

# CAN ARTIFICIAL ECONOMIES HELP US UNDERSTAND REAL ECONOMIES?

**Alan Kirman**

*GREQAM, Aix Marseille Université, EHESS*

---

This paper argues that the path followed by modern macroeconomic theory excludes the analysis of major endogenous movements in macroeconomic variables. Rather than persist with models based on the idea that the economy behaves like a rational individual we should build models of the economy as a complex system of interacting agents. Profiting from the advances made in computer science we can now build agent based or computational models which allow us to model the out of equilibrium behaviour of such systems. They allow us to remove many of the restrictive assumptions of standard models and to incorporate the heterogeneity of economic agents and the evolution of the network that governs the interactions between the individuals and firms in the economy. Such models can help fill the theoretical void with which policymakers declare that they have been faced in the current crisis.

*Keywords:* crisis, complex system, heterogeneity, networks.

---

*“First, we have to think about how to characterise the homo economicus at the heart of any model. The atomistic, optimising agents underlying existing models do not capture behaviour during a crisis period. We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. We need to entertain alternative motivations for economic choices. Behavioural economics draws on psychology to explain decisions made in crisis circumstances. Agent-based modelling dispenses with the optimisation assumption and allows for more complex interactions between agents. Such approaches are worthy of our attention.”*

Jean-Claude Trichet (2010).

Recently, considerable dissatisfaction has been expressed not only by Trichet but also by Bernanke, Turner and many others in policymaking positions, with economic theory in general and

macroeconomic theory in particular. This leads to two questions, why does macroeconomic theory seem to have become, as Trichet and others suggest, so irrelevant particularly in times of crisis and to what extent are agent based or computational economic models a more satisfactory alternative?

Before answering these questions it is worth observing that computational or algorithmic models have a long and distinguished tradition in economics. One has only to think of Von Neumann's work and of Scarf's contributions, particularly to the computation of economic equilibria and the contributions of Dantzig and Kuhn in using mathematical programming, to see this. What agent based modelling is doing is to renew the tradition of using an algorithmic approach to model economic phenomena. The recent, rapid development of computer science explains the resurgence of this approach. Now computational models have two significant advantages. Firstly there is a wealth of data on the behaviour of individuals and from this we can categorise different behavioural types and use these as a basis for building agent based models. Secondly, given the progress that has been made in computational power and capacity we are now capable of simulating the dynamics of very large systems of heterogeneous interacting agents. This is, for example, the ambition of the EU project, Futur-ICT.

## 1. The evolution of modern macroeconomics

But to return the first question as to how macroeconomic theory seems to have become so irrelevant, it is worth looking at the recent evolution of the discipline of economics. Economic theory has developed, as a criterion for "rigorous" analysis that our models should be based on the intellectual bedrock of the "rational individual" or homo oeconomicus. The rationality of economic agents is not defined by the intuitive idea that individuals do not act against their own interest, but rather, that they have preferences, which satisfy the axiomatic structure typified by that used in the Arrow Debreu model. For firms it is even simpler, they choose that combination of inputs and outputs which maximises their profit. If we take time into account we assume that our agents, consumers and firms, have infinite horizons and that they discount the future appropriately. Lastly if the future is uncertain, they know the nature of that uncer-

tainty, they know the probability distribution of future events, they have “rational expectations”. This is the basis for the underlying theoretical model of modern economics the “general equilibrium model” initiated by Walras, improved by Pareto and honed to perfection by their successors and which culminated in the Arrow Debreu model and was closed by assuming rational expectations. The task of the economist in this tradition is, therefore, to make assumptions about individual preferences and technologies and to build models, particularly macroeconomic models on that basis. One then finds the equilibrium of the system and examines the characteristics of such equilibrium states. Whenever objections as to the realism of the model are made, the response is to modify some aspect of it to take the criticism into account but not to put the whole exercise in doubt.

The problems with this approach are well known and need not be elaborated here. Suffice it to say that we know that restricting ourselves to only making assumptions on individual preferences and technologies will allow us to say nothing about how one might get to an equilibrium nor whether such a state is unique. Thus, all that we can say is that, under our assumptions, an equilibrium will exist. But, since the idea that the only way to construct “scientific” models is to limit ourselves to assumptions on individuals, all that we can do is assume the economy to be in equilibrium all the time. This is precisely where modern macroeconomics has arrived. The answer to building models which allow us to say something about how economies function out of equilibrium and how they might move to equilibrium seems to be simple, add assumptions about the way in which people interact and the institutions that govern them, and this is precisely what Lucas suggested.

*“Applications of economic theory to market or group behaviour require assumptions about the mode of interaction among agents as well as about individual behaviour.” Lucas (1988).*

However, curiously, rather than do this, and maybe change the basic approach to economics, the solution that has been adopted is to assume that the economy as a whole acts like a rational individual, an assumption for which there is no theoretical justification, (see Kirman (1992)). Perversely, the idea of enlarging the model to incorporate assumptions about how it is organized has been considered to be “unscientific” whilst the assumption that the economy

acts like an individual was not. Not only was this intrinsic contradiction ignored, the underlying assumption that we should only make assumptions about individuals was actually elevated to the status of a principle. Lucas, some years before making the observation just mentioned, explicitly rejected the idea of adding parameters to the basic model to allow even for an adjustment process. In fact at that time he said,

*“Now, I am attracted to the view that it is useful, in a general way, to be hostile toward theorists bearing free parameters, so that I am sympathetic to the idea of simply capitalizing this opinion and calling it a Principle.”* Lucas (1980, p. 709).

But, by doing so he made it impossible for economists, who followed his lead, to study out of equilibrium phenomena. Since, with the assumptions that he considered scientific, all that we could show was that an equilibrium exists, the economy should be studied in that state. Even if one considers dynamics, the idea would be that the economy simply evolves through a sequence of equilibrium, thus business cycles are equilibrium phenomena. The fact that, at another point of time, Lucas suggested that we needed additional assumptions on the organisation of the economy in addition to the assumptions on individuals, did not deviate macroeconomic theorists from the path which he had encouraged them to pursue.

Following this theoretical path has had important consequences for the way in which macroeconomics has been developed. Despite Lucas' observations, it is generally assumed in macroeconomic models that the way in which the economy or markets are organised, as such, has little impact on economic outcomes. Individuals participate in anonymous markets in which they are price takers and little is said about who sets the prices and how. When exceptions are made to this basic structure it is significant that economists refer to “imperfect competition” and market “imperfections”. Thus there is a benchmark model in which individuals interact only through the price system and other situations in which individuals react to each other are thought of as deviations from the norm. Direct interaction, and the externalities that go with it are either declared to be the subject of game theory or are incorporated with difficulty into a modified GE model.

Our attraction for the idea of economics as a “science” which stems from Walras' obsession with showing that we could develop a

general internally consistent mathematical model of the economy, has driven us into a corner. The attitude of many theoretical economists to real world economic problems is directly in the spirit of Bourbaki from whom Debreu took his inspiration. As Bourbaki<sup>1</sup> said,

*“Why do applications [of mathematics] ever succeed? Why is a certain amount of logical reasoning occasionally helpful in practical life? Why have some of the most intricate theories in mathematics become an indispensable tool to the modern physicist, to the engineer, and to the manufacturer of atom-bombs? Fortunately for us, the mathematician does not feel called upon to answer such questions.”*  
Bourbaki, (1949, p. 2).

Thus, in that spirit, the furthering of economy theory was seen as an avenue to more advanced models and not as a pursuit of explanations of economic phenomena. We became totally preoccupied by the properties of the economy in an equilibrium state. But, given the important results established in the '70s<sup>2</sup> it became clear that we had, within our “scientific” models, to abandon the concern with how the equilibrium prices are established and how the economy evolves towards equilibrium. There was almost no consideration of the idea that the economy might never be in equilibrium in the standard sense. So theorists have concentrated on the properties, in particular, the efficiency, of equilibrium states. They have insisted on the rigour of the analysis, but much less on the realism of the assumptions. In the end, the mathematical road that we followed petered out some 40 years ago in pure theory and has only remained in macroeconomic theory.

## 2. An alternative approach

Keynes once remarked, and this remark has been widely cited recently, that economists should become more like dentists, using such knowledge of the economy that they have acquired to improve the health of the patient particularly in times of crisis. Colander (2011) refers to this as, an “engineering”, rather than a “scientific” approach. Rather than developing general theories which have the ambition of giving us a clear, if simplified, vision of the economy as

---

1. Nicolas Bourbaki was, of course, a pseudonym for a group of mathematicians mainly based in Paris.

2. The results in question are those of Sonnenschein (1972), Mantel (1974) and Debreu (1974) himself.

a whole and independent of context, he argues that we should concentrate on developing models capable of providing us with recommendations for specific situations. Unconcerned with the basic physics that underlies the tools that he uses, the engineer tries to solve problems, often relying on generous approximations to give him self a margin of safety.

I would like to argue that we should not necessarily abandon the idea of a general model, after all Keynes called his major contribution, “The General Theory...”. This does not mean that we should aim at a model of everything, but rather, that we can establish some basic principles on which our models should be built. However, if we are to have such basic foundations they should be radically different to those that we are in the habit of using. Our basic model, as suggested in Miller and Page (2007), Epstein (2007) and Kirman (2010) should be one of a complex system of interacting agents who learn and adapt (for an early exposition, see e.g Aoki (1996)). Their behaviour and their interaction with each other generates aggregate phenomena, from which they again learn or to which they adapt. There are positive feedbacks, and we have no reason to believe that such a system will converge to a stationary equilibrium. We have therefore to study economies that are out of equilibrium and how they evolve. In such systems the aggregate behaviour is not like that of an individual and the way in which individuals react to each other will be an important feature of the evolution of the economy. Two things should be emphasised. Firstly, we cannot assume the aggregation problem away as is currently done in macroeconomics. Secondly, we need to understand out of equilibrium dynamics.

I claim therefore that we have to turn things inside out and bring direct interaction to the centre of the stage. Furthermore, I claim that we should radically simplify our models of individuals and that, in so doing, we may still observe interesting and complicated aggregate behaviour which is, however, the result of the aggregation itself and not of the complicated behaviour of some “representative individual”. We should treat the economy as a complex system and, as in other disciplines, we should not expect the system to behave like an individual.

The way to do this is by building models of simple individuals who follow simple rules and interact with each other just as molecules in biological systems or particles in physical systems. This is at

the heart of agent based models or computational economics. Indeed, the now widely accepted definition of this approach is that given by Leigh Tesfatsion<sup>3</sup>, (2002) where she says the following,

*“Agent based computational economics is the computational study of economic processes modelled as dynamic systems of interacting agents.”*

The fact that we are talking about a computational approach does not mean that we are abandoning any attempt to obtain analytical results, but does mean that a reasonable way to proceed is to try to obtain formal results in simple models and, where this proves too difficult in more general models, to use simulations and to see whether the results persist in those contexts. The formal analysis is more likely to come from statistical physics (see e.g. Blume (1993)), discrete mathematics and computer science, than the sort of mathematics that we use, in general, in theoretical economics. This does not make it any less rigorous. Lucas' principle, as he stated it at the time, was based on the false premise that our assumptions on individuals are, in some sense, scientific. In fact, those assumptions have no special status. They come from the introspection of economists and not from the long and careful examination of how individuals actually behave. We have become familiar with them and this has made them acceptable. But there is no reason that we should not adopt different formal models that were originally used to explain the behaviour of systems of interacting particles or molecules.

The usual argument against this is that humans have intentions and are forward looking and cannot therefore be modelled as one would model molecules or inanimate particles. This misses the essential point, if we can describe the rules that an individual follows and the way, in which he interacts with other individuals, we can use the formal models developed elsewhere to understand what the outcomes will be. We do not need to know what the deep psychological motives for an individual's actions are. Consider the argument that individuals are forward looking, and think of the problem of forecasting. In all our models individuals map past history into forecasts of the future. Once they have a forecast of the

---

3. Those who are interested in the agent based modelling approach cannot do better than to go to Tesfatsion's website devoted to this issue, <http://www.econ.iastate.edu/tesfatsi/umulmark.htm>

future they take an action, so in the end, they just have an algorithm, which maps past history into actions. There is nothing intrinsic which prevents us from building simple agents or robots that do this. We can choose what we consider to be the appropriate level of sophistication for the mapping from the past to actions; we can also model the reactions of other agents to an individual's choice of actions. What is more we can let the agent learn about the rules that he uses and we can find out if our simple creatures can learn to be the sophisticated optimisers of economic theory.

In doing this we are not straying far from what has been recommended by our illustrious predecessors and the leaders of our profession. The first idea that I am suggesting is that we have to treat the economy as a complex system. But Herb Simon (1962) already described a complex system when explaining how he thought economic theory should develop and he said,

*"Roughly by a complex system I mean one made up of a large number of parts that interact in a non-simple way. In such systems, the whole is more than the sum of the parts, not in an ultimate metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole. In the face of complexity, an in-principle reductionist may be at the same time a pragmatic holist."*  
Herbert Simon (1962, p. 267).

The second argument that I would make is that we should dispense with the a priori assumptions about rationality and optimisation, which are so central to economic theory. But, if you think that this might be heretical consider what Robert Lucas (1988), had to say on the subject:

*"In general we view or model an individual as a collection of decision rules (rules that dictate the action to be taken in given situations) and a set of preferences used to evaluate the outcomes arising from particular situation-action combinations. These decision rules are continuously under review and revision: new decisions are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not. I use the term "adaptive" to refer to this trial-and-error process through which our modes of behaviour are determined."*

However, Lucas then goes on to argue that we can safely ignore the dynamics of this process since,

*"Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to*

*work over a range of situations and hence are no longer revised appreciably as more experience accumulates."*

In general, however, one cannot assume convergence to some equilibrium but one has to look at the dynamic evolution of the economy resulting from the interaction between agents. One is also interested in knowing how the state of the system evolves over time and not only whether it settles down to what might be thought of as some sort of equilibrium. Here I am taking a different position from Lucas and arguing that one cannot assume that all the adaptation has taken place in the past but that we are faced, in economics, with many situations in which individuals are constantly adapting to change and thereby generating change. Thus, not only the relevant time scale but also the process itself is very different from that relevant for biological evolution, which is too often used by simple analogy. Indeed when explaining the difference between standard and computational or agent based economic models, Farmer and Foley (2010) explain,

*"An agent-based model is a computerized simulation of a number of decision-makers (agents) and institutions, which interact through prescribed rules. The agents can be as diverse as needed — from consumers to policy-makers and Wall Street professionals — and the institutional structure can include everything from banks to the government. Such models do not rely on the assumption that the economy will move towards a predetermined equilibrium state, as other models do. Instead, at any given time, each agent acts according to its current situation, the state of the world around it and the rules governing its behaviour."* Farmer and Foley (2010, p.685) .

In fact, the economy is a noisy system that may not show any tendency to settle to a steady state. Thus the argument that individuals learn to achieve equilibria does not take into account the fact that the environment about which they are learning is largely composed of other agents who are also learning. This undermines the basic arguments made by Evans and Honkapohja, (2001) who argue that learning in macroeconomics leads to equilibrium states.

If we take the view that most of the dynamics of the economy is due to the interaction between the heterogeneous agents in the economy, this means taking a very different view of business cycles and crises. Rather than putting these down to some exogenous, (technological) shock, the shocks would be idiosyncratic ones which affect individuals or firms and are transmitted to others. The system

is not occasionally knocked off course and then returns to its steady state path but internally generates movements and from time to time, phase changes. As Wilhem Buiter, a former member of the Bank of England's Monetary Policy Committee and now chief economist of Citigroup, says,

*"Those of us who worry about endogenous uncertainty arising from the interactions of boundedly rational market participants cannot but scratch our heads at the insistence of the mainline models that all uncertainty is exogenous and additive."* Buiter (2009).

### 3. Business cycles and crises

What is necessary then is to build models in which much of the volatility of macroeconomic variables is accounted for by microeconomic shocks. A step in this direction has been made by Gabaix (2011) who suggests that simply making assumptions about the heterogeneity of firms more consistent with the observed data, can make a major difference. He points out, in a recent paper, that if one accepts, consistently with the empirical evidence, that the distribution of firm sizes in the U.S. follows a power law, then the idiosyncratic movements of the largest 100 firms appear to explain about one-third of variations in output growth. The underlying assumption about firm sizes, that he refers to as the "granular" hypothesis, suggests that a number of macroeconomic questions can be clarified by looking at the behaviour of large firms. He actually details the history of a number of major incidents at the firm level that were translated into movements of aggregate output. The underlying idea is not new and in the past several economists have proposed mechanisms that generate macroeconomic shocks from purely microeconomic causes. A pioneering paper is by Jovanovic (1987), whose models generate non-vanishing aggregate fluctuations owing to a multiplier proportional to  $\sqrt{N}$ , the square root of the number of firms in the economy. However, Jovanovic's results have been criticised as empirically implausible. Yet a number of economists followed the route he mapped out. For example Scheinkman, and Woodford (1994) applied the physical theory of self-organizing criticality to explain aggregate fluctuations. Their approach however, generates fluctuations which are more fat-tailed than in reality, with infinite means. Again Nirei (2006) proposed a model where aggregate fluctuations arise from the (s,S) rules deve-

loped by Scarf and others at the firm level. Each of these examples is close, in spirit, to the complex systems approach that I am recommending here. Still we have a large residual to explain in terms of the overall size of aggregate shocks. This leads naturally to the second point, that individual shocks may be transmitted to other individuals producing contagion effects, and thus relatively small shocks may be amplified, through the network of linkages between firms. This suggests that we have to pay much more attention to the network structure of the economy than is typically done.

#### 4. Networks

As I have said, it is remarkable that Lucas observed that we had to develop hypotheses about the organisation of the interaction between agents in direct contradiction with the principle that he invoked when he argued that we should not add any assumptions to those we make about the characteristics of individuals. Nevertheless, it is directly in line with the basic argument of this paper, which is that we have to model the economy as a system in which rather simple individuals, organisations or enterprises interact with each other. The complicated outcomes that we observe at the aggregate level are not due to the complicated nature of the individuals but rather to the structure of the system within which they interact. However, once one argues that it is the interaction between agents that is primordial in determining the aggregate outcomes then one has also to be much more specific about the way in which those interactions are structured.

In particular the network of relationships between individuals, banks and even nations are of primary importance. They do not figure in macroeconomic models but have been extensively studied by economists (see e.g. Jackson (2008), Goyal (2007), and Vega Redondo (2007).)

This means modelling the network of links between the individuals and institutions, specifying the nature of the nodes and links and, in addition, establishing criteria for determining their robustness. Here, however, we find an interesting problem with the economist's approach to networks. Economists wish to develop a very general theory and, in particular, one which is based on individual maximising behaviour. As Goyal (2007) says,

*“The distinctiveness of the economics approach to networks lies in the different methodology used. These differences can be traced to a substantive methodological premise in economics: social and economic phenomena must be explained in terms of the choices made by rational agents.”*

In fact if, as I have claimed, and others do, (see Haldane and May (2011), that we have to view networks as an integral part of the vision of the economy as a complex adaptive system, what we need is a very different basis for our models. Again Lucas' recommendation that we interest ourselves in the structure of interactions, reveals the difficulties in dealing with phenomena such as networks and trying to stick to the conventional macroeconomic approach. Firstly, it underlines the economist's quest for a very general abstract model which will encompass many of the empirical phenomena that we observe and, secondly, the need that theoretical macroeconomists feel to base the model on the same micro-foundations that have been shown to be inadequate as a basis for a general, empirically verifiable model.

A different approach is needed. For example if we now couple the role of networks with the explanations of the origins of aggregate shocks, proposed by Gabaix (2011) we can move a long way to explaining a large part of the volatility of macroeconomic variables and the onset of crises. As Gabaix says,

*“ It would be interesting to exploit the hypothesis that the financial crisis was largely caused by the (ex post) mistakes of a few large firms, e.g., Lehman and AIG. Their large leverage and interconnectedness amplified into a full-fledged crisis, instead of what could have been a run-of-the-mill (sic) that would have affected in a diffuse way the financial sector.” Gabaix (2011, p.764).*

Thus what we need to do is to integrate considerations of the interconnectedness of the network into explaining macroeconomic evolution. This has been the subject of a considerable theoretical and empirical literature. Durlauf (1993) generated macroeconomic uncertainty with idiosyncratic shocks and local interactions between firms. His results are driven by the nonlinear interactions between firms. This sort of result based on diffusion across a network, coupled with the « granularity » of firm size advanced by Gabaix (2011) is a powerful basis for examining the dynamics of the macro-economy. The argument would then be that the skewed size distribution of firms together with their strong inter-linkage under-

lies much of macroeconomic fluctuations. The interdependence may be due to the input output structure of the economy but might also be linked to the ownership structure of the large firms in an economy. If we are interested in global fluctuations then we need, for the latter explanation, a detailed analysis of the ownership structure of the largest international firms, or Trans-national corporations (TNCs) as they are sometimes called. Vitali *et al.* (2011). use a large data base on the share holdings of firms to establish the nature of the network of international ownership. Their findings, which have provoked considerable discussion, are remarkable. As the authors explain

*“Nearly 40 percent of the control over the economic value of TNCs in the world is held, via a complicated web of ownership relations, by a group of 147 TNCs in the core, which has almost full control over itself.”* Vitali *et al.* (2011).

Unsurprisingly, three-quarters of these companies are banks.

## 5. The emergence of the networks structure

An important observation is that there is no evidence that this structure was intentional, there was no giant conspiracy, rather the network evolved endogenously in this way. The network is an emergent phenomenon, characteristic of complex systems. However, concentrated power in the hands of a few has clear implications for global financial stability as recent events have shown. What is worth observing is that, starting from the empirical evidence, the authors were able to build a picture of the structure of the network and then, to emphasise the implications of that structure for the stability of the network. Building on this approach could potentially help policymakers and economists to find ways to stabilize financial markets.

The important thing to notice here is that two statistical characteristics of the network of the international network of firms allow one to draw conclusions as to the evolution and fragility of the international economy. This does not depend on finding micro-founded explanations of those statistics although a number of efforts have been made to do This is at the opposite end of the spectrum to the idea that fluctuations at the macroeconomic level are essentially due to large unspecified (technological?) shocks to the economy as a whole.

Why do I describe this as belonging to the realm of “agent based” or computational modelling? What we are looking at here is a system of different interacting agents, and one where the differences between the agents can be measured. Starting from the distribution of the sizes of the firms in the particular system in question, one already obtains a measure of the aggregate volatility induced by idiosyncratic shocks. If one then couples this with the network structure of the firms, one can then model the aggregate impact of individual shocks. But both the size distribution of firms and the nature of the network are initially inferred from the data, so this is an inductive rather than a deductive approach. The robustness of the conclusions to the specification of the distribution and of the network structure can be examined by simulating a model in which the distribution of firm sizes is varied and where the network structure can be modified.

To take another example, the properties of the international financial network have been examined by Haldane, the director of the Bank of England responsible for financial stability (Haldane (2009)). In this case, the nodes correspond to countries and the size of the nodes to the total amount of foreign assets held by the country corresponding to the node in question. A link between countries means that at least one of the two holds the assets of the other and these links are weighted by the sum of the mutually held assets. Typically one would define a minimum threshold for such assets to constitute the basis for a link. One can calculate a number of statistics to characterise the structure of the graph, the empirical degree distribution for example, and the proportion of the total weight of all the links made up by the total of the weights associated with the links emanating from the largest nodes. Whilst the connectivity of the global financial network has increased remarkably in recent years (see Nier et al. 2007), the degree distribution<sup>3</sup> has changed and has become more skewed with a few nodes having very high degree and a group of nodes becoming very central. To quote Haldane (2009) of the Bank of England, when talking about these developments in the banking network before the global financial crisis, he says:

*“This evolution in the topology of the network meant that sharp discontinuities in the financial system were an accident waiting to happen. The present crisis is the materialisation of that accident.”*  
Haldane (2009, p. 4).

Again note that the reasoning is inductive. Start with an empirical phenomenon, establish its features and then examine the consequences for the evolution of the system of those features. Here what one is looking at is a network that emerged from a particular evolution of trading relationships which were mutually advantageous. What we see is how it can become fragile, without those who participate in it realizing what is going on. The importance of this for economists is clear. Interaction and the networks through which it operates have to be analysed since they play a large role in determining aggregate economic phenomena. Furthermore, understanding the evolution of the structure of the networks that make up the economy is not just an intellectual exercise; it is important for very practical reasons and policy makers are coming to appreciate this. For as Haldane says,

*“Deregulation swept away banking segregation and, with it, decomposability of the financial network. The upshot was a predictable lack of network robustness. That is one reason why Glass-Steagall is now back on the international policy agenda. It may be the wrong or too narrow an answer. But it asks the right question: can network structure be altered to improve network robustness? Answering that question is a mighty task for the current generation of policymakers. Using network resilience as a metric for success would help ensure it was a productive one.”* Haldane (2009).

When models that would address these questions are proposed, they are often described as following an engineering methodology rather than a scientific one. Rather than demanding the total internal consistency which characterises current economic models such a methodology would use a much broader and looser set of assumptions that would blend economic and non-economic considerations. In this view, all aspects of the problem necessary to arrive at an answer to the economic problem posed would be included in the applied economist's research. Thus, for example, if judgments about tradeoffs of individual's welfare were necessary, the economic engineer would develop as his objective a method for making those judgments as possible. Again this sort of approach is not new. Earlier economists, who took a more engineering approach, were quite willing to develop models involving interpersonal welfare comparisons as Colander (2007) points out. He gives the example, of Irving Fisher (1927) and Ragnar Frisch (1932) who developed a statistical method for making interpersonal comparisons of wants; they justi-

fied their models by pragmatism. Fisher posed the rhetorical question about whether the necessary assumptions can be used, and answered: “To all these questions I would answer ‘yes’—approximately at least. But the only, or only important, reason I can give for this answer is that, in actual practical human life, we do proceed on just such assumptions.” He continues: “Philosophical doubt is right and proper, but the problems of life cannot, and do not, wait.” (Fisher, 1927). As Colander says, maximizing a non-operational social welfare function is not a policy goal of engineering research.

Whilst many would accept to accept the idea that an engineering approach may be useful in providing solutions to, or understanding of, very specific economic problems they would ask that it should also allow us to better understand more general economic phenomena. A first example of an area where this approach may be particularly useful is that of the market.

## 6. Markets and their organisation

Of all economic institutions the market is probably the most ancient and the most historically documented. If any feature of the economy is emphasised in analysing economic phenomena it is surely the market. Indeed, as Braudel observed,

*“Ce très vieux type d’échange se pratiquait déjà à Pompei, à Ostie ou à Tingad la Romaine, et des siècles, des millénaires plus tôt : la Grèce ancienne a eu ses marchés; des marchés existent dans la Chine classique comme dans l’Égypte pharaonique, dans la Babylonie où l’échange était si précoce... En Ethiopie, les marchés par leurs origines se perdent dans le temps.”* Braudel (1979).

Yet as Douglas North remarked,

*“It is a peculiar fact that the literature on economics...contains so little discussion of the central institution that underlies neoclassical economics—the market.”* North (1977, p.710).

One has only to think of the standard vision, in economic theory, of a market to see why there is such a gulf between what Braudel is describing and modern economic theory. What is described is a system in which the actors act according to a system of rules, which constrains them, and this, in turn, generates the aggregate economic outcomes. These actors are anonymous and their relations with others are not considered. Financial markets are often analysed on the basis of such a vision. Yet, those who participate in,

who regulate or study actual market mechanisms have a very different view. For example Aboulafia argues that markets are essentially social institutions in his well-known study of financial markets, indeed he says,

*“Markets are socially constructed institutions in which the behavior of traders is suspended in a web of customs, norms, and structures of control...Traders negotiate the perpetual tension between short-term self-interest and long-term self-restraint that marks their respective communities.”* M. Aboulafia (1997).

Kuhn goes further and argues that individual relationships and trust are necessary for the functioning of markets. For him, it is clear that,

*“Markets are not self-operating, objective mechanical objects. They are, rather, a complex set of constraints, rules, rights, regulations, and laws, guiding human participants in making their multiple, various trades, purchases, and exchanges. The motivating force that generates benign market outcomes is the willingness of all to obey the guidelines and deal openly—transparently—with each other. Invisible to the naked eye are the common social bonds of trust among all, strangers and acquaintances alike. The bonds of trust are what create and sustain truly efficient, effective markets.”* Kuhn (1995).

In another context Alan Greenspan, Chairman at the time of the Federal Reserve, has remarked that,

*“It is hard to overstate the importance of reputation in a market economy. To be sure, a market economy requires a structure of formal rules—a law of contracts, bankruptcy statutes, a code of shareholder rights—to name but a few. But rules cannot substitute for character. In virtually all transactions, whether with customers or with colleagues, we rely on the word of those with whom we do business. If we could not do so, goods and services could not be exchanged efficiently. Even when followed to the letter, rules guide only a small number of the day-to-day decisions required of corporate management. The rest are governed by whatever personal code of values corporate managers bring to the table.”* Greenspan (2003).

This poses a problem for those who would like to model the way markets really function. The anonymous market poses few problems, for one just has to specify the rules which individuals follow when they are faced with the prices given by some unspecified market mechanism. However if we take the previous remarks seriously, we are faced with the idea that individuals build up relations of confidence with each other and this seems more like a subject for psychologists or, at least, for behavioural economists.

Furthermore, if we specify who interacts with whom in markets, we simply have to specify the graph in which the various buyers and sellers are linked and we return to the sort of network analysis I have already described. Moreover, if we are to explain how these links form and are sustained, the task is even more difficult. An argument that is often used is that in the large anonymous markets of today, relationships are not longer important, therefore we do not need to worry about the network linking traders together nor how it is formed. The quotes that I have given suggest that this, even today, is far from being the case. Traders in financial markets such as the Forex market use a very limited number of other traders, despite the fact that they are faced with thousands of quotes at any point in time. Again, in a recent paper, Puri *et al.* (2011), analyzed the importance of retail consumers' banking relationships for loan defaults using a dataset of over one million loans by savings banks in Germany. They find that loans of retail customers, who have a relationship with their savings bank prior to applying for a loan, default significantly less than customers with no prior relationship. Thus relationships play a very significant role.

Two remarks are worth making here. Firstly, not long ago such an analysis would have been almost impossible but, as I have remarked, the abundance of data together with the progress in computational capacity now allows us to undertake such exercises. Secondly, the same advances now permit us to build models in which individuals learn with whom they wish to interact and within which one can study the consequences of such interactions. In this way, artificial markets can contribute to the quest for the explanation of some of the features of the complex market structures that we observe. Conventional macroeconomic models are not concerned with the details of how markets function, but a better knowledge of market microstructure may be very useful in explaining some of the evolution of financial markets, for example. However, what we are talking about here is the emergence of certain phenomena and the dynamic evolution of the structure of the relations within markets, where, by their very nature, many transactions are on a repeated basis. Such problems do not sit well in the context of models that aim at the analysis of steady states of systems in which agents only interact through some anonymous unspecified "market".

However, artificial markets on their own are what their name suggests, artificial. The three traditional approaches for economists are, theoretical, experimental and empirical. The idea here is that the fourth approach that I am recommending, that of using agent based models to construct artificial markets which are then simulated, can complement the other approaches each of which has its weaknesses.

What are the drawbacks of theoretical models? The first and most important weakness is that they have to be greatly simplified in order to make them analytically tractable. Miller and Page (2007) in describing the purification of economic models, cite a Chinese philosopher, who says,

*“Water which is too pure has no fish.” Ts'ai Ken Tan.*

That is, in reducing our models to a minimum to be able to solve them analytically, for example to characterise their equilibria we may actually purify away the phenomena we are interested in. The second weakness is that the assumptions are often made for analytic tractability rather than for economic realism. Artificial markets can help here by providing results in more general analytically intractable situations and then seeing if these results coincide with those obtained in the simpler case which could be handled analytically.

A second approach that has developed rapidly in recent years is that of experimental economics. Leaving aside the fact that macroeconomics may not be the most suitable subject for laboratory experiments,<sup>4</sup> one could ask, in general, what are the limitations of experiments? Once again, this time, in order to make the situation understandable for the subjects, one has to simplify. Furthermore, the situation with which the subjects are faced is extremely unnatural. Often they believe that they have a problem to solve for which there is a « right » answer, thus rather than reacting naturally, they try to outguess the experimenter. The first lesson, it seems to me, that we learn is that, even when faced with a well specified problem in a controlled environment, subjects frequently do not behave as theory would predict. Thus, my own view is that this teaches us that individuals are noisier and less consistent than we assume them to

---

4. Although it is only fair to observe that the formation of bubbles in asset markets and the formation of expectations have been the subject of a considerable number of experiments, (see e.g. Duffy (2008)).

be and that we should incorporate this into our models. Some economists are more ambitious and would like to know if, other than using simplified theoretical models, one could not develop another benchmark for the rationality against which to evaluate subjects' behaviour in experiments. Again artificial models could be used to provide such a benchmark.

Finally, why can one not content oneself with working with empirical data directly? Doing this can enable us to establish some « stylised facts » or statistical regularities but gives us no idea as to the structure that generated them. It is precisely to get an understanding of how the system functions, that we typically build a model, and then we are faced again with the choice between a theoretical model and its agent-based counterpart. Although we can usefully employ both approaches, the typical theoretical models is developed before looking at the facts, while, what I would argue is that we should use salient features of the empirical data as our benchmark, and then construct models, which reproduce some of these. This is the essence of the agent based approach which is essentially data driven and more inductive than deductive.

As a very simple example, in Weisbuch *et al.* (2000), and Kirman and Vriend (2001) we wished to explain the strong loyalty of many buyers to sellers in the Marseille fish market. We first developed an extremely primitive theoretical model in which people simply learn from their previous experience and then, in consequence, change their probability of visiting different sellers as a result of their experience. We then went on to simulate more elaborate versions of the model and were still able to reproduce the salient feature. Models of this sort that attribute very little computational ability or general reasoning capacity to individuals are capable of reproducing specific features of real markets. Since then a literature on this sort of model for these markets has developed. (see e.g. Sapio *et al.*, 2011).

As a further illustration, consider another example of a market, but this time for a financial asset, (Anand *et al.* (2010)), where we once again started with an empirical phenomenon, the collapse of the price of asset backed securities early in the current crisis. We first constructed a simple theoretical model to capture the essence of the phenomenon and then ran simulations of a more general dynamic model in which the agents act in the same way as in the theoretical

model to see if the model evolves to the states predicted by the theory.

Our goal was to model the general mechanism whereby investors, as a rule, trade securities without giving due diligence to fundamental information that is, they do not check on the toxicity of the asset. The rationale motivating investors, is simply that it is profitable to adopt this rule, because other investors have already adopted it.

The market consists of agents, who, in the case of the sub-prime crisis, we can think of as the banks who were both the issuers and the investors in these Asset Backed Securities,(ABS). Each agent decides whether or not to follow a rule, which is to purchase an ABS, relying on signals from the rating agencies, without independently evaluating the fundamental value of underlying assets. If enough other participants do so, the agent becomes convinced, not irrationally, that the ABS is highly liquid and hence easy to trade.

Let us assume that the ABS is toxic with a certain probability. By toxic I mean, for example, that the underlying asset was incorrectly graded and that the original borrower of loan has already defaulted or has a higher probability of defaulting. Agents are linked together with trading partners in a financial network. This captures the fact that the secondary market for trading ABS and other credit derivatives is not centralized but instead takes place over-the-counter with traders in one firm directly calling up other traders to sell their securities.

When an agent  $i$  receives an offer to buy a new ABS, she considers whether or not to follow the rule. The line of reasoning she pursues is to first determine the probability that, if she adopts the rule and subsequently attempts to re-sell the security, the potential buyer, agent  $j$  will refuse to buy the security. This will be because agent  $j$  does not follow the rule and, as such, may verify that the underlying asset is toxic and, hence, not liquid. Each agent now calculates the expected gain to him of following the rule given the rules chosen by the neighbours in his network and adopts the rule if the expected pay-off is higher than that obtained by not adopting it and checking.

It is not difficult to find the equilibria of this simple market, in terms of the rule being followed, and there are two, one of which is always an equilibrium, and the other which only appears above a certain critical value for the probability of default on the underlying

asset. In the first equilibrium no banks check on the underlying assets whilst in the second all banks do so. Now in order to test the stochastic stability of the two equilibria we ran simulations in which agents noisily learn (they use reinforcement learning, (see Bush and Mosteller, 1955 or for a more sophisticated version Camerer and Ho, 1999), which rule is more profitable. What transpires from the simulations, is that the system always converges to the no-checking equilibrium if the probability of default is low enough, but a small increase in that probability, can lead the market to collapse into the equilibrium in which everyone checks. Thus a small change in one of the parameters of the model can lead to catastrophic consequences at the aggregate level.

Indeed, what we did was to examine the co-evolution of the default rates on mortgages and the prices of securities backed by those mortgages. The default rates steadily increased but this was not reflected in the price of assets until they suddenly collapsed and the interbank market froze. Thus, a continuous change at one level led to a discontinuous change at the aggregate level. Whilst we could establish the existence of the equilibria of the model analytically, we had to resort to simulations to see to which equilibria the learning process converged.

This underlines an important message. As soon as we are interested in real economic phenomena we cannot avoid examining how the economy behaves out of equilibrium and the characteristics of the states through which it passes, or to which it settles. This sort of "bounded rationality" approach has received a lot of attention but is often dismissed for its lack of rigour. In reality, the analysis of the evolution of the "state" of the market in the model can be perfectly rigorous given the specific choice of rules for the agents. Yet, it is still the case that building artificial markets or economies, in which agents have simple rules of behaviour, is not widely accepted in economics. The reason seems to me simple; choosing rules of thumb for agents is regarded as *ad hoc*. However, as I have already explained, we have come to accept that the restrictions that we impose on the preferences of individuals, unlike other behavioural rules, are not *ad hoc*. Therefore, if we replace those assumptions, which, by their very nature, cannot be empirically tested, by other rules, we are subject to the criticism that we lose the rigour of "proper micro foundations."

One of the arguments of the proponents of agent based modelling is that, unlike the more standard economic models, their approach is largely innocent of theoretical pre-suppositions. In many agent based models, the individuals learn, as Lucas (1988) would have them do, which rules to use, and this is not dictated a priori. However, it should be noted that the very specification of the rules amongst which the agent chooses has an impact on the outcomes. Ideally, one would like to start, as in the “artificial life” approach, with agents who are totally ignorant (see Arthur et al. (1997)). However, this would imply that they would somehow generate a set of rules with which they would experiment. This pushes the analysis back many stages to a very fundamental level. What is done in most agent based models is to provide the agents with a set of rules and simply note that this, to some extent, conditions the outcomes of the process.

Still we are faced with the criticism that artificial markets, are not “scientific”. Let me simply repeat that choosing the best element of a well defined preference order is not necessarily a reasonable assumption when both the order and the set of alternatives are highly complicated, and that something is to be gained from simplifying our account of individuals' behaviour in complicated situations<sup>5</sup>. Whether the specification of the set of rules available to individuals is more ad hoc than the standard assumptions on preferences and technologies is a subject for legitimate debate.

The message here is rather simple. Markets are an important feature of all economies. Each market is characterised by an organisation and structure that will have an impact on the outcomes observed. In general it is difficult to capture all but the simplest feature of such markets in theoretical markets. Even in the case of the simplest markets, those for perishable goods, standard models do not seem to be well adapted to shedding light on the nature of the economic outcomes that one might expect. Curiously enough the particular example which I have mentioned, that of the Marseille fish market does exhibit rather a lot of regularity at the aggregate level. Nevertheless this is not due to individuals behaving, in isolation in a regular way as in the standard competitive model.

---

5. I have developed this sort of argument at length in Kirman (2006) where I suggest that we have gone down the wrong route in modelling demand.

The complicated organisation of this sort of model breaks any simple link between individual and aggregate behaviour. A number of the specific features of this market, such as the special trading relationships that have developed are difficult to account for in the standard framework.

Artificial or agent based markets are particularly useful in studying situations where the interaction and organisation make simple theoretical analysis too difficult. To repeat, it is not legitimate, I would argue, to dismiss these models as “*ad hoc*”. Firstly, one can develop a theoretical model in a very restricted case and then simulate the model to see if the conclusions hold up in a more general case. Secondly, one can use a simplified version of the artificial market in which the solution should be obvious to see if it functions correctly before moving on to the more general framework in which the situation is more difficult to predict. This allows us to do more than simply confirm standard theoretical results but also to detect those features which emerge from the additional structure in the artificial markets. Finally, armed with this information, one can then, return to the empirical data to check them. In this approach therefore there is a constant feedback between the data and the model construction. The data is not just used to test or validate the theoretical model, but plays an active part in its conception and construction.

## 7. Conclusion

The sort of argument made in this paper in favour of agent based models and computational models in general is often interpreted as an argument against a “scientific approach” to economics. I would argue that this is based on a false notion of what science consists of. Whereas economists have insisted, in the recent past, on a very particular approach to the development of formal economic models it is now time to explore the possibility of different but no less rigorous avenues. As mathematics moves into a more computational mode, economics cannot afford to stand aside and insist on the sort of Bourbakian axiomatics that have dominated the field in recent years. Many leading mathematicians, such as Avi Wigderson at the Institute for Advanced Study in Princeton, have argued forcefully that a computational revolution is taking place in that discipline.

Whereas computer science was regarded as marginal in the field of mathematics in the past, theoretical computer science is now considered to be an integral part of the field. The dismissal of the computational approach by many economic theorists overlooks this and, as I said at the outset, it also forgets that there has been a long and distinguished part of the evolution of economic theory that focused on an algorithmic approach to economic problems. What agent based modelling is doing, is to renew the tradition of using an algorithmic approach to model economic phenomena. Its advantages are clear since it focuses on the essentially dynamic nature of the economy and allows for the explicit introduction of heterogeneity into the models, rather than vainly trying to reduce aggregate activity to that of an individual. None of this is an argument for a less analytical approach to economics but it is an argument for entertaining the possibility of other types of analysis, without any a priori restriction on the field in which they originated. We were wedded to physics and then to mathematics in the past and it seems likely that computer science is more likely to play an increasing role in constructing economic models in the future. Developing and using such models is surely to be preferred to a situation in which theoretical models are abandoned in times of crisis and policymaking reverts to “judgement and experience” alone, to cite Trichet (2010). Agent based modelling is not just an intellectual exercise. As Farmer and Foley (2009) say, « Policy-makers can thus simulate an artificial economy under different policy scenarios and quantitatively explore their consequences. ». Although we will not be able to predict precisely the onset of the next crisis at least we may be better prepared to deal with it.

## References

- Aboulafia M., 1997. *Making Markets: Opportunism and Restraint on Wall Street*. Cambridge: Harvard University Press.
- Anand, K., A. Kirman and M. Marsili, 2010. "Epidemics of Rules, Information Aggregation Failure and Market Crashes", *European Journal of Finance*. (forthcoming).
- Aoki M., 1996. *A New Approach to Macroeconomic Modelling*. New York: Cambridge University Press.

- Arthur B. *et al.*, 1997. "Asset Pricing Under Endogenous Expectations in an Artificial Stock Market." in *The Economy as an Evolving Complex System II*, Edited by W. B. Arthur, S.N. Durlauf, and D. Lane, Addison Wesley.
- Blume L., 1993. "The Statistical Mechanics of Strategic Interaction." *Games and Economic Behaviour*, 5: 387-424.
- Bourbaki N., 1949. "Foundations of Mathematics for the Working Mathematician." *The Journal of Symbolic Logic*, vol. 14 (1).
- Braudel F., 1979. *Civilisation matérielle, économie et capitalisme. Tome II : Les Jeux de l'échange*. A. Colin Paris
- Buiter W., 2009. "The Unfortunate Uselessness of Most 'State of the Art' Academic Monetary Economics." *Financial Times*, March 3.
- Bush R.-R., & F. Mosteller, 1955, *Stochastic Models for Learning*. New York: Wiley.
- Camerer & Ho, 1999. *Experience-Weighted Attraction learning in normal-form games*. *Econometrica*, 67: 827-873.
- Colander D., 2007. *The Making of an Economist Redux*, Princeton, Princeton University Press.
- Colander D 2011. "Is the Fundamental Science of Macroeconomics Sound?," *Review of Radical Political Economics*, vol. 43(3): 302–309, September.
- Durlauf, S., 1993. "Non Ergodic Economic Growth". *Review of Economic Studies*, 60: 349-366.
- Duffy J. 2008. "Macroeconomics: A Survey of Laboratory Research". Chapter prepared for the *Handbook of Experimental Economics*, Vol. 2, J. Kagel and A.E. Roth, Eds.
- Epstein J. M., 2007. *Generative Social Science: Studies in Agent-Based Computational Modeling*. Princeton: Princeton University Press.
- Evans G.W., & Honkapohja, S., 2001. *Learning and Expectations in Macroeconomics*. Princeton and Oxford: Princeton University Press.
- Farmer J. D., & D. Foley, 2009, "The Economy Needs Agent-Based Modeling." *Nature*, 460: 685-686.
- Fisher I., 1927. "A Statistical Method for Measuring 'Marginal Utility' and Testing the Justice of a Progressive Income Tax" in *Economic Essays*. Contributed in Honor of John Bates Clark .
- Frisch, R., 1932. *New methods of measuring marginal utility*, Tübingen, Verlag von J. C. B. Mohr (Paul Siebeck), p. 142.
- Gabaix X., 2011. "The Granular Origins of Aggregate Fluctuations," *Econometrica, Econometric Society*, vol. 79(3), 733–772.
- Goyal S., 2007. *Connections: An introduction to the economics of networks*. Princeton, NJ: Princeton University Press.

- Greenspan A., 2003, "Corporate governance". Remarks at the 2003 Conference on Bank Structure and Competition, Chicago, Illinois May 8.
- Haldane A., 2009, "Rethinking the financial network". Speech delivered at the Financial Student Association, Amsterdam.
- Haldane A. and G. & M. Robert, 2011. "Systemic risk in banking ecosystems". *Nature* 469, 351-355
- Jackson M., 2008. *Social and economic networks*, Princeton, NJ: Princeton, University Press.
- Jovanovic, B., 1987. "Micro Shocks and Aggregate Risk". *Quarterly Journal of Economics*, 102: 395-409.
- Kirman A., 1992. "What or whom does the representative individual represent?". *Journal of Economic Perspectives*, 6 (2): 117-136
- , and N. Vriend, 2000, "Evolving Market Structure: A Model of Price Dispersion and Loyalty." *Journal of Economic Dynamics and Control*, 459-502.
- , 2006, "Demand Theory and General Equilibrium: From Explanation to Introspection, a Journey down the Wrong Road." Annual supplement, *History of Political Economy*.
- , 2010. *Complex Economics: Individual and Collective Rationality*, Routledge, London, 2010
- Kuhn J., 2005. "On Today's Business Ethics". EPIC, New York: Columbia University.
- Lucas R., 1980. "Methods and Problems in Business Cycle Theory". *Journal of Money Credit and Banking* 12(4): 696-715.
- Lucas R., 1988. "Adaptive Behaviour and Economic Theory." *Journal of Business*, 59: 401-426.
- Mantel R., 1974. "On the characterisation of aggregate excess demand". *Journal of Economic Theory*, 7: 348-53.
- Miller J. & S. Page, 2007. *Complex Adaptive Systems*. Princeton: Princeton University Press.
- Nirei M., 2006. "Threshold Behavior and Aggregate Critical Fluctuations". *Journal of Economic Theory*, 127: 309-322.
- Nier, E., J. Yang, T. & Yorulmazer and A. Alentorn, 2007. Network models and financial stability. *Journal of Economic Dynamics and Control*. vol. 31: 2033-2060.
- North D., 1977. "Markets and other allocation systems in history". *Journal of European Economic History*, 6: 703-16.
- Puri, Manju, J. Rocholl and S. Sascha, 2011. *On the Importance of Prior Relationships in Bank Loans to Retail Customers*. Available at SSRN: <http://ssrn.com/abstract=1572673>

- Robbins L., 1935. *An Essay on the Nature and Significance of Economic Science*. London Macmillan.
- Sapio S, A. Kirman & G. Dosi, eds, 2011. "Special Section on Fish Markets". *Journal of Economic Behavior and Organisation*, 30: 1-68.
- Scheinkman J. A. & M. Woodford, 1994. "Self-Organized Criticality and Economic Fluctuations". *The American Economic Review* Vol. 84, 2: 417-421
- Simon H, 1962. "The Architecture of Complexity". *Proceedings of the American Philosophical Society* 106: 467-482
- Solow R., 2007. "Reflections on the Survey". In David Colander, *The Making of an Economist, Redux*. Princeton: Princeton University Press.
- Sonnenschein H., 1972, "Market excess demand functions". *Econometrica*, 40: 549-63.
- Trichet J.-C., 2010. "Reflections on the nature of monetary policy non-standard measures and finance theory." Speech by President of the ECB, Opening address at the ECB Central Banking Conference Frankfurt, 18 november 2010.
- Vega-Redondo F., 2007. *Complex Social Networks*. *Econometric Society*. Monograph Series, Cambridge: Cambridge University Press.
- Vitali S., J-B. Glattfelder & S. Battiston, 2011. *The Network of Global Corporate Control*. PLoS ONE 6(10): e25995. doi:10.1371.
- Weisbuch G., A. Kirman & D. Herreiner, 2000. "Market Organisation and Trading Relationships." *Economic Journal*, 110: 411-436.