes are shorter and we decrease their

[16] For instance, a bilateral test of the NPG condition on the parameter $\gamma\pi$ is build upon the null hypothesis $|\gamma\pi|=0$ against the alternative $|\gamma\pi|\neq 0$, while the unilateral test is build upon the null hypothesis $\gamma\pi=0$ against the alternative $\gamma\pi>0$ which is a more adequate testing hypothesis in the sustainability context.

[17] Both growth-adjusted measures of real interest rate are computed using real GDP and GDP deflator from OECD Economic Outlook database, hence divergences only come from the nominal rate.

[18] This case illustrates why testing for stationary debt-output ratio may not be meaningful, since it does not always rule out Ponzi Scheme: with a negative average growth-adjusted real interest rate, fiscal policy could be running a Ponzi Scheme with a stationary debt-output ratio.

expected duration while keeping the expected duration of sustainable regimes constant and equal to their estimated value. According to our estimates, France's gross public debt-to-GDP ratio would reach an average value of 195% across fiscal regimes. Yet, this expected debt-to-GDP ratio is far too large to reject the null of an explosive debt-GDP ratio with a good level of confidence, see table 3. Assuming that $d_{NS} = 0$, we obtain the underlying debt-to-GDP target ratio $b_S^* = 69\%$ towards which public debt converges during sustainable regimes[19].

In table 5, we perform the same exercize but using the average apparent interest rate on public debt. Results are radically different from those presented in table 4 since the average apparent rate is negative over the period 1963-2013, which is explained by a long period of financial repression in France until the 1980s[20]. As a result, France's public debt would have an ergodic average of 41% of GDP in the baseline scenario (based on point estimates obtained in table 2). These results call for two comments. First, when using the apparent interest rate, debt-to-GDP ratio is stationary as long as $\gamma\pi > r - y$. Second, it could seem paradoxical that $\mathbb{E}[b_t]$ decreases when d_S (respectively d_{NS}) decreases (respectively increases). It results from the fact that α_{NS} is closer to zero than α_S: when d_S (resp. d_{NS}) decreases (resp. increases), then

$$-\mathbb{E}[\alpha(z_t)] = -\left(\alpha_S \frac{d_S}{d_S + d_{NS}} + \alpha_{NS} \frac{d_{NS}}{d_S + d_{NS}}\right)$$

decreases towards zero.

In table 6, we compute expected debt-to-GDP ratios depending on different growth-adjusted real interest rates. Our experiment illustrates, in particular, the potential risk of explosive public debt dynamics if the growth-adjusted real interest rate increased in the future, provided France's fiscal policy continues to fit into our estimated Markov-switching policy rule.

5 Conclusions

This paper introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes, where by unsustainable regime we mean a periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violation of Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, and fiscal regimes thus matter for fiscal sustainability analysis.

The Regime-switching MBS test is then applied to French data over a 51-year horizon and compared to standard MBS tests. Contrary to standard tests, our Regime-switching MBS test rejects the null of Ponzi Game but with a low level of confidence. Under the assumption of a fiscal limit on the primary surplus-to-GDP ratio, the relevant sustainability condition would be the debt-stabilizing condition. Then, the rejection of the null of an explosive public debt-to-GDP ratio depends on the measure of the growth-adjusted real interest rate: the apparent rate or the market rate. Using the average apparent rate on public debt, our test rejects the null of an explosive public debt-to-GDP ratio. Yet, due to financial

[19] This level cannot be compared to Maastricht criterion of 60% of gross public debt. Indeed, we used the OECD's gross government financial liabilities in our estimates rather than Maastricht gross public debt, for data availability reasons. These two measures of gross public debt differ in terms of debt instruments and valuation methods. As a result, Maastricht debt is generally much lower than gross governement financial liabilities.

[20] Aloy et al. (2014) conduct a counterfactual exercize on French public debt in which they assume that the financial repression prevailing in France until the 1980s would have continued after.They find a substantial cumulated decrease in the cost of debt service of around 30% between 1999 and 2010.

Table 4: Expected regime durations and Debt-GDP ratios using market long-term interest rate

Scenario 1: Increasing expected duration of sustainable regimes						
d_S	π_S	π_{NS}	$\gamma\pi$	NPG condition	Bounded debt-GDP	$\mathbb{E}[b_t]$
2	0.24	0.76	-1.4419%	Violated	No	Explosive
4	0.38	0.62	-0.1262%	Violated	No	Explosive
7.22	0.53	0.47	1.1803%	Satisfied	Yes	195%
10	0.61	0.39	1.8956%	Satisfied	Yes	127%
15	0.70	0.30	2.7144%	Satisfied	Yes	98%
30	0.82	0.18	3.8211%	Satisfied	Yes	81%
60	0.90	0.10	4.5345%	Satisfied	Yes	74%
$+\infty$	1.00	0.00	5.400%	Satisfied	Yes	69%
Scenario 2: Decreasing expected duration of unsustainable regimes						
d_{NS}	π_S	π_{NS}	$\gamma\pi$	NPG condition	Bounded debt-GDP	$\mathbb{E}[b_t]$
20	0.27	0.73	-1.1981%	Violated	No	Explosive
15	0.32	0.68	-0.6621%	Violated	No	Explosive
10	0.42	0.58	0.1851%	Satisfied	No	Explosive
6.4	0.53	0.47	1.1803%	Satisfied	Yes	195%
4	0.64	0.36	2.1986%	Satisfied	Yes	113%
2	0.78	0.22	3.4521%	Satisfied	Yes	85%
1	0.88	0.12	4.3075%	Satisfied	Yes	76%
0	1.00	0.00	5.4000%	Satisfied	Yes	69%

Notes: Debt-output ratios are computed from equation (35). For scenarios 1 and 2, we use average market long-term interest rate $r = 3\%$, average real growth rate $y = 2.68\%$ and $r - y = 0.32\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_S and for $d_{NS} = 6.4$. In scenario 2, we compute expected debt-output ratios under various values of d_{NS} and for $d_S = 7.22$. All others parameters are constant and equal to point estimates obtained in table 2.

Table 5: Expected regime durations and Debt-GDP ratios using apparent interest rate

Scenario 1: Increasing expected duration of sustainable regimes						
d_S	π_S	π_{NS}	$\gamma\pi$	NPG condition	Bounded debt-GDP	$\mathbb{E}[b_t]$
0	0.00	1.00	-3.5800%	Violated	No	Explosive
2	0.24	0.76	-1.4419%	Violated	Yes	37%
4	0.38	0.62	-0.1262%	Violated	Yes	40%
7.22	0.53	0.47	1.1803%	Satisfied	Yes	41%
10	0.61	0.39	1.8956%	Satisfied	Yes	41%
30	0.82	0.18	3.8211%	Satisfied	Yes	41%
60	0.90	0.10	4.5345%	Satisfied	Yes	41%
$+\infty$	1.00	0.00	5.4000%	Satisfied	Yes	42%
Scenario 2: Decreasing expected duration of unsustainable regimes						
d_{NS}	π_S	π_{NS}	$\gamma\pi$	NPG condition	Bounded debt-GDP	$\mathbb{E}[b_t]$
$+\infty$	0.00	1.00	-3.5800%	Violated	No	Explosive
60	0.11	0.89	-2.6155%	Violated	Yes	21%
30	0.19	0.81	-1.8380%	Violated	Yes	35%
10	0.42	0.58	0.1851%	Satisfied	Yes	40%
6.4	0.53	0.47	1.1803%	Satisfied	Yes	41%
4	0.64	0.36	2.1986%	Satisfied	Yes	41%
2	0.78	0.22	3.4521%	Satisfied	Yes	41%
0	1.00	0.00	5.4000%	Satisfied	Yes	42%

Notes: Debt-output ratios are computed from equation (35). For scenarios 1 and 2, we use average apparent interest rate $r = -0.43\%$, average real growth rate $y = 2.68\%$ and $r - y = -3.11\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_S and for $d_{NS} = 6.4$. In scenario 2, we compute expected debt-output ratios under various values of d_{NS} and for $d_S = 7.22$. All others parameters are constant and equal to point estimates obtained in table 2.

Table 6: Growth-adjusted real rates and Debt-GDP ratios

$r-y$	Bounded debt-GDP	$\mathbb{E}b_t$
2.50%	No	Explosive
2.00%	No	Explosive
1.50%	No	Explosive
1.00%	Yes	754%
0.50%	Yes	244%
0.00%	Yes	146%
-0.50%	Yes	104%
-1.00%	Yes	81%
-1.50%	Yes	67%
-2.00%	Yes	57%
-2.50%	Yes	49%
-3.00%	Yes	44%

Notes: Debt-output ratios are computed from equation (35). We use point estimates of γ_S, γ_{NS}, α_S, α_{NS}, and respective expected durations of regime d_S and d_{NS} from table 2. Then, we set $r = 3\%$ and compute expected debt-output ratios for various real GDP growth rate.

repression, the average apparent rate was significantly lower than the market long-term interest rate before the mid-1980s. Thus, using the average market rate, we cannot reject the hypothesis of an explosive public debt-to-GDP ratio. Overall, our results indicates that the end of financial repression in the 1980s dramatically increased the growth-adjusted real interest rate, and therefore tightened the requirements for a stable public debt-to-GDP ratio. We also provide evidence that French fiscal policy actually tightened after 1995, in the ongoing process toward the European Monetary Union.

Future research may now move towards the analysis of the interactions between monetary policy and fiscal policy and their consequences on fiscal sustainability. In contrast with early attempts (Davig and Leeper (2011)), a euro-area country like France cannot be described by a domestic monetary policy. Theoretical research is thus required to match domestic fiscal policy with a federal monetary policy. Beyond that, the question of fiscal sustainability in a Regime-switching MBS framework could be embedded in an open-economy framework. It would introduce another determinant of fiscal sustainability, namely cooperative or non-cooperative fiscal behaviours.

A Appendix

A.1 No-Ponzi Game condition

We show that a strictly positive long-run feedback effect, i.e. (27), is a necessary and sufficient condition for the NPG (26) to hold, in a dynamically efficient economy and with a bounded innovation process $\mu(z_t)$, following Bohn (1998, see online appendix).

Using (17) and iterating on (14) yields:

$$b_{t+T} = \prod_{i=1}^{T} \frac{1+r_{t+i}}{1+y_{t+i}} \Big(1-(1+y_{t+i})\gamma(z_{t+i})\Big) b_t - (1+r_{t+T})\mu_{t+T}(z_{t+T}) - \sum_{k=1}^{T-1} (1+r_{t+k}) \left(\prod_{j=k+1}^{T} \frac{1+r_{t+j}}{1+y_{t+j}} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big) \right) \mu_{t+k}(z_{t+k}) \quad (43)$$

Then, using (25) one gets an expression for the transversality condition on debt-income ratio:

$$\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} = \mathbb{E}_t \prod_{i=1}^{T} \Big(1-(1+y_{t+i})\gamma(z_{t+i})\Big) b_t - \mathbb{E}_t a_{t,T} - \mathbb{E}_t \sum_{k=1}^{T-1} \left(\prod_{j=k+1}^{T} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big) \right) a_{t,k} \quad (44)$$

with $a_{t,k} = (1+y_{t+k})\tilde{Q}_{t,T+k}\mu_{t+k}(z_{t+k})$.

Using (24), we write the following approximation:

$$\mathbb{E}_t \prod_{i=1}^{T} \Big(1-(1+y_{t+i})\gamma(z_{t+i})\Big) \simeq \Big(1-(1+y)\gamma\pi\Big)^{T-N} \Phi_N \quad (45)$$

with $\Phi_N = \prod_{i=1}^{N} \big(1-(1+y)\gamma P^i Z_t\big)$ for N arbitrarily high enough. Use again this approximation in order to simplify the following sum:

$$\mathbb{E}_t \sum_{k=1}^{T-1} a_{t,k} \left(\prod_{j=k+1}^{T} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big) \right) \simeq \Big(1-(1+y)\gamma\pi\Big)^{T-N-1} \Omega + \sum_{k=N}^{T-1} \mathbb{E}_t a_{t,k} \Big(1-(1+y)\gamma\pi\Big)^{T-k-1} \quad (46)$$

with $\Omega = \sum_{k=1}^{N-1} \Phi_{N-1} \mathbb{E}_t a_{t,k}$ which is finite.

Hence, using these approximations, we get:

$$\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} \simeq \Phi_N \Big(1-(1+y)\gamma\pi\Big)^{T-N} b_t - \mathbb{E}_t a_{t,T} - \Big(1-(1+y)\gamma\pi\Big)^{T-N-1} \Omega - \mathbb{E}_t \sum_{k=N}^{T-1} a_{t,k} \Big(1-(1+y)\gamma\pi\Big)^{T-k-1} \quad (47)$$

Following Bohn (1998), two assumptions are required to find a sufficient condition for fiscal policy to satisfy its PVBC: dynamic efficiency and bounded process μ_t. Assuming dynamic efficiency implies income has a finite present value, that is:

$$\lim_{T\to+\infty} Y_t \sum_{i=1}^{T} \mathbb{E}_t \tilde{Q}_{t,i} = \overline{Y} \quad (48)$$

which implies $\lim_{T\to+\infty}\mathbb{E}_t\tilde{Q}_{t,T}=0$, by convergence of the serie $\sum_{i=1}^{T}\mathbb{E}_t\tilde{Q}_{t,i}$. In addition, assuming μ_t is bounded[21]:

$$\left|\mu_t\right|\leq M \tag{49}$$

then it directly implies $\lim_{T\to+\infty}\mathbb{E}_t a_{t,k}=0$, that is:

$$\forall\delta>0\,,\quad \exists N\in\mathbb{N}\quad|\quad \forall k>N\quad \left|\mathbb{E}_t a_{t,k}\right|\leq\delta \tag{50}$$

Rearranging (47) and taking absolute value yields:

$$\left|\mathbb{E}_t\tilde{Q}_{t,T+1}b_{t+T}-\Phi_N\Big(1-(1+y)\gamma\pi\Big)^{T-N}b_t+\Big(1-(1+y)\gamma\pi\Big)^{T-N-1}\Omega+\mathbb{E}_t a_{t,T}\right|\simeq\left|\sum_{k=N}^{T-1}\mathbb{E}_t a_{t,k}\Big(1-(1+y)\gamma\pi\Big)^{T-k-1}\right| \tag{51}$$

Assume that $\gamma\pi>0$. If present-value of future income is finite and if innovation μ_t is a bounded process, then using triangle inequality yields:

$$\begin{aligned}\left|\mathbb{E}_t\tilde{Q}_{t,T+1}b_{t+T}-\Phi_N\Big(1-(1+y)\gamma\pi\Big)^{T-N}b_t+\Big(1-(1+y)\gamma\pi\Big)^{T-N-1}\Omega+\mathbb{E}_t a_{t,T}\right| &\leq \delta\textstyle\sum_{k=N}^{T-1}\Big(1-(1+y)\gamma\pi\Big)^{T-k-1}\\ &\leq \frac{\delta}{(1+y)\gamma\pi}\left(1-\Big(1-(1+y)\gamma\pi\Big)^{T-N}\right)\\ &\leq \frac{\delta}{(1+y)\gamma\pi}\end{aligned} \tag{52}$$

implying:

$$\forall\epsilon>0\,,\quad \exists K\in\mathbb{N}\quad/\quad \forall T\geq K\quad \left|\mathbb{E}_t\tilde{Q}_{t,T+1}b_T\right|<\epsilon \tag{53}$$

with $\epsilon=\frac{\delta}{(1+y)\gamma\pi}$, which is equivalent to:

$$\lim_{T\to+\infty}\mathbb{E}_t\tilde{Q}_{t,T+1}b_{t+T}=0$$

A.2 Stationary debt-output ratio

Using the necessary and sufficient condition for a *strictly* stationary Markov-switching autoregressive process of order one, we show a strictly larger feedback effect than the average growth-adjusted real interest rate, i.e. (33), is a necessary and sufficient condition for the debt-output ratio process (32) to be strictly stationary and fluctuate around its ergodic mean (35).

Considering stochastic processes $\{x_t\}$ described by:

$$x_t=\phi_0+\phi(z_t)x_{t-1}+\varepsilon_t \tag{54}$$

where z_t is a discrete-time Markov process, defined on the state-space $z(\Omega)$. We know from Kesten (1973) that a sufficient condition for *strict stationarity* is:

$$\mathbb{E}\Big[\ln\big|\phi(z_t)\big|\Big]\equiv\sum_{i\in z(\Omega)}\ln\big|\phi(i)\big|\pi(i)<0 \tag{55}$$

[21] When μ_t is also regime-switching, this assumption implies boundedness in each regime.

which means that a globally stationary process $\{x_t\}$ can be locally (or periodically) non-stationary. This condition ensures that $\{x_t\}$ is strictly (or strongly) stationary implying its joint-probability distribution does not change over time. Strict stationarity only implies $\{x_t\}$ has a finite mean but does not imply necessarily a finite variance. Since weak stationarity requires finite variance, this condition is not sufficient for weak stationarity. For a finite variance, this process must verify a stronger condition. Define $\Phi \equiv \text{diag}\Big(\phi(i), \forall i \in z(\Omega)\Big)$ and $\rho(M)$ the spectral radius of any square-matrix M. Then, for this stricly stationary process to admit a unique stationary solution at second-order, it must satisfy the following condition:

$$\rho(\Phi^2 P) < 1 \tag{56}$$

where P is the transition matrix of the underlying Markov-chain.

Applying it to equation (32) yields the following condition:

$$\mathbb{E}[\ln \phi(z_t)] = \mathbb{E}\left[\ln\left|\frac{1+r_t}{1+y_t}\right| + \ln\Big|1-(1+y_t)\gamma(z_t)\Big|\right] < 0 \tag{57}$$

Hence, taking unconditional expectation of r_t, y_t and $\gamma(z_t)$ and using usual approximation $\ln(1+x) \underset{x\to 0}{\sim} x$, we get:

$$\gamma\pi > \frac{r-y}{1+y} \simeq r - y \tag{58}$$

and process $\{b_t\}$ has an ergodic mean of:

$$\mathbb{E}[b_t] = \frac{-(1+r)\mathbb{E}[\alpha(z_t)]}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \tag{59}$$

A.3 Data on real interest rates and real GDP growth rate

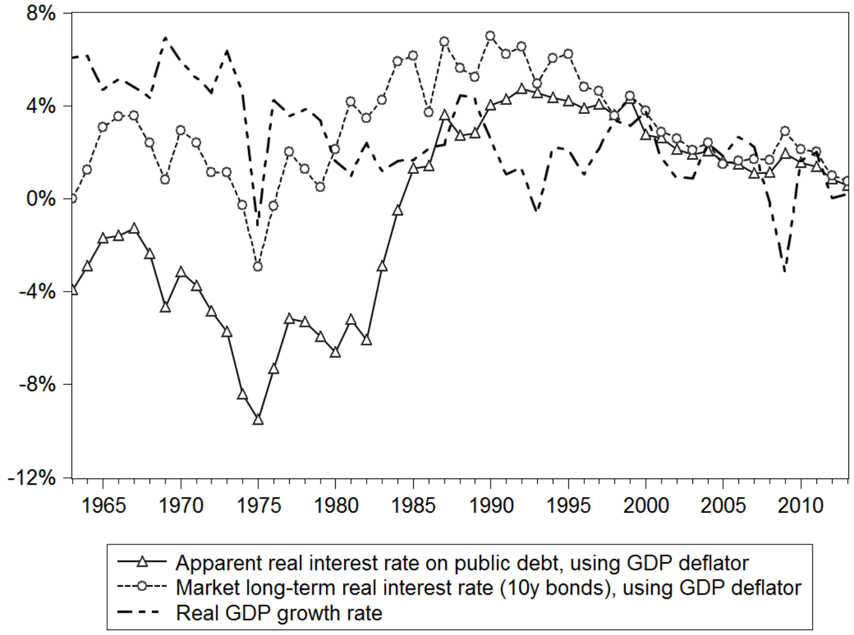
Table 7 presents descriptive statistics on real interest rate measures and real GDP growth. Figure 3 plots each time series and growth-adjusted real interest rates.

Table 7: Descriptive statistics on real interest rates and real GDP growth, 1963-2013

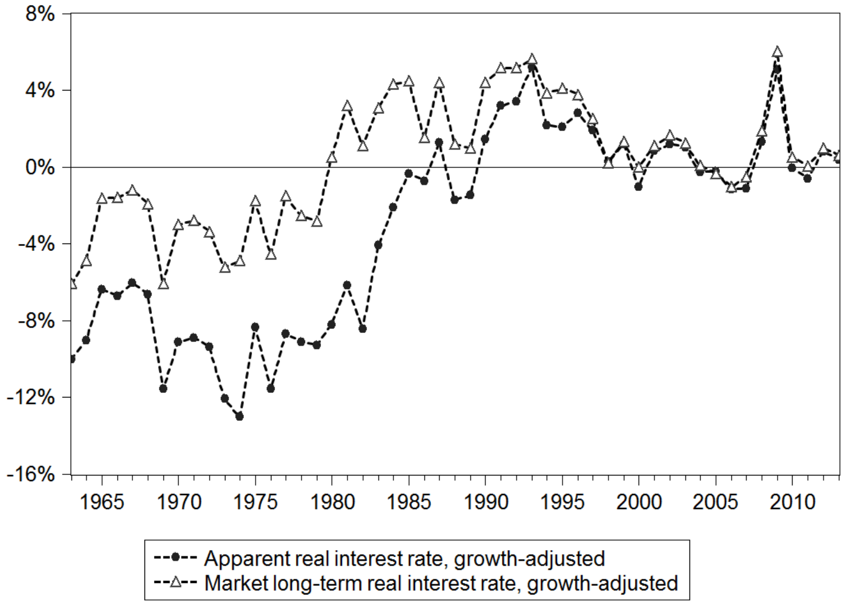
	Apparent real rate	Market long-term real rate	Real GDP growth rate
Mean	-0.43%	3.00%	2.68%
Median	1.13%	2.86%	2.31%
Maximum	4.75%	6.99%	6.91%
Minimum	-9.49%	-2.94%	-3.11%
Std. Dev.	4.0%	2.2%	2.1%
Skewness	-0.515	-0.060	-0.125
Kurtosis	2.025	2.759	2.994
Jarque-Bera normality test	4.276	0.154	0.132
p-value	0.118	0.926	0.936
Sum	-0.221	1.530	1.366
Sum Sq. Dev.	0.080	0.023	0.022
Observations	51	51	51

Figure 3: Apparent and market real interest rates and real GDP growth rate

(a) Real interest rates and real GDP growth



(b) Growth-adjusted real interest rates



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