Working paper

2016-17

CAPITAL ACCUMULATION AND THE DYNAMICS OF SECULAR STAGNATION

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June 2016

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Abstract

We characterize the dynamics of secular stagnation as a permanent regime switching from a full employment equilibrium to an underemployment equilibrium. In the latter, the natural interest rate is negative, and the economy is in deflation. Due to the non negativity condition imposed on policy rate, the zero lower bond (ZLB) applies which prevents targeting inflation. The secular stagnation equilibrium is achieved in a standard overlapping generations model with capital accumulation where two market imperfections are introduced: credit rationing and downward nominal wage rigidity. To figure out how to escape the secular stagnation trap, we study the impact of various macroeconomic policies. Raising the inflation target is only effective if the central bank has enough credibility. By supporting aggregate demand, fiscal policy can help the economy get out of the secular stagnation trap. However, this policy reduces the incentive to accumulate capital: there is a trade-off between exiting secular stagnation and depressing potential GDP. Dynamic multipliers are upper than one unless the fiscal stimulus is too strong. We also shed light on an asymmetry in the dynamics: recovery takes longer than falling into recession.

Keywords: Secular Stagnation, Capital Accumulation, Zero Lower Bound.

JEL codes: D91, E31, E52

1 Introduction

In the conventional wisdom convyed by the benchmark New Keynesian macroeconomic model, the deep economic and financial recession that hit the US and Europe in 2008 should have been followed by a much quicker recovery than observed. Indeed, eight years later, the economic state of the US and Europe is still marked by slow growth and underemployment, with low inflation or even deflation, raising fears of a scenario *à la* Japanese. This too-slow or missed recovery has been widely discussed in the wake of the famous speech by Larry Summers at the International Monetary Fund (IMF) in 2013, under the label of "secular stagnation" (see Summers, 2013, 2014, 2016; Krugman, 2013; Bernanke, 2015, among many others). The concept of secular stagnation was coined in 1938 in a speech by A. Hansen, which was published in 1939. In stressing insufficient investment and a declining population in the United States, he was worried contrarily to the Keynesian approach that the economy would not recover spontaneously from a lack of demand. In Hansen's view, a state of secular stagnation results when an abundance of savings relative to demand for credit pushes up the "natural" interest rate (defined following Wicksell, 1898, as the real rate compatible with full employment) below zero. As a consequence, if the real interest rate remains permanently above the natural rate, the result is a chronic shortage of aggregate demand and investment, with a weakened growth potential.

Assuming secular stagnation is fit to describe the current economic state is still controversial. Though, it has received strong support from recent empirical studies evidencing the persistent negativity of natural interest rate since, or not long after, the onset of the recession (Barsky et al., 2014; Laubach and Williams, 2003 and 2015; Cúrdia, 2015; Pescatori and Turunen, 2015). This negativity has also had important consequences for the use of monetary policy to fight the deep recession. This is particularly clear if one considers a simple Taylor rule (1993) in which the nominal rate coincides with the sum of the natural rate and the inflation target[1]. The nominal rate, after first being aggressively cut, was led afterwards into negative territory by the natural interest rate. However, nominal interest rates cannot, in practice, be forced by central banks to be "too negative". Otherwise, private agents would benefit from keeping their savings in the form

[1] In Cúrdia et al. (2015), such a rule would be called preferably a W rule in reference to Wicksell (1898).

of banknotes. Consequently, the nominal interest rate has reached, or approached, its zero lower bound[2] (ZLB), leaving conventional monetary policy toothless. The monetary authorities had to implement non-conventional policies, called quantitative easing.

Developing a framework that can account for a persistent constrained monetary policy is not an easy task. In standard Ramsey models with infinite-life agents, a liquidity trap can only be a temporary phenomenon (Krugman, 1998; Eggertsson and Krugman, 2013; Werning, 2012). This is well explained by Michau (2015). As the binding ZLB forces the real interest rate to be above the household discount rate, household consumption keeps growing. This is inconsistent with the stationarity required in secular stagnation. To bypass this difficulty, Michau (2015) incorporates a preference for wealth in the Ramsey model. The dynamics can then be characterized by an overaccumulation of capital and a negative natural interest rate. Alternatively, Eggertsson and Mehrotra (2014) propose an overlapping generation (OLG) model based on the savings behaviour of agents with a finite lifespan in a context of a rationed credit market. As is well known, the OLG framework can allow a persistent excess in savings. As for the monetary policy, the central bank sets the nominal rate by following a Taylor rule. Both models, through the interaction between the ZLB and downward wage rigidity, allow to characterize a secular stagnation equilibrium and have the great merit of clarifying the mechanisms behind a fall into long-term stagnation. From this perspective, they contribute to update the macroeconomic analysis of equilibrium multiplicity and persistence of crises.

In this paper, we propose a tractable theoretical model of secular stagnation based on Eggertsson and Mehrotra (2014). We develop an overlapping generations model by incorporating imperfections affecting the credit market (borrowing constraint) and the labor market (downward wage rigidity). The central bank conducts monetary policy according to a Taylor rule subject to a ZLB on the nominal interest rate. Contrarily to Eggertsson and Mehrotra (2014), we also assume that the accumulation of physical capital is a prerequisite to any productive activity. As secular stagnation is by definition a medium-/long-term concept, we think it is important to consider capital accumulation in characterizing a secular stagnation steady state. To that purpose, individuals when young must first borrow on the financial market to buy capital which is productive the

[2]The main policicy rate of the US Fed was 0.25% since December 2008, and increased by 0.25 point only in December 2015. The main policy rate of the ECB was 0.05% since September 2014, then 0% since March 2016.

following period. Similarly to Eggertsson and Mehrotra (2014) a tighter credit rationing can induce an immediate fall in aggregate demand and an excess in savings over investment opportunities. Consequently, the real interest rate falls. If full employment requires a negative natural interest rate, the economy sinks into a state of persistent deflation, characterised by underemployment and a low output.

Deprived of capital accumulation, the dynamics in Eggertsson and Mehrotra (2014) is characterized by adjustments without transition from one steady state to another. In contrast, we shed light on an asymmetry in the dynamics. If the credit constraint loosens, then capital converges to its pre-crisis level. However, exiting the crisis takes longer than entering it. In other words, capital is rapidly destroyed during the crisis, whereas it takes longer to reaccumulate and rebuild it in the recovery phase. This characteristic suggests that economic policies used to fight secular stagnation must be undertaken as soon as possible. In addition, we show that the level of deflation following the shock overshoots its steady state level. Indeed, as the level of capital already installed cannot be adjusted initially, there is an excess supply, which in turn results in a higher deflation.

From a monetary policy perspective, we confirm the result of Eggertsson and Mehrotra (2014). An increase in the inflation target can be helpful to exit secular stagnation but only if the central bank is credible enough. Indeed, starting from a unique secular stagnation equilibrium, we show that an increase in the inflation target can bring up the full employment equilibrium. However, the secular equilibrium equilibrium does not disappear and both equilibria are locally determined.

From a fiscal policy perspective, any increase in aggregate demand induces inflationary pressure and can help the economy out of secular stagnation. However, we highlight a clear trade-off between exiting secular stagnation and depressing potential GDP. This emphasises that, if a too small fiscal stimulus cannot allow pulling the economy out of secular stagnation, a too large fiscal stimulus can be counterproductive in terms of economic efficiency. Dynamic multipliers are upper than one unless the fiscal stimilus is too strong. Furthermore, our model exhibits the following Keynesian paradoxes: increasing productivity or reducing rigidities can be counterproductive in fighting unemployment and deflation in a state of secular stagnation. Accordingly, intermediate fiscal stimuli appear in our model as the most efficient policy for escaping secular stagnation.

In addition to Eggertsson and Mehrotra (2014) and Michau (2015), our model is also related to the few other models that aim to explain the persistence of the crisis as a potential permanent ZLB situation. In Kocherlakota (2013), with overlapping generations or a credit constraint, a fall in the price of land can generate a secular stagnation type of equilibrium. Nevertheless, note that for the result to hold, some kind of upward nominal wage rigidity is required. In Caballero and Farhi (2014), with a perpetual youth OLG model with no capital accumulation, it is the shortage of safe assets that can bring the economy to the ZLB and the associated recession. Accordingly, one priviledged way to stimulate aggregate demand and exit recession is for the government to issue safe public debt, and eventually to buy risky private assets with the proceeds. In this model, unlike in ours, an increase in the inflation target can also be efficient to get out of secular stagnation. Finally, in Benigno and Fornaro (2015), who rely on an endogenous growth model with innovation activities, pessimistic expectations are key to explain the fall into recession. Afterward, the economy may be persistently or permanently trapped, because weak growth depresses aggregate demand, pushing the nominal interest rate against the ZLB, while depressed demand reduces profits, hence investment in innovation. In this context, contrarily to ours, any policy that enhances productivity growth can be efficient for exiting the stagnation trap.

This article consists in four parts. In the second section, we present our overlapping generations model with capital accumulation, market imperfections and a Taylor rule. In section three, we characterize the economic dynamics (dynamic time paths and steady states) and the secular stagnation equilibrium. There are three configurations. If the long-run equilibrium is unique, two cases must be considered: either full employment with an inflation target, or secular stagnation with underemployment and deflation. In both cases, the equilibria are globally determined with the dynamics characterized by a unique saddle path. Finally, in a third configuration, the two preceding equilibria coexist with a third equilibrium of full employment and missed inflation target. This equilibrium is undetermined and the other two are determined. These determined steady states are not unique. Therefore, they are determined only locally, not globally. Section four discusses the economic policy issues (monetary and fiscal) bound up with exiting the secular stagnation trap. The last section concludes.

2 The model

The economy is composed of four types of agents: individuals, firms, a governement and a central bank.

Following Samuelson (1958) and Diamond (1965), we assume that individuals live for three periods: they are successively young, middle-aged-workers then retired. The number of young individuals, N_t at date t, is growing at the constant rate n so that:

$$N_t = (1+n)\, N_{t-1} \tag{1}$$

Competitive firms produce one good, which is both a consumption good and an investment good, by using two factors: labor and capital. The government finances public expenditure by taxing workers (with a balanced budget), and the central bank determines the nominal rate of interest to control inflation.

Accordingly, there are four markets in the economy: good, labor, capital and credit.

2.1 Individuals and credit rationing

During the first period of their lives, individuals borrow to invest I_{t-1} in capital. One period later, the investment is sold to firms with a return equal to R_t^k. When people are active, they offer inelastically an amount of work $\bar{l}$ normalized to unity, $\bar{l} = 1$, and work for a real wage rate w_t. They consume c_t and save such that a_{t-1}^m is their real net asset. They also pay back their loans plus interest and a lump-sum tax T. In the final period of life, people consume d_{t+1}. Assuming that each individual effectively works an identical duration $l_t \leq \bar{l}$, the budgetary constraints are as follows:

$$\begin{cases} a_{t-1}^y = -I_{t-1} \\ c_t + a_t^m = w_t l_t - T + R_t^k I_{t-1} + R_t a_{t-1}^y \\ d_{t+1} = R_{t+1} a_t^m \end{cases} \tag{2}$$

where a_t^y denotes the net real asset at date t of a young individual and R_t the real interest factor.

Note that purely for analytical simplicity, we assume as usual in this type of literature that individuals do not consume in the first period of life (see for example Boldrin and Montes, 2005, and Docquier et al., 2007). Furthermore, it is assumed that there is no altruism, and so individuals start in life with zero assets. The preferences of an individual born in period $t-1$ are therefore characterized by the following utility function:

$$U_{t-1} = \log c_t + \beta \log d_{t+1} \tag{3}$$

where β denotes the psychological discount factor. It is easily shown that the optimal behavior of the consumer, obtained by utility maximization of eq. (3) under budgetary constraints (2), yields the following optimal asset:

$$a_t^m = s\left(w_t l_t - T + R_t^k I_{t-1} + R_t a_{t-1}^y\right) \tag{4}$$

where $s = \frac{\beta}{1+\beta}$ denotes the saving rate.

To characterize the imperfection of financial markets, we assume following Aiyagari (1994), Eggertsson and Krugman (2012), Eggertsson and Mehrotra (2014) or Coeurdacier et al. (2015) that the credit market is rationed as[3]:

$$-a_{t-1}^y \leq \frac{D}{R_t} \tag{5}$$

Such a constraint does not focus on the loanable proportion, but on households' ability in the following period to repay their loans, i.e. to repay the capital borrowed plus interest. If this constraint bites (of course this is assumed when $R_t^k > R_t$), we then have $a_{t-1}^y = -\frac{D}{R_t}$.

2.2 Firms

From the production side, we assume that the good is produced in a competitive sector characterized by a Cobb-Douglas technology with constant return to scale, such that $F\left(K_t, N_{t-1} l_t\right) = A K_t^\alpha \left(N_{t-1} l_t\right)^{1-\alpha}$, where $\alpha < 1$ and A denotes the total factor productivity (TFP). The profit maximization then yields:

[3] Microeconomic theories of credit rationing are based mostly on the non-observability either of the individual effort (moral hazard) or of skills (adverse selection) (see Stiglitz and Weiss, 1981; Aghion and Bolton, 1997; Piketty, 1997). They all share the common explanation that greater collateral allows one to borrow more.

$$w_t = A(1-\alpha)k_t^{\alpha}l_t^{-\alpha},\ l_t \leq 1 \tag{6}$$

and

$$R_t^k = A\alpha k_t^{\alpha-1}l_t^{1-\alpha} + (1-\delta) \tag{7}$$

where $k_t = \frac{K_t}{N_{t-1}}$ is the level of capital per worker and δ the depreciation rate of capital, $\delta \in (0,1]$.

2.3 Wage bargaining and nominal rigidity

Each generation of workers negotiates a contract. We assume that at the beginning of each period a wage negotiation defines the profile of nominal wages throughout the period of activity. For simplicity, we define $W_t(0)$ and $W_t(1)$ as the levels of nominal wages at the beginning and end of period t. Assuming an aversion to a decline in nominal wages during the period, the wage at the end of the period is determined according to:

$$W_t(1) = \max\left(\tilde{W}_t, W_t^*\right) \tag{8}$$

where $W_t^* = A(1-\alpha)P_t k_t^{\alpha}$ denotes the full employment wage rate and $\tilde{W}_t = \gamma W_t(0) + (1-\gamma)W_t^*$, $\gamma \in (0,1)$ characterizes the aversion to the decline in nominal wages, or the degree of downward rigidity of wages. Assuming that wage bargaining leads to setting a constant level of the real wage over the period, $w_t = \frac{W_t(0)}{P_{t-1}} = \frac{W_t(1)}{P_t}$, we then have:

$$w_t = \max\left(\frac{(1-\gamma)A(1-\alpha)k_t^{\alpha}}{1-\frac{\gamma}{\Pi_t}}, w_t^*\right) \tag{9}$$

where w_t^* denotes the full employment real wage rate and Π_t the inflation factor at date t. We observe straightforwardly that in this configuration, if the economy is in deflation, then the negotiated real wage level is above its full employment level: $w_t = \frac{(1-\gamma)(1-\alpha)k_t^{\alpha}}{1-\frac{\gamma}{\Pi_t}} \geq (1-\alpha)k_t^{\alpha}$ if $\Pi_t \leq 1$. Indeed, in case of deflation, maintaining both purchasing power and full employment means a lower nominal wage. If the required drop is reduced, and especially if the aversion to a nominal wage decline is strong, then the real wage becomes stronger than the one that would allow full employment.

2.4 Central bank: Taylor rule and inflation target

We assume that the monetary authorities want to control inflation. Following Eggertsson and Mehrotra (2014), we express the Taylor rule as:

$$1 + i_t = \max\left(1, (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \tag{10}$$

where i_t denotes the nominal interest rate at date t, $\Pi^* \geq 1$ the official inflation target and $\phi_\pi > 1$ an inflation gap aversion parameter. When $1 + i_t = (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}$, the Taylor rule will operate and, in this sense, we can say that monetary policy is active. By contrast, when $i_t = 0$, the central bank is constrained by the nominal zero lower bound. In this case, we say that monetary policy is inactive. According to equation (10), we can highlight a level of inflation

$$\Pi_{kink} = (1 + i^*)^{\frac{-1}{\phi_\pi}}\,\Pi^* \tag{11}$$

such that $i_t \geq 0 \Leftrightarrow \Pi_t \geq \Pi_{kink}$. In addition, for the inflation target to be reached in the unconstrained regime we set $1 + i^* = R_{eq}\Pi^*$, where R_{eq} denotes the natural interest rate at steady state.

2.5 Equilibrium with market imperfections

By assuming that the credit constraint is binding, i.e. $R_t^k > R_t$, an equilibrium is defined by clearings in the capital market

$$N_{t-1}k_t = N_{t-1}I_{t-1} \tag{12}$$

and in the credit market

$$N_t a_t^y + N_{t-1}a_t^m = 0 \tag{13}$$

By contrast, knowing that wages are downardly rigid, the labor market does not necessarily clear. Equations (6) and (9) then yield:

$$l_t = \min\left(1, \mathcal{L}\left(\Pi_t\right)\right) \tag{14}$$

where $\mathcal{L}\left(\Pi_t\right) = \left(\frac{1-\frac{\gamma}{\Pi_t}}{1-\gamma}\right)^{\frac{1}{\alpha}}$ with $\mathcal{L}' = \frac{\gamma}{\alpha\Pi^2}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{1}{\alpha}-1} > 0$. We then observe that the underemployment of labor is particularly important whenever deflation is strong.

3 Characteristics of the secular stagnation equilibrium

3.1 The supply-demand equilibrium of good and the dynamics of capital

The supply of good per worker is obviously determined by:

$$y_t^s = Ak_t^\alpha \min\left(1, \mathcal{L}\left(\Pi_t\right)^{1-\alpha}\right) + (1-\delta)\, k_t \tag{15}$$

From the constancy of scale returns, we immediately check that the demand per worker is equal to $y_t^d = \frac{Y_t}{N_{t-1}} = w_t l_t + R_t^k k_t$. Knowing that at equilibrium $k_t = I_{t-1}$ (eq. 12), we deduce from equation (4) that $w_t l_t + R_t^k k_t = \frac{1}{s} a_t^m - R_t a_{t-1}^y + T$. Using equilibrium relations (13) and (14), the dynamics of the population (1), the credit rationing (5) and the public balanced budget $G = T$, where G denotes the public spendings by number of workers at any date t, the aggregate demand per worker can be expressed as:

$$y_t^d = \frac{1+n}{s} I_t + D + G \tag{16}$$

Typically, this increases with its two components, private investment and public demand. It also decreases with the saving rate s and increases with the population growth rate n. The aggregate demand is also impacted through the credit constraint (5). In particular, if D is lowered, consumption of the elderly is reduced so that $d_t = R_t a_{t-1}^m = -R_t (1+n)\, a_{t-1}^y = (1+n)\, D$.

In the good market, the supply-demand equilibrium $y_t^s = y_t^d$ (with eq. 12) then determines the following backward dynamics of the capital stock:

$$k_{t+1} = \frac{1}{1+n} \mathcal{S}\left(k_{t,} \Pi_t\right) \tag{17}$$

where $\mathcal{S}\left(k_t, \Pi_t\right) = sAk_t^\alpha \min\left(1, l\left(\Pi_t\right)^{1-\alpha}\right) - D - G + (1-\delta)\, k_t$ with $\mathcal{S}_k' = sR^k > 0$, $\mathcal{S}_\Pi' = \frac{sw}{(1+n)} \mathcal{L}' > 0$ if $\Pi < 1$ and $\mathcal{S}_\Pi' = 0$ if $\Pi \geq 1$.

If $\Pi_t \geq 1$, then the dynamics is $k_{t+1} = \frac{1}{1+n} \mathcal{S}\left(k_t, 1\right)$. In this case, it is easy to show that if $D + G$ is small enough, there exists a unique stable steady state with full employment k_{FE} for any $k_0 > k_{unst} > 0$, where the second steady state k_{unst} is unstable. The dynamic properties of the capital accumulation process can be characterized by studying $\Delta k_{t+1} =$

$k_{t+1} - k_t$. By definition, at the steady state k_{FE}, $\Delta k_{t+1} = 0$. Therefore, with decreasing productivity, if $k_t > k_{FE}$ then $\Delta k_{t+1} < 0$ and if $k_{FE} > k_t > k_{unst}$ then $\Delta k_{t+1} < 0$. Note that this process is independent of inflation, and thus the curve $\Delta k = 0$ is vertical in the plane (k, Π). If $\Pi_t < 1$, then the dynamics is expressed as $k_{t+1} = \frac{1}{1+n}\mathcal{S}(k_t, \Pi_t)$. When the economy falls into deflation, the nominal rigidity of wages eliminates the full employment equilibrium. Thus, a decline in the amount of work will reduce the marginal product of capital. In this case, the equilibrium level of capital will be lowered. The curve describing the locus $\Delta k_{t+1} = 0$ is growing in the plane (k, Π) when $\Pi < 1$ and $sR_t^k < 1$.

3.2 Taylor rule and stabilization of inflation

According to the Fisher equation, we have:

$$R_{t+1} = (1 + i_t)\frac{P_t}{P_{t+1}} = \frac{1 + i_t}{\Pi_{t+1}} \tag{18}$$

Combining equations (10) and (18), we obtain:

$$\Pi_{t+1} = \frac{\max\left(1, (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right)}{R_{t+1}} \tag{19}$$

Knowing that $k_{t+1} = I_t = \frac{D}{R_{t+1}}$, it yields from eq. (17) that:

$$\frac{1}{R_{t+1}} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D}. \tag{20}$$

Including this result in equation (19), the dynamics of inflation becomes:

$$\Pi_{t+1} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D}\max\left(1, (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \tag{21}$$

For clarity, to examine this forward dynamics of inflation, suppose that $\Pi_{kink} \geq 1$. In this case, if $\Pi_t \geq \Pi_{kink}$, then this equation becomes $\Pi_{t+1} = \frac{s}{(1+n)D}(1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\mathcal{S}(k_t, 1)$ and we obtain:

$$\Delta\Pi_{t+1} = 0 \Longleftrightarrow \Pi_t = \Pi^*\left(\frac{(1+n)k_{FE}}{\mathcal{S}(k_t, 1)}\right)^{\frac{1}{\phi_\pi - 1}} \text{ if } \Pi_t \geq \Pi_{kink} \tag{22}$$

We observe that when monetary policy is active ($\Pi_t \geq \Pi_{kink}$), then the stabilization of inflation requires that the level of inflation decreases when the level of capital is high (or the interest rate is low). To understand this configuration, rewrite the Fisher equation as $\Pi_{t+1} = \frac{1+i_t}{R_{t+1}}$. Stabilizing inflation $\Delta\Pi_{t+1} = 0$ is then equivalent to $\Pi_t = \frac{1+i_t}{R_{t+1}}$, or after log-linearization to $d\tilde{\Pi}_t = d\widetilde{1+i_t} - d\tilde{R}_{t+1}$. According to equation (17), increasing the level of capital in t yields more capital in $t+1$ and then a lower interest rate in $t+1$. Everything else being equal, i.e. with an unchanged nominal interest rate $d\widetilde{1+i_t} = 0$, the stabilization of inflation then requires that the level of inflation increases in t such that $d\tilde{\Pi}_t = d\tilde{\Pi}_{t+1} = -d\tilde{R}_{t+1} > 0$ when $d\tilde{k}_t > 0$. However, an active monetary policy yields an overreaction in the nominal interest rate such that $d\widetilde{1+i_t} = \phi_\pi d\tilde{\Pi}_t$ where $\phi_\pi > 1$. Hence, the initial effect of an increase in the level of capital through a decrease in the interest rate is reversed such that $d\tilde{\Pi}_t = d\tilde{\Pi}_{t+1} = \frac{d\tilde{R}_{t+1}}{\phi_\pi - 1} < 0$ when $d\tilde{k}_t > 0$. Accordingly, we can verify that when monetary policy is inactive ($\Pi_t < \Pi_{kink}$) and inflation is positive ($\Pi_t \geq 1$), this relationship is in the opposite direction. Indeed, in this case, $\Pi_{t+1} = \frac{\mathcal{S}(k_t,1)}{(1+n)D}$ which allows us to define:

$$\Delta\Pi_{t+1} = 0 \Longleftrightarrow \Pi_t = \frac{\mathcal{S}(k_t, 1)}{(1+n)\, D} \text{ if } 1 \leq \Pi_t \leq \Pi_{kink} \tag{23}$$

Finally, when monetary policy is active in the deflationary area ($\Pi_t < 1 \leq \Pi_{kink}$), we have $\Pi_{t+1} = \frac{\mathcal{S}(k_t,\Pi_t)}{(1+n)D}$ and we obtain:

$$\Delta\Pi_{t+1} = 0 \Longleftrightarrow \Pi_t = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)\, D} \text{ if } \Pi_t < 1 \leq \Pi_{kink} \tag{24}$$

Differentiating this equation in the neighborhood of $\Pi = 1$ yields $\left(1 - \frac{\mathcal{S}'_\Pi}{(1+n)D}\right) d\Pi_t = \frac{\mathcal{S}'_k}{(1+n)D} dk_t$. As is obvious, $\gamma = 0$ yields $\frac{d\Pi_t}{dk_t} > 0$. Indeed, in this case there is no wage rigidity, and the curve is strictly similar to the previous one. By contrast, if the wage rigidity is strong enough such that $\gamma > \frac{\alpha(1+n)D}{\alpha(1+n)D + (1-\alpha)sAk^\alpha_{\Pi=1}}$, the relation that links the level of capital to inflation in order to guarantee a stable level of inflation again decreases. In that case, the increase in the capital in t yields a sufficiently strong decrease in employment (due to the gap between the effective and the full employment real wage) so that the product in t decreases. Therefore, savings in t decreases as well as the capital in $t+1$ such that R_{t+1} increases. In all of these configurations, it is easy to show that for any given level of Π_t, $\frac{\partial\Delta\Pi_{t+1}}{\partial k_t} > 0$.

Three configurations can be illustrated by representing the curves Δk and $\Delta\Pi$ in the same phase plane (k, Π). If the equilibrium is unique, there are only two cases: full employment with inflation target (Fig. 1a) with (k_{FE}, Π^*) the stable steady state, or secular stagnation with underemployment and deflation (Fig. 1b) with (k_{Stag}, Π_{Stag}) the

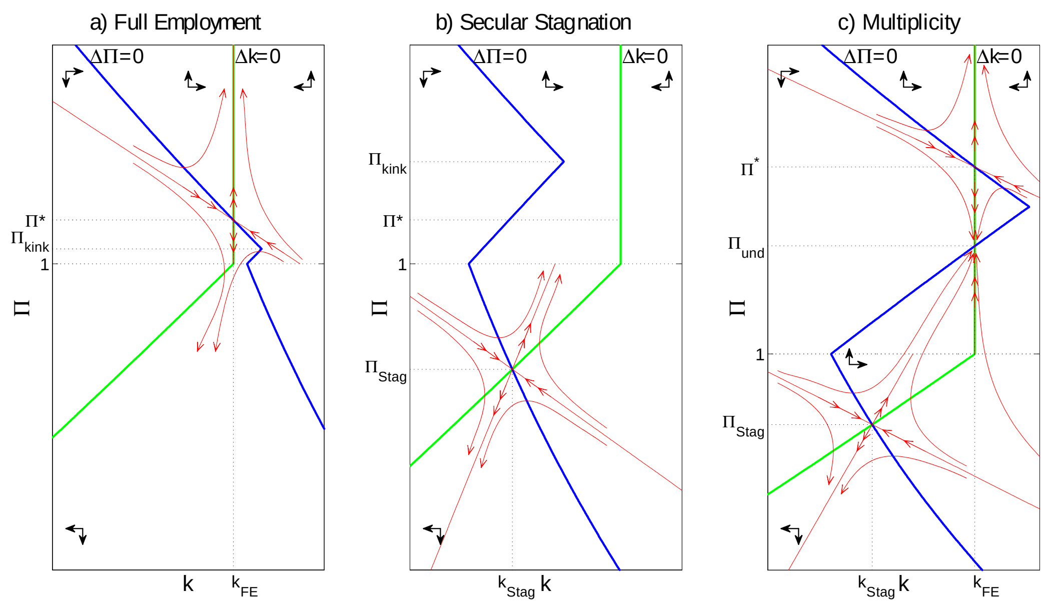

Figure 1: Features of the steady states

stable steady state. In both cases, the equilibria are globally determined with dynamics characterized by a unique saddle path. Finally, in a third configuration, the two preceding equilibria coexist with a third equilibrium of full employment and missed inflation target: (k_{FE}, Π_{und}). This equilibrium is undetermined and the other two are determined. These determined steady states are not unique. Therefore, they are only locally determined, not globally.

The question is that of the change from one dynamic time path to another, and in particular, what is it that may explain the fall into secular stagnation starting from a full-employment equilibrium. So we are naturally interested in the credit crunch as an eventual cause of the crisis.

3.3 Credit crunch and secular stagnation

What happens if the credit constraint is tightened? Paradoxically, as stressed in the Theorem 1 of Coeurdacier et al. (2015), a more constrained economy has a higher capital-to-efficient-labor ratio at full employment $\left(\frac{dk_{FE}}{dD} < 0\right)$, and a lower natural interest rate $\left(\frac{dR_{eq}}{dD} > 0\right)$. Indeed, from equation (17) and assuming with no loss of generality that $G = 0$, it follows that in the neighborhood of the saddle point steady state defined as

$k_{FE} = \frac{s}{1+n-s(1-\delta)}\left(Ak_{FE}^{\alpha} - D\right)$, the dynamics of capital is as follows: $dk_{t+1} = \varepsilon dk_t - \frac{s}{1+n}dD$, where $\varepsilon = \frac{s}{1+n}\left(A\alpha k_{FE}^{\alpha-1} + 1 - \delta\right) < 1$. This yields $\frac{dk_{FE}}{dD} = -\frac{s}{1+n}\frac{1}{1-\varepsilon} < 0$. As $k_{FE} = \frac{D}{R_{eq}}$, we deduce that at the steady state, $\frac{dR_{eq}}{R_{eq}} > \frac{dD}{D}$. These results come from the fact that the supply of savings is inelastic w.r.t. the interest factor while the present value of the borrowing constraint (loan application) has an elasticity of 1 w.r.t. the interest rate. A decrease in D automatically implies a decrease in the demand for credit which can be adjusted only by lower interest rates. Consequently, households are less indebted, increasing their future saving capacity and therefore the accumulation of savings.

Finally, it can be shown that if $D < \left(\frac{A}{\delta + \frac{1+n}{s}}\right)^{\frac{1}{1-\alpha}}$, then at the steady state we have $R_{eq} < 1$, i.e. the natural interest rate ($r_{eq} = R_{eq} - 1$) becomes negative, which can be problematic for monetary policy, as outlined in the introduction. Following a savings glut in the economy, monetary policy can be conducted so as to move toward the zero bound and thereby induce a type of long-term stagnation. Nominal wage rigidity combined with the positivity constraint of the nominal policy interest rate is a key element for displaying a situation of secular stagnation. The following lemma gives a necessary and sufficient condition for the existence of such an equilibrium (Walrasian disequilibrium).

Lemma 1 *A secular stagnation equilibrium exists and is locally determined iff* $R_{eq} < 1 \iff D < \left(\frac{A}{\delta + \frac{1+n}{s}}\right)^{\frac{1}{1-\alpha}}$. *If* $R_{eq} < \frac{1}{\Pi^*} \iff \Pi_{kink} > \Pi^*$, *then the secular stagnation equilibrium is the unique equilibrium.*

Proposition 2 *All things being equal, any sufficiently large decrease in* D *can bring the economy back into secular stagnation (Fig. 2).*

Figure 2 illustrates the fall into secular stagnation after a credit tightening at time $t = 0$[4]. Starting from the full employment steady state (the capital level is normalized to unity), if the credit crunch is sufficiently large, then the equilibrium interest rate is sufficiently negative and the conventional monetary policy can no longer be conducted actively. In this case, the secular stagnation regime is the unique equilibrium, and the economy plunges into recession with an underemployment of labor. Production is lower

[4] The parameters used to construct this figure are $\beta = 0.7$, $\alpha = 0.3$, $\gamma = 0.3$, $\phi_\pi = 2$, $\Pi^* = 1.02$, $n = 0$, $\delta = 1$, $A = 3.44$, $G = T = 0$, $D = 1.01$ before the shock and $D = 0.98$ after. This parameterization is mostly illustrative. At the full employment equilibrium, $R^k = 1.032 > R = 1.01$. At the secular stagnation equilibrium, $R^k = 1.037 > R = 1.028$. In both cases, the credit constraint (5) is then binding.

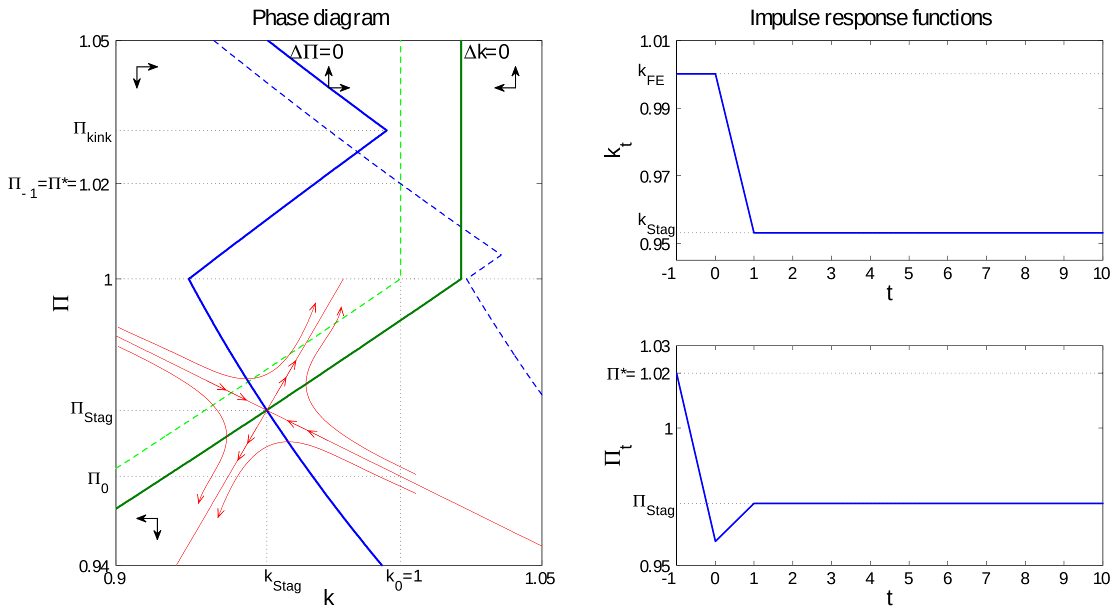

Figure 2: Dynamics of Secular Stagnation ($\Delta D < 0$)

than its potential level and the inflation rate is negative. In the first period of the shock, as the capital is predetermined, it does not change, $k_0 = k_{FE}$. However, as soon as the second period starts, it reaches its steady state level of secular stagnation[5], $k_t = k_{Stag}$ $\forall t \geq 1$. The fall into secular stagnation is very fast. We also observe an overshooting of the level of deflation just after the shock. Indeed, as the level of capital already installed cannot be adjusted initially, there is an excessive supply, which results in higher deflation: $\Pi_0 < \Pi_{Stag}$. Deflation then reaches its lower steady state level one period after the shock, $\Pi_t = \Pi_{Stag}$ $\forall t \geq 1$. Formally, it can be shown from the dynamics (21) that $\Pi_{Stag} = \frac{\mathcal{S}(k_{FE},\Pi_0)}{(1+n)D} = \frac{\mathcal{S}\left(k_{Stag},\Pi_{Stag}\right)}{(1+n)D}$. Therefore, as $\mathcal{S}'_k > 0$ and $\mathcal{S}'_\Pi > 0$ if $\Pi < 1$, $k_{FE} > k_{Stag}$ yields $\Pi_0 < \Pi_{Stag}$ (< 1). Note also that, as in Eggertsson and Merhotra (2014), and unlike in Eggertsson and Krugman (2012), the secular stagnation equilibrium is persistent as long as the credit crunch lasts. From this point of view, active policies against the credit crunch are crucial in the fight against secular stagnation.

Assume now that such a policy is so efficient that it had restored confidence in the financial markets. What then happens if the credit constraint returns to its original position? We return of course to the initial situation where the only determined equilibrium is characterized by full employment. Nevertheless, as illustrated in Figure 3, capital re-

[5]The local dynamics is associated with an eigenvalue equal to zero (see Appendix).

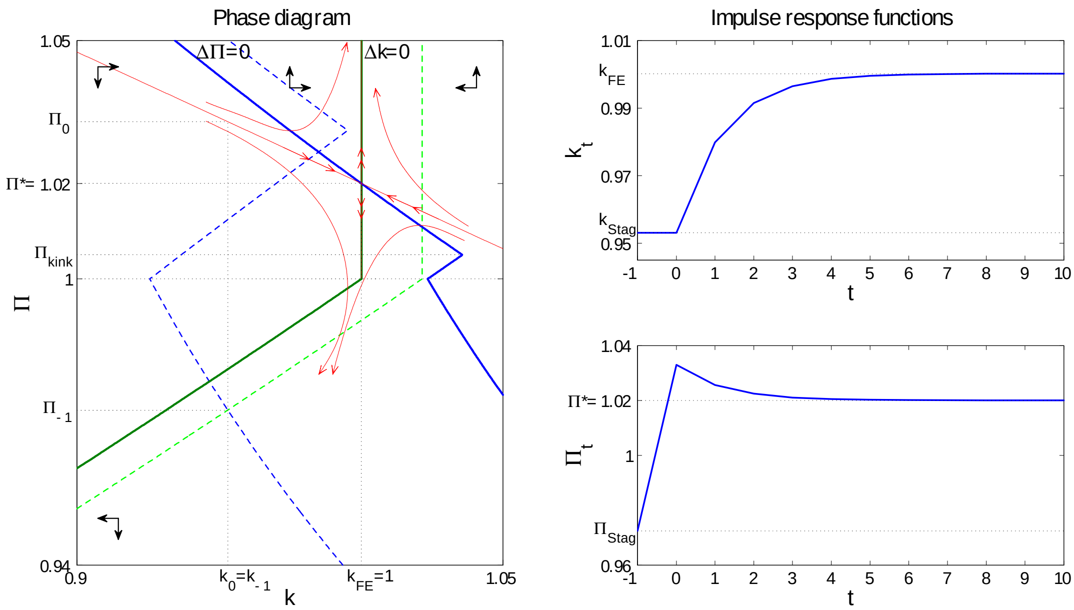

Figure 3: Dynamics of recovery

turns to its original level only after seven periods. In other words, the fall into secular stagnation takes place significantly faster than the recovery.

Proposition 3 *Capital accumulation induces an asymmetry in the dynamics such that recovery takes longer than it does to fall into secular stagnation (Figs. 2 and 3).*

This proposition suggests that economic policies to combat secular stagnation must be implemented within the shortest possible time, preferably even before the secular stagnation appears.

We observe again that the dynamics of inflation are characterized by an overshooting. However, in this case monetary policy is active, and an increase in ϕ_Π can mitigate the extent of the overshooting. To see this, consider first the autonomous dynamics of capital characterized by $k_{t+1} = \frac{1}{1+n}\mathcal{S}\left(k_t,1\right)$ in the neighborhood of the full employment equilibrium: $\frac{k_{t+1}-k_{FE}}{k_{FE}} = \eta\frac{k_t-k_{FE}}{k_{FE}}$ where $\eta < 1$. The stable manifold as illustrated in Fig. 3 is then characterized by the following equation $\frac{\Pi_t-\Pi^*}{\Pi^*} = -\frac{\eta}{\phi_\Pi-\eta}\frac{k_t-k_{FE}}{k_{FE}}$ (see Appendix), where $\lim\limits_{\phi_\Pi=\infty}\left(-\frac{\eta}{\phi_\Pi-\eta}\right) = 0 \iff \lim\limits_{\phi_\Pi=\infty}\left(\Pi_t-\Pi^*\right) = 0\ \forall t$. The characterization of the dynamics of secular stagnation with respect to capital accumulation is important not only to determine the speed of converge towards the secular stagnation steady state and its dynamic features (overshooting of deflation for example), but also to highlight the asymmetry.

It is worth noting that the existence of a secular stagnation equilibrium is not due solely to the effects of the credit tightening. In particular, without going further into the details of the model, a decrease in the growth of the labor force (n decreases) and an increase in life expectancy (which can be associated with an increase in β then with an increase in the saving rate s) also play a role in explaining secular stagnation. Secular stagnation may well have become the "new normal" (Summers, 2013). In addition to the stabilization of financial markets, any economic policy that could be effective in the fight against secular stagnation must be considered.

4 Macroeconomic policy

As secular stagnation can exist only as wages are downwardly rigid, we could naturally think that promoting growth and employment proceeds through increasing the flexibility of the labor market. However, in secular stagnation, this would have a paradoxical impact as a decrease in wage rigidity γ tends to reduce production and employment. This result might seem surprising. Indeed, when there is no rigidity, i.e. $\gamma = 0$, actual production is always equal to its potential. However, this result can be easily explained. In secular stagnation, a stronger nominal wage flexibility results in recessionary effects, because it generates deflationary pressures, and therefore, as monetary policy is constrained by the ZLB, an increase in the real interest rate $\left(R = \frac{1}{\Pi}\right)$. Demand, and then effective production, are reduced at equilibrium. Paradoxically, a higher nominal wage flexibility yields an increase in the real wage. Increasing labor market flexibility, unless this becomes total, then has counterproductive effects for the economy (Eggertsson and Krugman, 2012). Another Keynesian paradox, named "Paradox of toil" (Eggertsson, 2010) arises in our setting: as higher productivity generates deflationary pressures, it also leads to lower production and employment. Monetary and fiscal policies must then be considered in detail.

4.1 Monetary policy

Suppose that the economy is characterized by a unique deflationary secular stagnation equilibrium as shown in Figure 1b ($\Pi_{kink} > \Pi^*$). To get out of such an equilibrium, the monetary authorities can choose to increase the inflation target Π^*. In this case, we see that this increase in the inflation target also increases Π_{kink} but less than proportionately: $\frac{\frac{d\Pi_{kink}}{\Pi_{kink}}}{\frac{d\Pi^*}{\Pi^*}} = 1 - \frac{1}{\phi_\pi} < 1$. Starting from a situation characterized by $\Pi_{kink} > \Pi^*$, two configurations are then possible. First, the increase in Π^* is not sufficient and thus Π_{kink} remains below Π^*. In this case, the secular stagnation equilibrium is unique and unchanged, and the monetary policy is ineffective. Second, the increase in Π^* is sufficient such that Π^* becomes greater than Π_{kink}. In this case, as shown in Figure 4, the full employment equilibrium appears[6]. This does not mean that monetary policy will necessarily be effective. It is observed in fact that the secular stagnation equilibrium still exists. Therefore, nothing indicates that inflation expectations will automatically jump on the high saddle path converging to Π^*.

Proposition 4 *An increase in the inflation target, even if high enough, is not a sufficient condition for the monetary policy to be efficient to get out the economy from secular stagnation (Fig 4).*

Two properties need to be emphasized. First, the secular stagnation equilibrium is locally determined. Second, any inflation level $\Pi \in (\Pi_{Stag}, \Pi_{\sup})$ gives rise to a trajectory potentially convergent to $\Pi_{und} < \Pi^*$, where (k_{FE}, Π_{und}) is locally undertermined [7]. From this point of view, in line with Benhabib et al. (2001), we can consider that the Taylor rule involves a risk of destabilization, even though its primary ambition is to be stabilizing.

In such a configuration, anchoring private agents' inflation expectations in behaviour that will lead to the desired equilibrium becomes a difficult task for the monetary authorities. For inflation targeting to be effective, it is crucial in particular that the central bank have sufficient credibility (Woodford, 2004). So long as private agents do not believe the central bank when it announces a new inflation target, it is likely that the inflation obtained will not meet the target. The central bank's credibility is thus directly related to how well it has managed to achieve its targets in the past. In secular stagnation, the central bank cannot by definition meet a target (monetary policy is inactive). This property suggests that the central bank must react quickly enough to avoid the deflationary trap.

[6] In Figure 3, the inflation target becomes $\Pi^* = 1.07$ instead of 1.02 in Figures 1 and 2.

[7] In the appendix, we check that the normal equilibrium is associated locally to a higher eigenvalue to the unit and a lower eigenvalue, while the second balance is associated with two lower eigenvalues to unity.

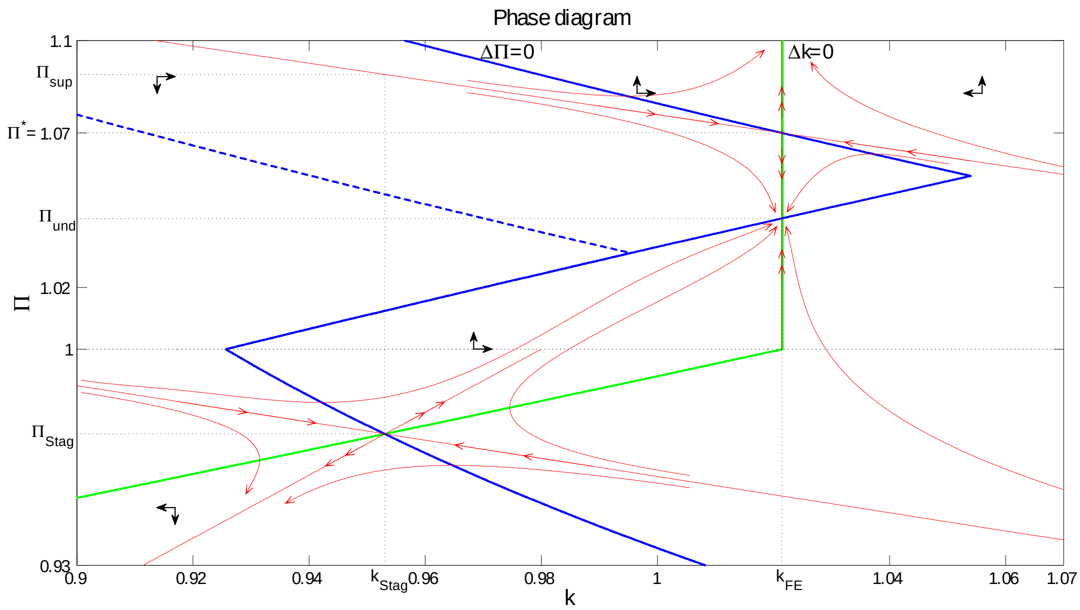

Figure 4: Secular stagnation, monetary policy and global indeterminacy ($\Delta\Pi^* = 0.05$)

4.2 Fiscal policy

It is easy to see that fiscal policy will have inflationary effects that, if strong enough, can help to escape secular stagnation.

Proposition 5 *There exist* $G_{\text{sup}} = AD^{\alpha} - \left(\frac{1+n}{s} + \delta\right) D$ *and* $G_{\text{inf}} = A\left(D\Pi^*\right) - \frac{1+n-s(1-\delta)}{s} D\Pi^*$, $G_{\text{sup}} \geq G_{\text{inf}}$, *such that:*

- $G < G_{\text{inf}}$ *yields that the secular stagnation equilibrium is unique and then globally determined,*
- $G > G_{\text{sup}}$ *yields that the full employment equilibrium is unique and then globally determined (Fig. 5).*

This Proposition sheds light on two threshold levels of public spending. If the fiscal stimuli are "too small" such that $G < G_{\text{inf}}$, the economy remains in secular stagnation (Fig. 6). Even if economic growth is enhanced and unemployment reduced, there remains some unemployment, and additional public spending is required to escape secular stagnation. This is the case if $G > G_{\text{sup}}$. However, it is worth noting that the associated increase in taxes reduces the incentives to save and then harms the capital accumulation and the long-term potential, $\frac{dk_{FE}}{dT} < 0$. Accordingly, there can be a trade-off between the exit from

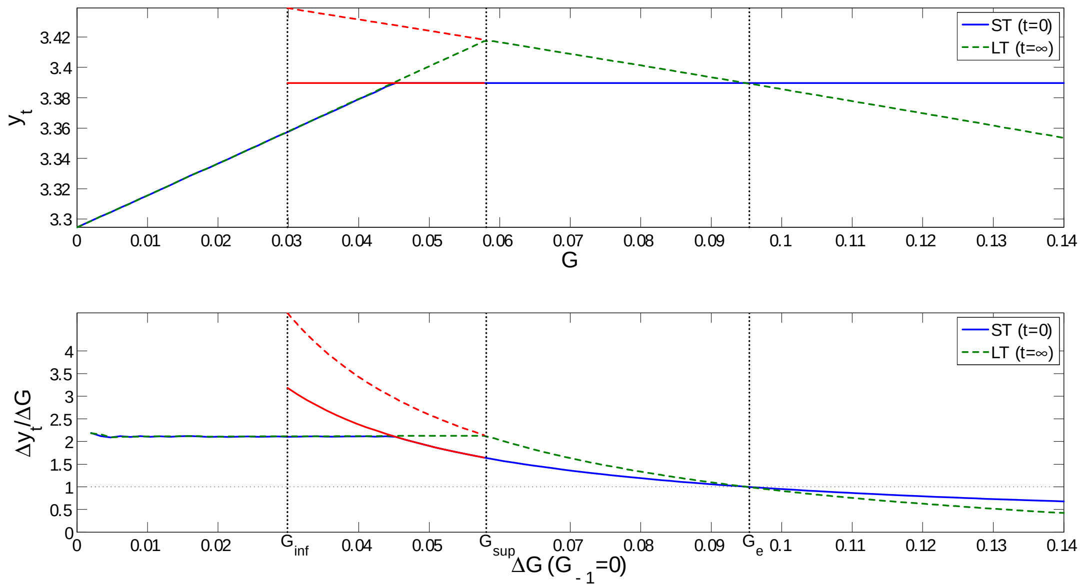

Figure 5: Impact of fiscal policy: production and multipliers (short and long run)

secular stagnation and the accumulation of capital. Note also that if $G_{\text{sup}} < G < G_{\text{inf}}$, the equilibrium is not determined. The economy can jump out of the secular stagnation to converge towards the full employment equilibrium (in red on Fig. 5), or can stay in it.

Lemma 6 *There exists a thereshold G_e such that if $G \geq G_e$ then the associated full employment level of capital is lower that the initial stagnation level, $k_{FE}(G \geq G_e) \leq k_{Stag0}$.*

Proposition 7 *A permanent increase in public spending, $\Delta G = G = G_t > 0\ \forall t \geq 0$, that allows the economy escaping secular stagnation, $G > G_{\text{sup}}$, has the following properties depending on its size:*

- *if $G < G_e$, the fiscal multipliers are all greater than one, $\frac{\Delta y_t}{\Delta G} > 1\ \forall t \geq 0$, and growing with time, $\frac{\partial \frac{\Delta y_t}{\Delta G}}{\partial t} \geq 0$ (Figs. 5 and 7),*
- *if $G \geq G_e$, the fiscal multipliers are all lower than one, $\frac{\Delta y_t}{\Delta G} \leq 1\ \forall t \geq 0$, and decreasing with time, $\frac{\partial \frac{\Delta y_t}{\Delta G}}{\partial t} \leq 0$ (Figs. 5 and 8).*

Hence, if $G_{\text{sup}} < G < G_e$, taxation is sufficiently low so that capital accumulation is promoted compared with the initial equilibrium with no public spending. Therefore, after period 0, the capital stock increases and the fiscal multiplier exhibits an increased

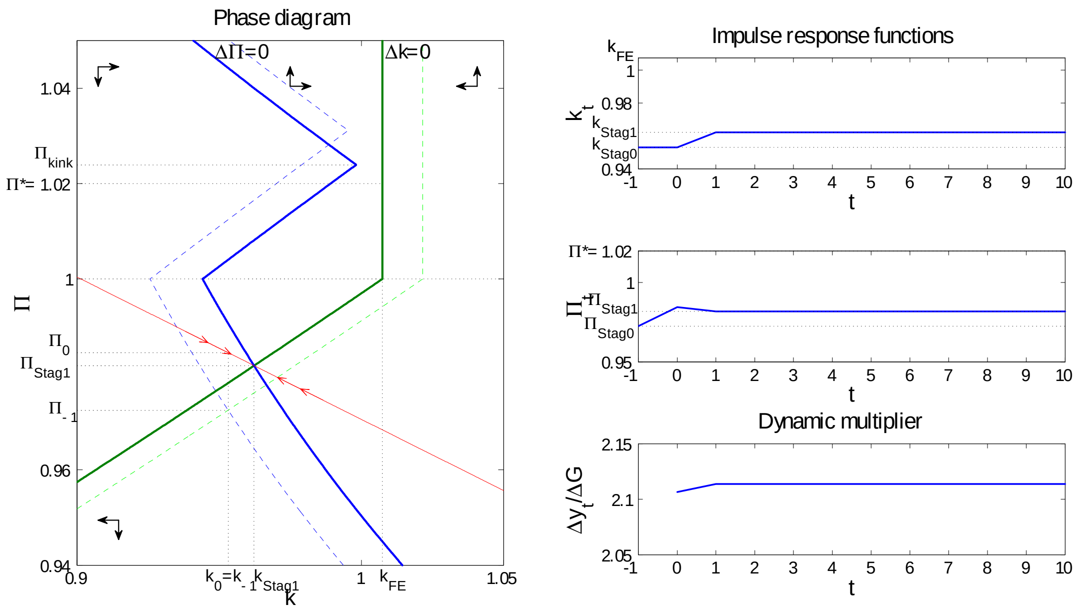

Figure 6: Fiscal policy: Impact of a "too small" fiscal stimulus ($G = 0.02 < G_{\text{inf}}$)

shape. In addition, the latter is always larger than one. Indeed, we know from equations (16) and (12) that at equilibrium $y_t = \frac{1+n}{s}k_{t+1} + D + G$, which yields $\frac{dy_t}{dG} = 1 + \frac{1+n}{s}\frac{dk_{t+1}}{dG}$. This policy is clearly efficient to escape secular stagnation. By contrast, if taxation is too large, the incentives to save are reduced, so that there is less capital at the full employment equilibrium compared with the initial secular stagnation equilibrium. The fiscal multiplier exhibits a decreasing shape and is lower than unity. This policy is clearly inefficient even if it helps escaping secular stagnation.

5 Conclusion

The secular stagnation hypothesis invites to reconsider traditional macroeconomic analysis and therefore the design of economic policy. In this article, we have developed a capital accumulation model that integrates two types of market imperfections that affect, respectively, the credit market (rationing) and the labor market (nominal rigidity). The lessons that can be drawn from this model are many.

The emergence of a nominal rate close to zero ("zero lower bound") raises fears of a loss of effectiveness of "conventional" monetary policy based mainly on the setting of an official interest rate. In a context where full employment equilibrium inflation and interest

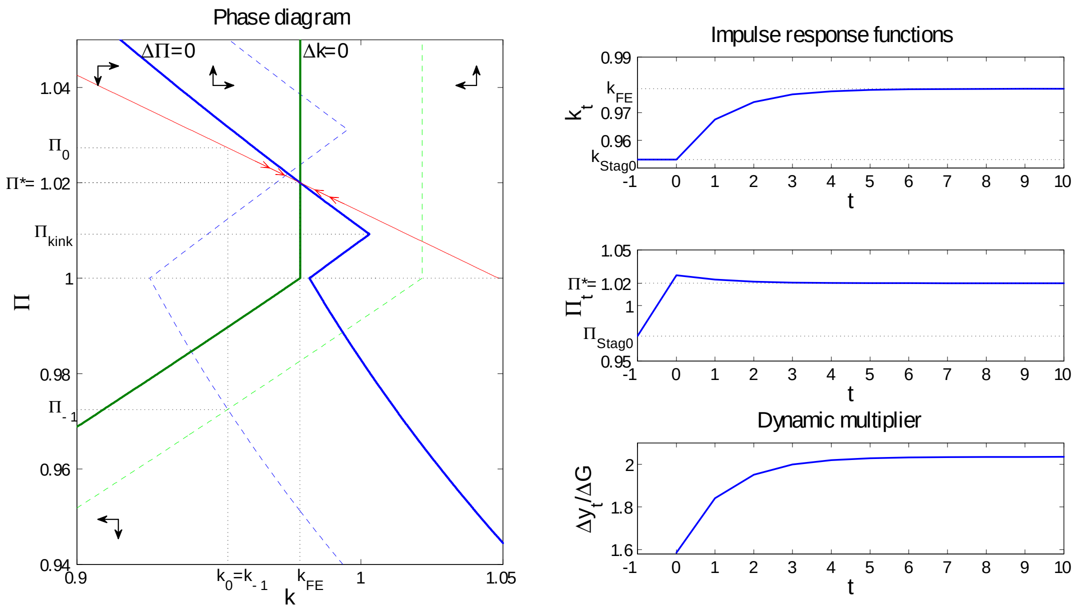

Figure 7: Fiscal policy: impact of a large stimulus ($G_{\text{sup}} < G = 0.06 < G_e$)

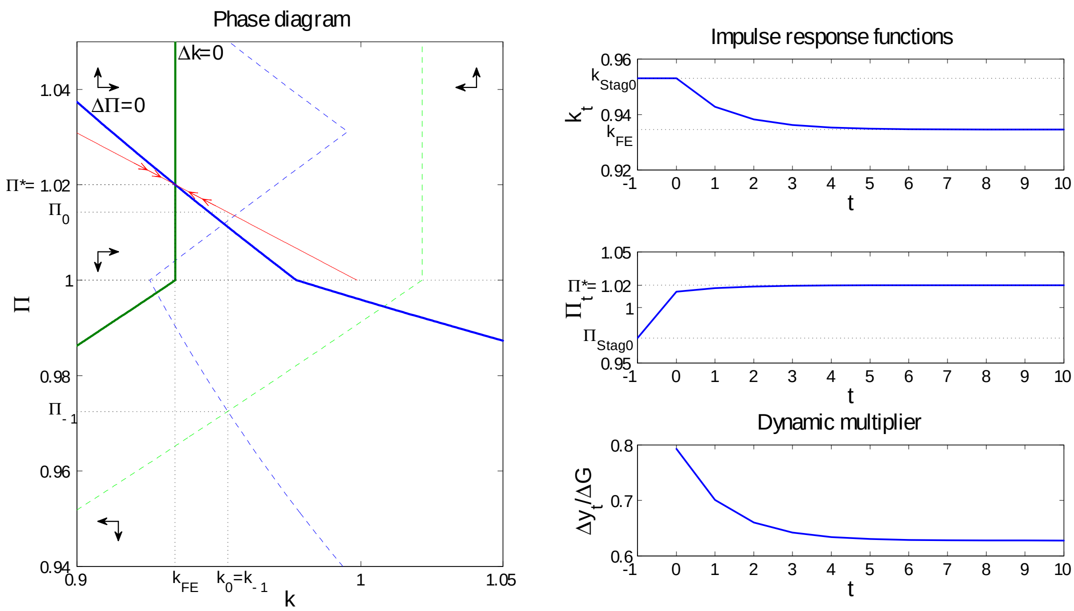

Figure 8: Fiscal policy: Impact of a "too large" fiscal stimulus ($G = 0.12 > G_e$)

rates are both negative, macroeconomic dynamics can lead to permanent unemployment trajectories characterizing a regime of secular stagnation.

In order to avoid the ZLB, it then urgently required to create inflation but also to avoid "bubbles" in speculative assets (Tirole, 1985). This might command specific regulation (Gali, 2014). The existence of a deflationary equilibrium calls into question the merits of monetary policy rules that are overly focused on inflation (Benhabib et al., 2001). The model that we develop also teaches us to beware of deflationary effects of increased productivity policies (increase A).

If reducing savings to raise the natural interest rate (for example, improving access to credit) is also an alternative idea, its impact on potential GDP is negative. There is a clear choice between escaping secular stagnation and depressing potential GDP. Dynamic multipliers are upper than one unless the fiscal stimilus is too strong.

One interesting solution could consist in financing infrastructure, education and R&D policies (increase A) by public borrowing (higher R_{eq}). Indeed, a strong investment policy (public or private) can satisfy twin objectives: supporting aggregate demand and developing potential production.

Two extensions of this model require special attention.

First, it would be useful to introduce a hysteresis effect due to a shock on the production or demand. Hysteresis is characterized by the persistence of consequences while their causes have disappeared. Time cost may result in lower skills of unemployed workers and the destruction of unused productive capital. The productive potential would then automatically shrink. Staying put in a state of crisis is particularly harmful, and the output gap needs to be closed quickly so as to avoid the accumulation of negative effects, although paradoxically a negative impact on productivity can have potentially favorable inflationary effects.

Second, this model neglects the international dimension by focusing only on domestic markets. Bernanke (2015) asserts that the opportunity for profitable investment outside the national borders reduces the relevance of the secular stagnation hypothesis. Nevertheless, it is undeniable that the opening of borders has profoundly changed the macroeconomic forces at work. Questioning the effect of international capital flows in terms of secular stagnation remains interesting and will be left for future research.

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Appendix: Study of the trajectories in the neighborhood of steady states

Case 1. Full employment and satisfied inflation target $(i \geq 0,\, l = 1)$:

The equation (19) can be written $\frac{D}{k_{t+1}} = \frac{1+i^*}{\Pi^{*\phi_\pi}} \frac{\Pi_t^{\phi_\pi}}{\Pi_{t+1}}$. After log-linearization of this equation and the equation (17), we obtain:

$$\tilde{\pi}_{t+1} = \phi_\pi \tilde{\pi}_t + \eta \tilde{k}_t \tag{25}$$

and

$$\tilde{k}_{t+1} = \eta \tilde{k}_t \tag{26}$$

where $\eta = \frac{\alpha A k^{\alpha-1} + 1 - \delta}{A k^{\alpha-1} - \frac{D+G}{k} + 1 - \delta}$. In matrix form, the local dynamics can be expressed as follows:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} \phi_\pi & \eta \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

where $\lambda_1 = \eta$ and $\lambda_2 = \phi_\pi$ are the two eigenvalues $(\det(M) = \eta\phi_\pi = \lambda_1\lambda_2$ and $tr\,(M) = \phi_\pi + \eta = \lambda_1 + \lambda_2)$. In the neighborhood of the steady state, we have $\eta < 1$ and $\phi_\pi > 1$, the full employment steady state with satisfied inflation target is a saddle point.

Case 2. Full employment with unsatisfied inflation target $(i = 0,\, l = 1)$:

In this case, the equation (17) and its linearized form (26) are still valid. By contrast, we have $\Pi_{t+1} = \frac{k_{t+1}}{D}$ and then:

$$\tilde{\pi}_{t+1} = \tilde{k}_{t+1} = \eta \tilde{k}_t \tag{27}$$

In this case, the local dynamics is characterized by the following system:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} 0 & \eta \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

where $\lambda_1 = \eta < 1$ and $\lambda_2 = 0$ are the two eigenvalues. In this case the dynamics are undetermined.

Case 3. Secular stagnation equilibrium $(i = 0,\, l < 1)$:

We deduce from (16):

$$\tilde{y}_t = \nu \tilde{k}_{t+1} - \frac{(1-\delta)\, s}{1+n} \nu \tilde{k}_t \tag{28}$$

where $\nu = \frac{1}{1+\frac{s}{1+n}\frac{D+G}{k}-\frac{(1-\delta)s}{1+n}}$. Furthermore as $y_t^s = Ak_t^{\alpha} l_t^{1-\alpha}$, we deduce $\tilde{y}_t = \alpha \tilde{k}_t + (1-\alpha)\, \tilde{l}_t$. Profit maximization gives $(1-\alpha)\, k_t^{\alpha} l_t^{-\alpha} = w_t$ and $l_t = \left(\frac{w_t}{1-\alpha}\right)^{\frac{-1}{\alpha}} k_t$. We find $\tilde{l}_t = \frac{-1}{\alpha}\tilde{w}_t + \tilde{k}_t$, and:

$$\begin{aligned} \tilde{y}_t &= \alpha \tilde{k}_t + (1-\alpha)\left(\frac{-1}{\alpha}\tilde{w}_t + \tilde{k}_t\right) \\ &= \tilde{k}_t - \frac{1-\alpha}{\alpha}\tilde{w}_t \end{aligned} \tag{29}$$

Moreover, given the rigidity of nominal wages expressed by $w_t = \gamma w_t \frac{1}{\Pi_t} + (1-\gamma)(1-\alpha)\, k_t^{\alpha}$, the log-linear form can be written as follows $\tilde{w}_t = \frac{\gamma}{\Pi}(\tilde{w}_t - \tilde{\pi}_t) + (1-\gamma)(1-\alpha)\,\alpha \frac{k^{\alpha}}{w}\tilde{k}_t$, where $\frac{k^{\alpha}}{w} = \frac{1-\frac{\gamma}{\Pi}}{(1-\gamma)(1-\alpha)}$. We find:

$$\tilde{w}_t = \alpha \tilde{k}_t - \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}}\tilde{\pi}_t \tag{30}$$

By introducing equation (30) in (29), we obtain:

$$\tilde{y}_t = \alpha \tilde{k}_t + \frac{1-\alpha}{\alpha}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}}\tilde{\pi}_t$$

From (28), we find:

$$\tilde{k}_{t+1} = \left(\frac{\alpha}{\nu} + \frac{(1-\delta)\, s}{1+n}\right)\tilde{k}_t + \frac{1-\alpha}{\alpha\nu}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}}\tilde{\pi}_t$$

Moreover, as $\Pi_{t+1} = \frac{k_{t+1}}{D}$, we have:

$$\tilde{\pi}_{t+1} = \tilde{k}_{t+1},$$

and we identify the dynamics in its matrix form:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1-\alpha}{\alpha\nu}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} & \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \\ \frac{1-\alpha}{\alpha\nu}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} & \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

$\det(M) = 0$ et $tr(M) = \frac{1-\alpha}{\alpha\nu}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} + \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n}$. If $\delta = 1$, then we deduce immediately that the two associated eigenvalues are $\lambda_1 = \frac{1-\alpha}{\alpha\nu}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} + \frac{\alpha}{\nu}$ and $\lambda_2 = 0$. A necessary and sufficient condition for a saddle-point equilibrium is $\lambda_1 > 1$. This condition is satisfied iff:

$$\nu < \alpha + \frac{1-\alpha}{\alpha}\frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}}$$

This condition is equivalent to the observation in the space (Y, Π) of a slope of demand $\left(\frac{1}{\nu}\right)$ which is greater than that of supply. This condition is always met when the secular stagnation equilibrium exists (see Lemma 1).